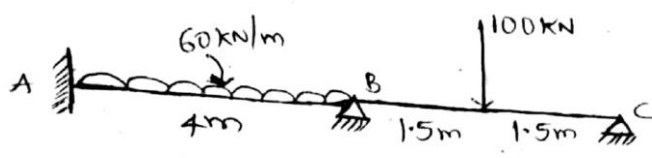
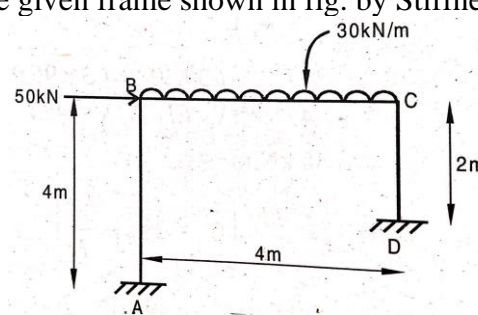


US  
N

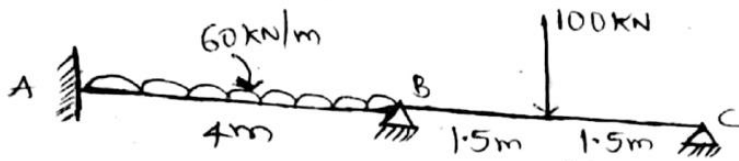
--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

Internal Assessment Test 4 –Feb. 2022

Sub:	Analysis of Indeterminate Structures					Sub Code:	18CV52	Branch:	Civil	
Date:	1/2/2022	Duration:	90 min's	Max Marks:	50	Sem / Sec:	5A		OBE	
<b>Answer TWO FULL Questions</b>								MARKS	CO	RBT
1.	<p>Analyze the Frame show in fig 1. Using Stiffness matrix method. Draw <b>BMD and SFD</b></p> 						[25]	CO1	L3	
2.	<p>Analyze the given frame shown in fig. by Stiffness matrix method.</p>  <p style="text-align: right;">fig 2</p>						[25]	CO2	L3	

Solution

1. Analyze the given continuous beam using stiffness matrix method of analysis



Steps

1. Fixed end moments

$$M_{FAB} = -80$$

$$M_{fba} = 80$$

$$M_{fbc} = -37.5$$

$$M_{fcb} = 37.5$$

2.  $[\Delta]$ ,  $[P]$ ,  $[P_L]$

- a.  $[\Delta] = \text{unknown displacement matrix} = \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}$

$$[P] = \text{moments acting in unit directions} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

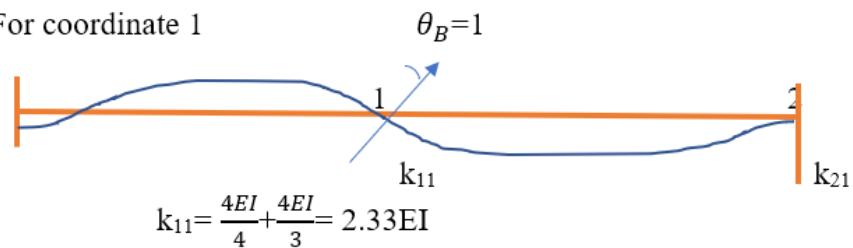
$$[P_L] = \text{joint moments} = \begin{bmatrix} M_{FBA} + M_{FBC} & \cdot \\ M_{FCB} & \cdot \end{bmatrix} = \begin{bmatrix} 80 - 37.5 & \cdot \\ 37.5 & \cdot \end{bmatrix} =$$

3. Stiffness matrix = to find the  $\theta_B, \theta_C$

- a. Applying the unit rotation along the 1 co-ordinate directions

$$[\Delta] = [k]^{-1} [P - P_L]$$

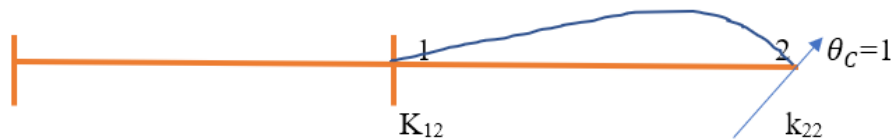
For coordinate 1



$$k_{11} = \frac{4EI}{4} + \frac{4EI}{3} = 2.33EI$$

$$k_{21} = \frac{2EI}{L} = 0.67EI$$

applying the unit rotation at co-ordinate 2



$$K_{12} = \frac{2EI}{L} = 0.67EI$$

$$K_{22} = \frac{4EI}{L} = 1.33EI$$

$$[k] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

$$[k]=EI \begin{bmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix}$$

$$[k]^{-1}=\frac{1}{EI} \begin{bmatrix} 2.33 & 0.66 \\ 0.67 & 1.33 \end{bmatrix}^{-1}$$

$$[\Delta] = [k]^{-1} [P-P_L]$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 2.33 & 0.66 \\ 0.67 & 1.33 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (80 - 37.5) \\ 0 - (37.5) \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} -11.89 \\ -22.36 \end{bmatrix}$$

$$\theta_B = -11.89/EI, \theta_C = -22.36/EI$$

4. Slope deflection equation:

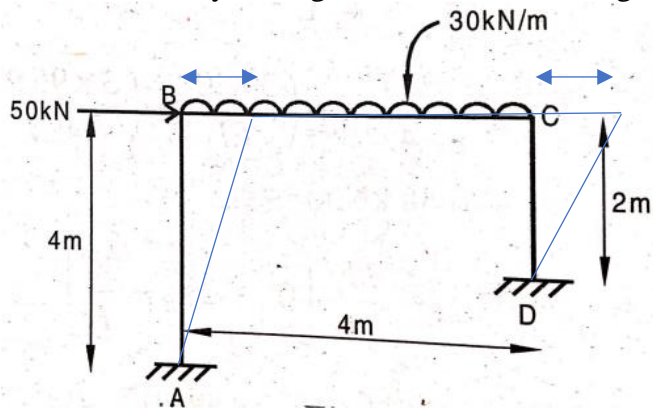
$$\begin{aligned} M_{AB} &= M_{FAB} + \frac{2EI}{l} (2\theta_A + \theta_B - \frac{3\delta}{l}) \\ &= -80 + 2EI/4(0 - 11.89/EI - 0) = -85.94 \text{ kN.M} \end{aligned}$$

$$\begin{aligned} M_{BA} &= M_{FBA} + \frac{2EI}{l} (2\theta_B + \theta_A - \frac{3\delta}{l}) \\ &= 80 + 2EI/4(2(-11.89/EI) + 0 - 0) = 68.11 \text{ kN.m} \end{aligned}$$

$$\begin{aligned} M_{BC} &= M_{FBC} + \frac{2EI}{l} (2\theta_B + \theta_C - \frac{3\delta}{l}) \\ &= -37.5 + 2EI/3(2(-11.89/EI) - 22.36/EI) = -68.16 \text{ kN.M} \end{aligned}$$

$$\begin{aligned} M_{CB} &= M_{FCB} + \frac{2EI}{l} (2\theta_C + \theta_B - \frac{3\delta}{l}) \\ &= 37.5 + 2EI/3(2(-22.36/EI) - 11.89/EI) = -0.05 \text{ kN.m} \end{aligned}$$

Problem 2: Analyze the given frame shown in fig. by Stiffness matrix method.



Unknowns = 3

1. Fixed end moment:

$$M_{fbc} = -40$$

$$M_{fcb} = 40$$

2.  $[\Delta]$ ,  $[P]$ ,  $[P_L]$

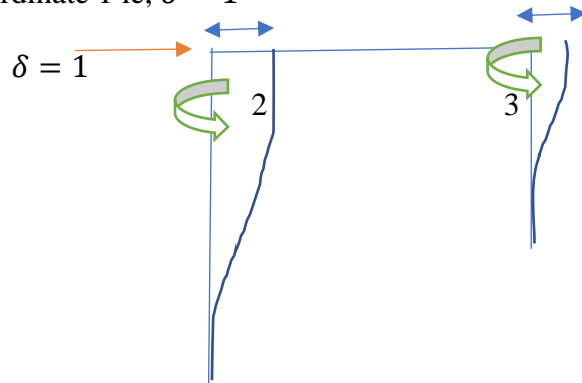
$$[\Delta] = \text{unknown displacement matrix} = \begin{bmatrix} \delta \\ \theta_B \\ \theta_C \end{bmatrix}$$

$$[P] = \text{External forces} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

$$[P_L] = \text{joint force(moments)} = \begin{bmatrix} 0 \\ M_{FBA} + M_{FBC} \\ M_{FCB} + M_{FCD} \end{bmatrix} = \begin{bmatrix} 0 \\ -40 \\ 40 \end{bmatrix}$$

3. Stiffness matrix= to find the  $\delta, \theta_B, \theta_C$

Applying the unit rotation along the co-ordinate 1 ie,  $\delta = 1$



$$k_{11} = \frac{12EI}{4^3} + \frac{12EI}{2^3} = 1.68EI$$

$$k_{21} = -\frac{6EI}{4^2} = -0.37EI$$

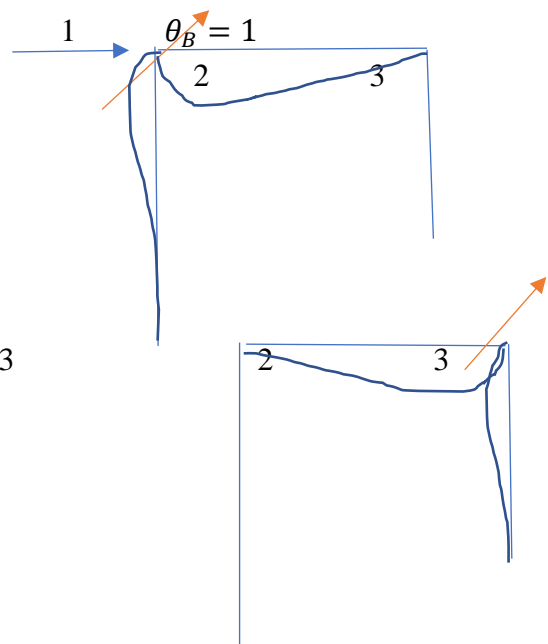
$$k_{31} = -\frac{6EI}{2^2} = -1.5EI$$

Applying the unit rotation along the co-ordinate 2, ie =

$$k_{12} = -\frac{6EI}{L^2} = -0.37EI$$

$$k_{22} = \frac{4EI}{L} + \frac{4EI}{L} = 2EI$$

$$k_{23} = \frac{2EI}{L} = 0.5EI$$



$$\theta_C = 1$$

Applying the unit rotation along the co-ordinate 3

$$K_{13} = -\frac{6EI}{L^2} = -1.5EI$$

$$K_{23} = \frac{2EI}{L} = 0.5EI$$

$$k_{33} = \frac{4EI}{L} + \frac{4EI}{L} = 3EI$$

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = [EI] \begin{bmatrix} 1.68 & -0.37 & -1.5 \\ -0.37 & 2 & 0.5 \\ -1.5 & 0.5 & 3 \end{bmatrix}$$

$$[\Delta] = [k]^{-1} [P - P_L]$$

$$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 1.68 & -0.37 & -1.5 \\ -0.37 & 2 & 0.5 \\ -1.5 & 0.5 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 50 \\ 40 \\ -40 \end{bmatrix}$$

$$\delta = 35.68/EI, \theta_B = 26.6/EI, \theta_C = 0.1/EI$$

4. Substitute the above values in the slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l} \left( 2\theta_A + \theta_B - \frac{3\delta}{l} \right)$$

$$M_{AB} = -0.3$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l} \left( \theta_A + 2\theta_B - \frac{3\delta}{l} \right)$$

$$M_{BA} = 13.33$$

For Span BC,  $l =$

$$M_{BC} = M_{FBC} + \frac{2EI}{l} \left( 2\theta_B + \theta_C - \frac{3\delta}{l} \right)$$

$$M_{BC} = -13.33$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l} \left( \theta_B + 2\theta_C - \frac{3\delta}{l} \right)$$

$$M_{CB} = 53.33$$

For Span CD,  $l =$

$$M_{CD} = M_{FCD} + \frac{2EI}{l} \left( 2\theta_C + \theta_D - \frac{3\delta}{l} \right)$$

$$M_{CD} = -53.33$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l} \left( 2\theta_D + \theta_C - \frac{3\delta}{l} \right)$$

$$M_{DC} = -53.33$$