

Internal Assessment Test 2 – January 2022

Sub:	Fluid Mechanics	Sub Code:	18CV33	Branch:	CV
Date:	25.01.2022	Duration:	90 mins	Max Marks:	50
		Sem/Sec:	III		OBE

Answer all questions.

Provide neat sketches wherever necessary

MARK
S C RB
 O T

1	<p>State Impulse-Momentum equation and give an application. List all the forces present in fluid motion and list the forces used for Euler's equation.</p> <p><i>Answer:</i></p> <p>It is based on law of conservation of momentum on the momentum principle which states that the net force acting on a fluid mass = change in momentum of flow / unit time. It is that the force acting on a fluid mass is given by Newton's second law of motion.</p> <p>$F = ma$ where a is same acting in the same distance as force F</p> <p>$a = \frac{dv}{dt}$</p> <p>$F = m \frac{dv}{dt}$</p> <p>$= \frac{d(mv)}{dt}$</p> <p>$F = \frac{d(mv)}{dt}$ ————— momentum principle</p> <p>$F \cdot dt = d(mv)$ ————— impulse-momentum equation.</p>	[10]	CO 4	L1
---	---	------	---------	----

Acc to Newton's second law of motion the net force F_x acting on a fluid in the direction of x is equal to mass of the fluid element multiplied by the acc in the x -dir. Thus mathematically

$\therefore F_x = \rho \cdot \Delta x \cdot \frac{dV}{dt}$ where ρ is the density of the fluid, Δx is the length of the fluid element, and $\frac{dV}{dt}$ is the acceleration in the x -direction.

- In the fluid flow, the following forces act on a fluid element:
- (i) F_g , gravity force.
 - (ii) F_p , the pressure force.
 - (iii) F_v , force due to viscosity.
 - (iv) F_t , force due to turbulence (eddies).
 - (v) F_c , force due to compressibility.

Thus, in eqn, the net force

$$F_x = (\rho \Delta x) \left[(F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x \right]$$

2 State and prove Bernoulli's equation for motion of fluid along a stream line. List the assumptions for the equation.

Bernoulli's equation states that in a steady, incompressible flow, the sum of pressure head $\left(\frac{p}{\rho g}\right)$, velocity head $\left(\frac{v^2}{2g}\right)$ and datum head (z) is a constant.

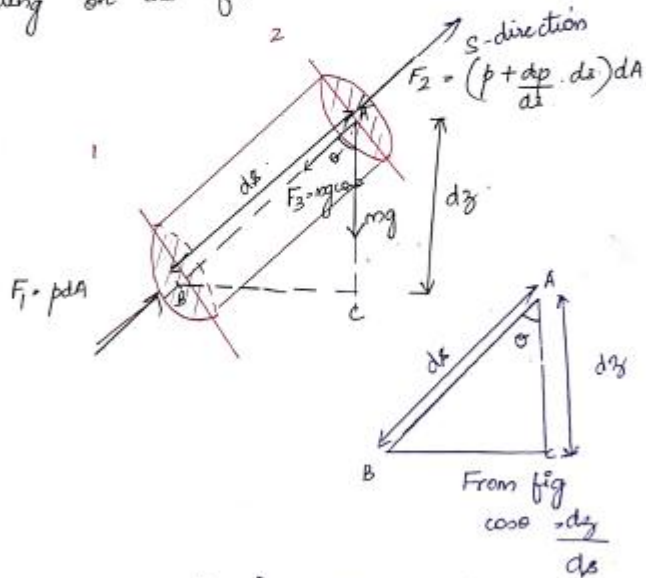
[10]

CO
4

L3

~~1.~~
Bernoulli's Equation and Bernoulli's Equation from Euler's Equation

Consider a stream line in which flow is taking place in s-direction as shown in figure. Consider a cylindrical element of cross section dA and length ds . The forces acting on the fluid element are:



1) Force due to pressure at section -1

$$F_1 = p dA \quad (1)$$

2) Force due to pressure at section -2

$$F_2 = \left(p + \frac{dp}{ds} \cdot ds \right) dA \quad (2)$$

3) Weight of the fluid element along s-direction

$$F_3 = mg \cos \theta$$

$$m = \text{mass} = \rho \times \text{volume}$$

$$= \rho \times dA \times ds \quad \text{--- (3)}$$

$$\cos \theta = \frac{dz}{ds} \quad F_3 = \rho dA ds g \frac{dz}{ds} \quad \text{--- (4)}$$

Using equation of motion along s-direction

$$\sum F_s = ma_s$$

$$F_1 - F_2 - F_3 = ma_s \quad \text{--- (5)}$$

where a_s = acc due to gravity along s-direction

$$a_s = \frac{dv}{dt} \quad \text{But } v \text{ is a function of } s \text{ and } t$$

$$a_s = \frac{dv}{dt} + \frac{dv}{ds} \cdot \frac{ds}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} + \frac{dv}{dt} = 0$$

Assuming steady flow

$$\frac{dv}{dt} = 0, \quad \frac{dv}{ds} = \frac{dv}{ds} \quad (\text{dependent only on distance})$$

$$\frac{ds}{dt} = v$$

$$a_s = \frac{dv}{ds} \cdot v \quad \text{--- (6)}$$

Substituting (1), (2), (3), (4) and (6) in (5);
we have

$$p dA - \left(p + \frac{dp}{ds} ds\right) dA - \rho dA ds g \frac{dz}{ds} - \rho dA ds \frac{dv}{ds} \cdot v$$

$$\cancel{p dA} - \cancel{p dA} - \frac{dp}{ds} ds dA - \rho dA ds g \frac{dz}{ds} - \rho dA ds \frac{dv}{ds} \cdot v$$

Taking all terms on one side,

$$\frac{dp}{ds} ds dA + \rho dA ds g \frac{dz}{ds} + \rho dA ds \frac{dv}{ds} \cdot v = 0$$

dividing by ds throughout, we have

$$dp ds dA + \rho dA ds g \frac{dz}{ds} + \rho dA ds dv = 0$$

dividing by $\rho g = \rho dA ds g$ throughout, we have

$$\frac{dp}{\rho g} + \frac{dz}{ds} + \frac{v dv}{g} = 0$$

This is Euler's Equation

Integrating the terms, we have

$$\int \frac{dp}{\rho g} + \int dz + \int \frac{v dv}{g} = 0$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant} \rightarrow \text{Bernoulli's equation}$$

Note
 $\frac{v^2}{2g}$ = velocity head
 kinetic head
 z = potential head
 datum head
 Head = $\frac{\text{Energy}}{\text{Weight}}$

Assumptions (for derivation of Bernoulli's eqn)

- 1) The liquid is ideal & incompressible
- 2) the flow is steady & continuous
- 3) The flow is along a streamline; it is one-dimensional
- 4) vel is uniform over a sectⁿ & is equal to the mean vel.
- 5) the only forces acting on the fluid are gravity forces & pressure forces.

3

a) What is static and stagnation pressure in a pitot tube.

At a stagnation point the fluid velocity is zero. In an incompressible flow, stagnation pressure is equal to the sum of the free-stream static pressure and the free-stream dynamic pressure. Stagnation pressure is sometimes referred to as pitot pressure because it is measured using a pitot tube.

b) Water flowing through a pipe having diameter 30cm and 15cm at the bottom and upper end respectively. The intensity of pressure at the bottom is 29.43N/cm² and the

[10]

CO
4

L3

pressure at the upper end is 14.715N/cm^2 . Determine the difference datum head if the rate of flow through the pipe is 50l/s .

a) Bottom (1) Upper (2)
 $d_1 = 30\text{cm} = 0.3\text{m}$ $d_2 = 15\text{cm} = 0.15\text{m}$
 $p_1 = 29.43\text{N/cm}^2$ $p_2 = 14.715\text{N/cm}^2$
 $= 29.43 \times 10^4 \text{N/m}^2$ $= 14.715 \times 10^4 \text{N/m}^2$
 $Q = 50\text{l/s} = 50 \times 10^{-3} \text{m}^3/\text{s}$
 $\rho = 1000 \text{kg/m}^3$
 $V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi}{4} d_1^2}$ $V_2 = \frac{Q}{A_2} = \frac{Q}{\frac{\pi}{4} d_2^2}$
 $= \frac{50 \times 10^{-3}}{\frac{\pi}{4} \times 0.3^2}$ $V_2 = \frac{50 \times 10^{-3}}{\frac{\pi}{4} \times 0.15^2}$
 $V_1 = 0.707 \text{m/s}$ $V_2 = 2.828 \text{m/s}$
 Applying Bernoulli application at section (1) & (2)
 $\frac{p}{\rho g} + \frac{V^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$
 $\frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{0.707^2}{2 \times 9.81} + Z_1 = \frac{14.715 \times 10^4}{1000 \times 9.81} + \frac{2.828^2}{2 \times 9.81} + Z_2$
 $30 + 0.0254 + Z_1 = 15 + 0.407 + Z_2$
 $Z_2 - Z_1 = 14.618 \text{m}$ Difference = 14.618m

4 a) Write the equation of force along x and y direction for a pipe bend and explain the terms in the equation.

$$F_x = p_1 A_1 - p_2 A_2 \cos \theta - \rho * Q * (v_2 \cos \theta - v_1)$$

$$F_y = -p_2 A_2 \sin \theta - \rho * Q v_2 \sin \theta$$

$$F_r = \sqrt{F_x^2 + F_y^2}$$

$$\tan \phi = \frac{F_y}{F_x} \text{ wrt to x axis}$$

b) 250l/s of water is flowing in a pipe having diameter 300mm . If the pipe is bent by 135° , find the magnitude and direction of the resultant force on the bend, when the pressure of water flowing is 39.24N/cm^2 .

[10] CO4 L3

4b) $Q = 250 \text{ l/s} = 0.25 \text{ m}^3/\text{s}$
 $d_1 = d_2 = d = 200 \text{ mm} = 0.2 \text{ m}$, $A_1 = A_2 = A = \frac{\pi}{4} \times 0.2^2 = 0.0707 \text{ m}^2$
 $p_1 = p_2 = P = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$
 $\theta = 135^\circ$, $v_1 = v_2 = v = \frac{Q}{A} = \frac{0.25}{0.0707} = 3.536 \text{ m/s}$
 $F_x = \rho g (v_1 - v_2 \cos \theta) + p_1 A_1 - p_2 A_2 \cos \theta$
 $F_y = -\rho g z_2 A \cos \theta - p_2 A_2 \sin \theta$
 $F_x = 1000 \times 0.25 \times (3.536 - 3.536 \cos 135^\circ) + 39.24 \times 10^4 \times 0.0707$
 $\quad - 39.24 \times 10^4 \times 0.0707 \times \cos 135^\circ$
 $F_x = 48868.79 \text{ N}$
 $F_y = -1000 \times 0.25 \times 3.536 \times \sin 135^\circ - 39.24 \times 10^4 \times 0.0707 \times \sin 135^\circ$
 $\quad = -20292.1195 \text{ N}$
 $F_R = \sqrt{F_x^2 + F_y^2}$
 $F_R = 52895.198 \text{ N}$
 $\tan \theta = \frac{F_y}{F_x} \Rightarrow \theta = \tan^{-1} \frac{20292.1195}{48868.79} = 22.5^\circ$

- 5 The 20cm x 10cm venturimeter is provided in a vertical pipe line carrying oil of specific gravity 0.8, the flow being upwards. The difference in elevation of the throat section and entrance section of the venturimeter is 50cm. The differential U tube mercury manometer shows a gauge deflection of 40cm. Calculate i) the discharge of oil, ii) the pressure difference between the entrance and throat section. Take $C_d = 0.98$. (Use the equations carefully. The direction of flow is an important parameter to be considered.)

[10]

CO4 L4

8

(a)

$$d_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$$A_1 = \frac{\pi d_1^2}{4} = 0.0314 \text{ m}^2$$

$$d_2 = 10 \text{ cm} = 0.1 \text{ m}$$

$$A_2 = \frac{\pi d_2^2}{4} = 0.00785 \text{ m}^2$$

$$z_1 = 0, z_2 = 80 \text{ cm} = 0.8 \text{ m}$$

$$z = 40 \text{ cm} = 0.4 \text{ m}$$



$$\left(\frac{p_1}{\rho g} + z_1 \right) - \left(\frac{p_2}{\rho g} + z_2 \right) = h - z \left(\frac{v_2}{v_1} - 1 \right)$$

$$S_{01} = 126, S_{02} = 0$$

$$h = z \left(\frac{S_{01}}{S_{02}} - 1 \right) = 0.4 \times \left(\frac{126}{0.8} - 1 \right)$$

$h = 5.4 \text{ cm}$ of oil

$$h = \left(\frac{p_1}{\rho g} + z_1 \right) - \left(\frac{p_2}{\rho g} + z_2 \right) \Rightarrow \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g} \right) + 0 = 0.5 = 5 \text{ cm}$$

Difference in pressure: $(p_1 - p_2) = (5.4 \text{ cm} + 0.5) \times 0.8 \text{ g/cm}^3 \times 9.81$

$$= 54.512 \text{ N/m}^2$$

$$Q = \frac{C_d \rho A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = 0.0091 \text{ m}^3/\text{s}$$