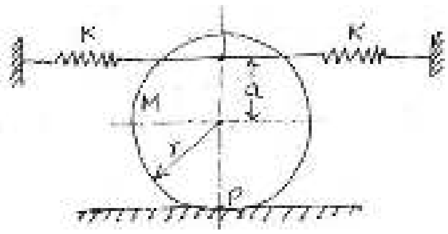
Internal Assessment Test II – Dec. 2021

Sub: Dynamics of Machines **Code:** 18ME53

Date: 20/12 / 2021 **Duration:** 90 mins **Max Marks:** 50 **Sem:** V **Branch:** MECH

Note: Answer all.

		Marks	OBE	
			CO	RBT
1	Define i) Degree of freedom ii) Simple Harmonic Motion	2	CO4	L1
2	Explain i) Types of Vibration	8	CO4	L1
3	Add the following motions analytically and check the solution graphically $x_1 = 4\cos(\omega t + 10^\circ)$ $x_2 = 6\sin(\omega t + 60^\circ)$	10	CO4	L2
4	Determine the natural frequency of the system shown in Fig. 1  Fig. 1	10	CO5	L3
5	A torsional pendulum has to have a natural frequency of 5 Hz. What length of steel wire of diameter 2 mm should be used for this pendulum? The inertia of the mass fixed at free end is 0.0098 kg - m^2. Take G = 0.85×10^{11} N/m^2	4	CO5	L2
6	A shaft carries four masses A. B. C. and D of magnitude 200kg. 300kg. 400kg and 200kg respectively and revolving at radii 80mm, 70mm. 60mm and 80mm in planes measured from A at 300mm. 400mm and 700 mm. The angle between the crank measured anticlockwise are A to B 45°. B to C 70° and C to D 120° the balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100mm between X and Y is 400mm and between Y and D is 200mm. If the balancing planes revolved at a radius of 100mm find their magnitudes and angular position.	16	CO3	L3

1. **Degree of freedom, F:** The minimum number of independent coordinates required to determine completely the positions and motions of all parts of a system at any instant of time defines the number of degrees of freedom of the system.

Simple Harmonic Motion (SHM): Any motion which repeats itself in equal intervals of time is known as periodic motion. Simple harmonic motion (SHM) is the simplest form of periodic motion. A simple harmonic motion is a reciprocating motion. The motion is periodic and its acceleration is always directed towards the mean position and is proportional to the displacement from the mean position.

1.b Types of Vibration

Vibrations in a system can be classified into three categories; free, forced and self-excited. Free vibration of a system is the vibration that occurs in the absence of any force, where damping may or may not be present.

An external force that acts on the system causes forced vibrations. Self-excited vibrations are periodic and deterministic.

1. Free and Forced Vibrations

Free Vibration: If a system, after an initial disturbance, is left to vibrate on its own, the ensuing vibration is known as free vibration. No external force acts on the system. The oscillation of a simple pendulum is an example of free vibration.

Forced Vibration: If a system is subjected to an external force (often, a repeating type of force), the resulting vibration is known as forced vibration.

<u>Machine tools, electric bells</u> etc.. are the suitable examples of forced vibration.

If the frequency of the external force coincides with one of the natural frequencies of the system, a condition known as resonance occurs, and the system undergoes dangerously large oscillations. Failures of such structures as buildings, bridges, turbines, and airplane wings have been associated with the occurrence of resonance.

2. Damped and Undamped Vibrations

If the vibratory system has a damper then there is a Reduction in amplitude over every cycle vibration since the energy of the system will be dissipated due to friction. This type of vibration is called damped vibration.

If the vibratory system has no damper, then the vibration is called undamped vibration.

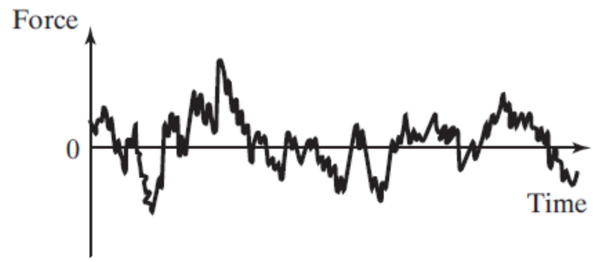
3. Linear and Nonlinear Vibration

If all the basic components of a vibratory system the spring, the mass, and the damper behave linearly, the resulting vibration is known as *linear vibration*. If, however, any of the basic components behave nonlinearly, the vibration is called *nonlinear vibration*.

4. Deterministic and Random Vibrations

If the magnitude of the excitation force or motion acting on a vibrating system is known then the excitation is known as deterministic. The resulting vibration is called the deterministic vibration If the magnitude of the excitation force or motion acting on a vibrating system is unknown, but the averages and deviations are known then the excitation is known as non-deterministic. The resulting vibration is called random vibrations.

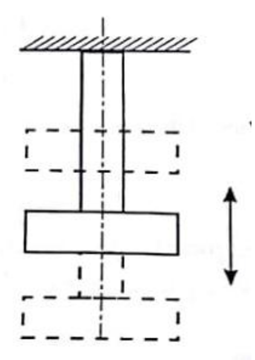
(a) A deterministic (periodic) excitation



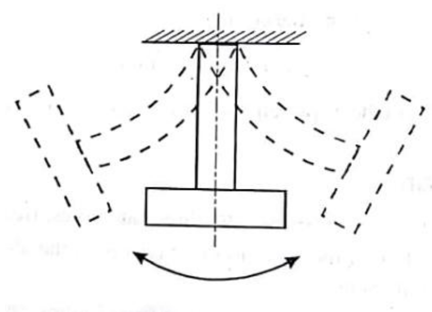
(b) A random excitation

5. Longitudinal, Transverse and Torsional Vibrations

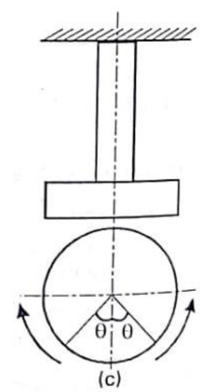
When the particles of the shaft or disc moves parallel to the axis of shaft, then the vibrations are known as longitudinal vibrations and are shown in Figure (a).



(a)



(b)



(c)

When the particles of the shaft or disc moves approximately perpendicular to the axis of the shaft, then the vibrations are known as transverse vibrations and is shown in Figure (b).

When the particles of the shaft or disc moves in a circle about the axis of the shaft i e if the shaft gets alternately twisted and untwisted on account of vibratory motion, then the vibrations are known as torsional vibrations and is shown in Figure(c).

3 $x_1 = 4\cos(\omega t + 10^\circ)$; $x_2 = 6\sin(\omega t + 60^\circ)$

Sol :

$$x = A\sin(\omega t + \theta)$$

$$x = x_1 + x_2$$

$$A\sin(\omega t + \theta) = 4\cos(\omega t + 10^\circ) + 6\sin(\omega t + 60^\circ)$$

$$A\sin\omega t \cdot \cos\theta + A\cos\omega t \cdot \sin\theta = 4\cos\omega t \cdot \cos 10^\circ - 4\sin\omega t \cdot \sin 10^\circ + 6\sin\omega t \cdot \cos 60^\circ + 6\cos\omega t \cdot \sin 60^\circ$$

$$\sin\omega t(A\cos\theta) + \cos\omega t(A\sin\theta) = \sin\omega t(-4\sin 10^\circ + 6\cos 60^\circ) + \cos\omega t(4\cos 10^\circ + 6\sin 60^\circ)$$

$$\sin\omega t(A\cos\theta) + \cos\omega t(A\sin\theta) = \sin\omega t(2.305) + \cos\omega t(9.135)$$

$$A\cos\theta = 2.305 \rightarrow (1)$$

$$A\sin\theta = 9.135 \rightarrow (2)$$

Squaring & adding $A^2\cos^2\theta + A^2\sin^2\theta = 2.305^2 + 9.135^2$

$$\boxed{A = 9.42}$$

② ÷ ①

$$\frac{A \sin\theta}{A \cos\theta} = \frac{9.135}{2.305}$$

$$\tan\theta = 3.963$$

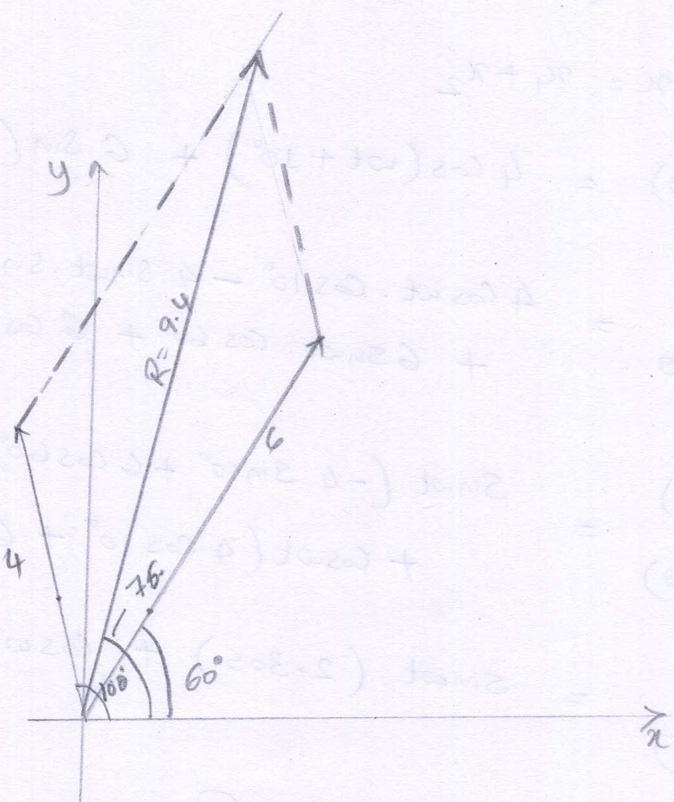
$$\boxed{\theta = 75.84^\circ}$$

$$\therefore \quad x = 9.42 \sin(\omega t + 75.84^\circ)$$

Graphical Method

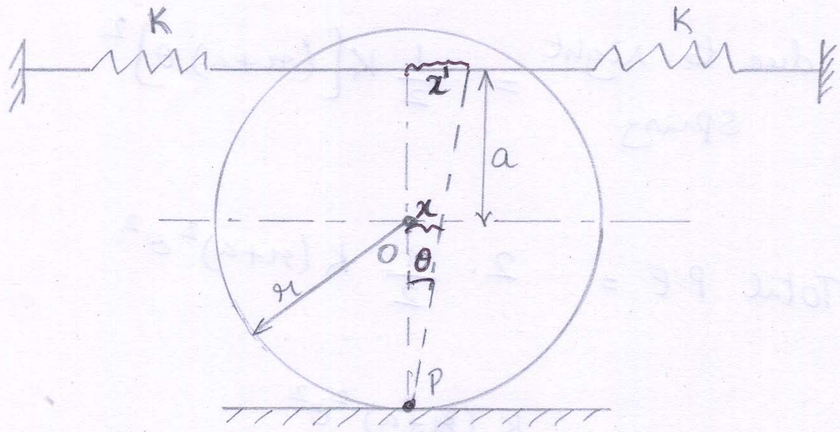
$$x_1 = 4\cos(\omega t + 10^\circ) = 4\sin(\omega t + 100^\circ)$$

$$x_2 = 6\sin(\omega t + 60^\circ)$$



$$x = 9.4 \sin(\omega t + 76^\circ)$$

3.



From fig $x = r\theta$; $x' = (r+a)\theta$

<u>Energy method</u>

$$K.E = \text{Rotational } K.E \ (KE_{Rot}) + (K.E)_{Translation}.$$

$$(K.E)_{Rot} = \frac{1}{2} I \dot{\theta}^2 = \frac{1}{2}\left[\frac{1}{2}Mr^2\right]\dot{\theta}^2$$

$$(K.E)_{Trans.} = \frac{1}{2}M\dot{x}^2 = \frac{1}{2}M(r\dot{\theta})^2 = \frac{1}{2}Mr^2\dot{\theta}^2$$

$$\therefore \text{Total } K.E = \frac{1}{2}\left[\frac{1}{2}Mr^2\right]\dot{\theta}^2 + \frac{1}{2}Mr^2\dot{\theta}^2$$

$$= \frac{1}{4}Mr^2\dot{\theta}^2 + \frac{1}{2}Mr^2\dot{\theta}^2$$

Potential Energy = P.E due to left spring + P.E due to right Spring

$$\text{P.E due to left Spring} = \frac{1}{2}K\left[(r+a)\theta\right]^2$$

P.E due to right spring $= \frac{1}{2} K\left[(r+a)\theta\right]^2$

$\therefore$ Total P.E $= 2 \cdot \frac{1}{2} K(r+a)^2\theta^2$

$= K(r+a)^2\theta^2$

According to energy method,

$$K.E + P.E = \text{Constant}$$

$$\frac{d}{dt}(K.E + P.E) = 0$$

$$\frac{d}{dt}\left\{\left[\frac{1}{4}Mr^2\dot{\theta}^2 + \frac{1}{2}Mr^2\dot{\theta}^2\right] + K(r+a)^2\theta^2\right\} = 0$$

$$\frac{1}{4}Mr^2\,2\dot{\theta}\ddot{\theta} + \frac{1}{2}Mr^2\,2\dot{\theta}\ddot{\theta} + K(r+a)^2\,2\theta\dot{\theta} = 0$$

$$\left[\frac{1}{2}Mr^2 + Mr^2\right]\ddot{\theta} + 2K(r+a)^2\theta = 0$$

$$\left[\frac{3Mr^2}{2}\right]\ddot{\theta} + 2K(r+a)^2\theta = 0$$

$$\ddot{\theta} + \frac{4K(r+a)^2}{3Mr^2}\theta = 0$$

$$\therefore \omega_n = \sqrt{\frac{4K(r+a)^2}{3Mr^2}}\ r/s.$$

Natural frequency $f_n = \frac{1}{2\pi}\cdot\omega_n = \frac{1}{2\pi}\sqrt{\frac{4K(r+a)^2}{3Mr^2}}$ Hz.//

5.a. <u>Given</u> :-

$f_n = 5\ Hz$; $d = 2mm = 0.002m$;

$I = 0.0098\ kg\text{-}m^2$

$G = 0.85 \times 10^{11}\ N/m^2$.

$l = ?$

Natural frequency of Pendulum $\omega_n = \sqrt{\frac{k_t}{I}}$

$$f_n = \frac{1}{2\pi} \cdot \omega_n$$

$$\omega_n = 2\pi \cdot f_n = 2\pi(5) = 10\pi\ r/s$$

$$\omega_n = 10\pi\ rad/sec$$

W.K.T $\frac{T}{J} = \frac{G\theta}{l} \Rightarrow k_t = \frac{T}{\theta} = \frac{GJ}{l}$

$$J = \frac{\pi d^4}{32}$$

$$= \frac{\pi (0.002)^4}{32}$$

$$J = 1.57 \times 10^{-12}\ m^4$$

$$\omega_n^2 = \frac{k_t}{I}$$

$$(10\pi)^2 = \frac{GJ}{I \cdot l}$$

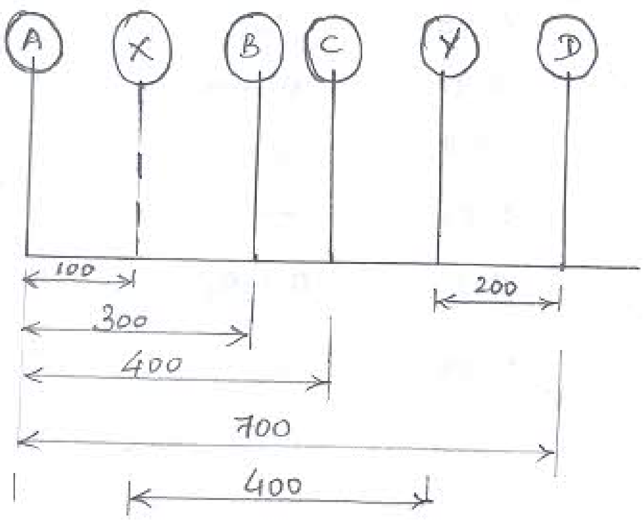
$$986.96 = \frac{0.85 \times 10^{11} \times 1.57 \times 10^{-12}}{0.0098 \times l}$$

$$\boxed{l = 0.0138\ m}$$

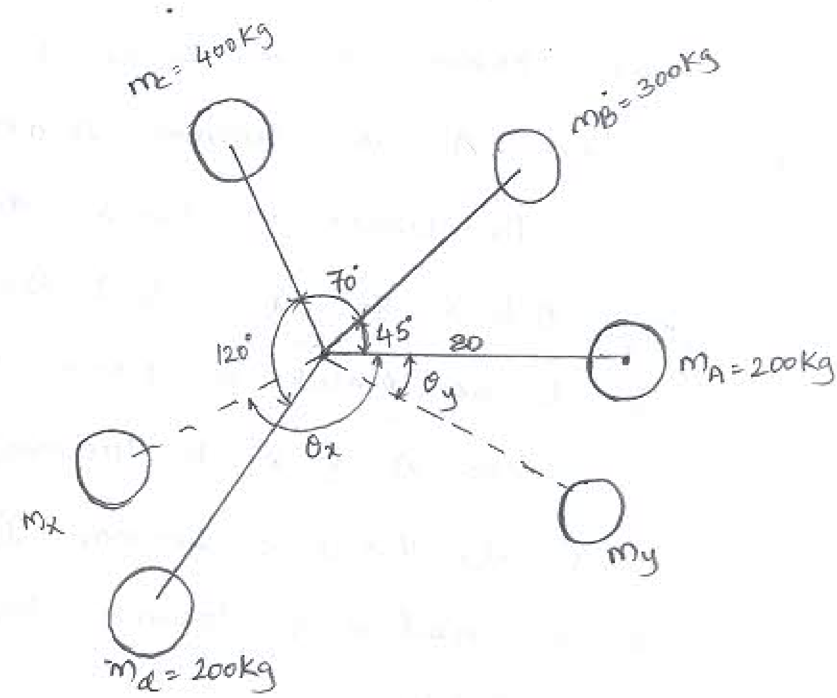
(P) A Shaft Carries four masses A, B, C & D of Magnitude 200kg, 300kg, 400kg & 200 kg respectively and revolving at radii 80 mm, 70mm, 60mm & 80mm in planes measured from A at 300mm, 400mm & 700mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° & C to D 120°. The balancing masses are to be placed in planes X & Y. The distance b/w the planes A & X is 100 mm, between X & Y is 400mm and b/w Y & D is 200mm. If the balancing masses revolve at a radius of 100mm, find their magnitudes & angular positions.

All dimensions in mm.

Sol: Position of planes



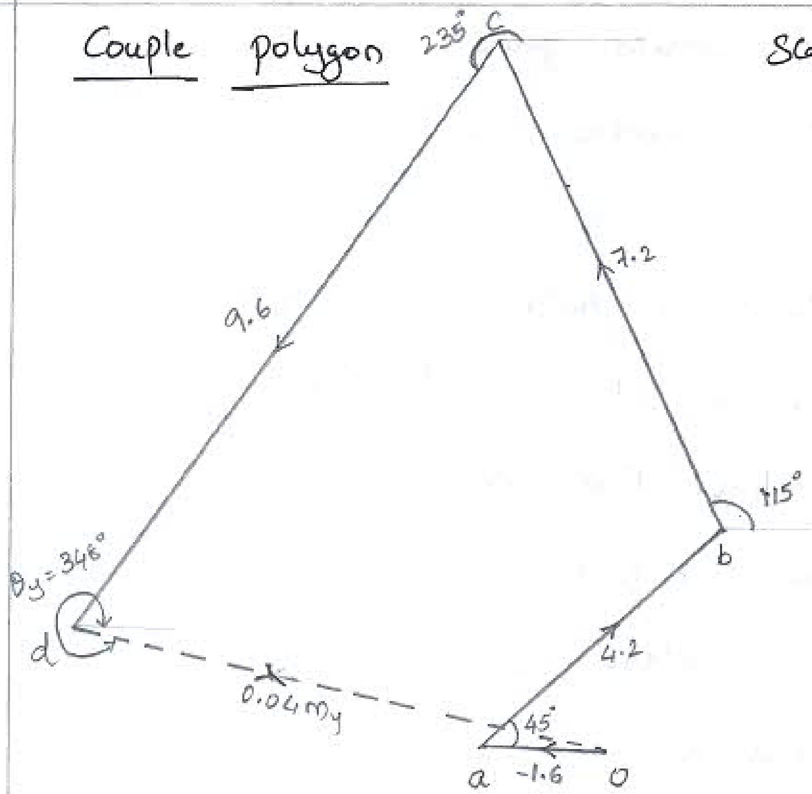
Space diagram   Scale 1cm = 20mm



Planes	Masses (m) Kg	Radius (r) m	Cent. force ÷ ω^2 (mr) Kg-m	Distance from R.P (l) m	Couple ÷ ω^2 (mrl) Kg-m^2
A	200	0.08	16	-0.1	-1.6
x	m_x	0.1	$0.1m_x$	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_y	0.1	$0.1m_y$	0.4	$0.04m_y$
D	200	0.08	16	0.6	9.6

Couple polygon

Scale 1 cm = 1 kg-m^2



$0.04 m_y = od$.

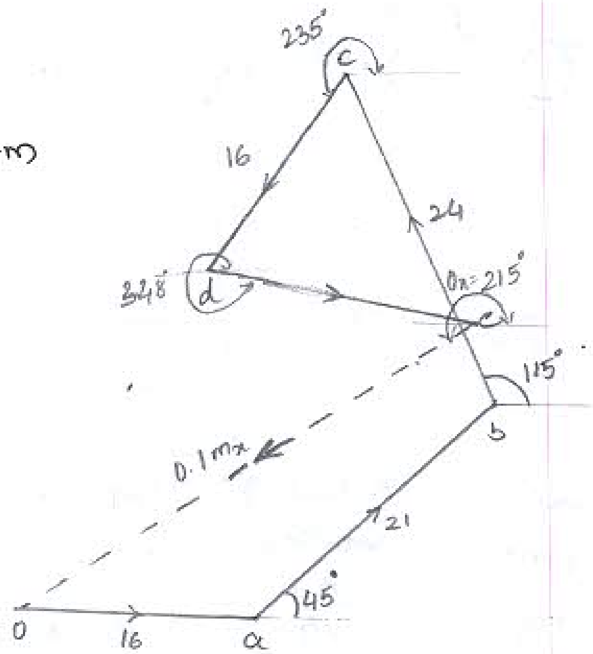
$0.04 m_y = 7.3$ kg-m^2

$m_y = 182.5$ kg.

$\theta_y = 348°$

Force polygon

Scale 2 cm = 1 kg-m



$oe = \frac{7.1}{2} = 3.55$

$0.1\ m_x = 35.5$ kg-m

$m_x = 355$ kg

$\theta_x = 215°$