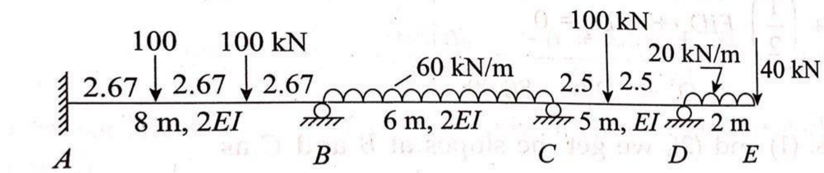
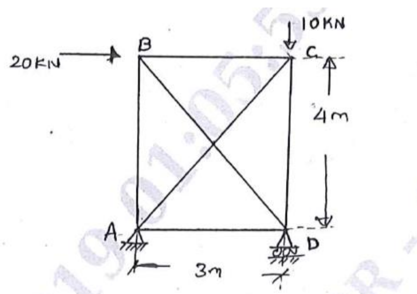
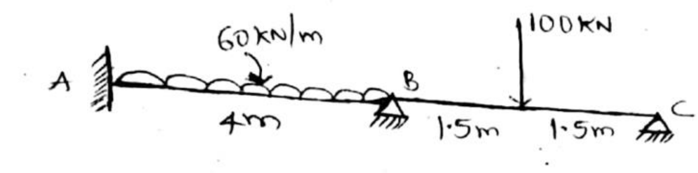
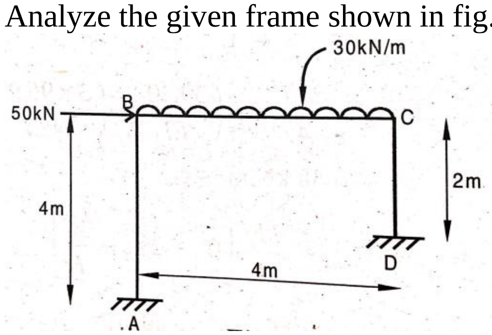
USN										

Internal Assessment Test 5 –Feb. 2022

Sub:	Analysis of Indeterminate Structures					Sub Code:	18CV52	Branch:
Date:	5/2/2022	Duration:	90 min's	Max Marks:	50	Sem / Sec:	5A	

Answer TWO FULL Questions

1.	Analyze the given continuous beam using stiffness matrix method. The support B and C sinks by 10mm and 5mm respectively. Take E=200Gpa, I= $80*10^6 mm^4$  fig 1			
2.	Analyze the truss shown in fig 2. by flexibility matrix method. Assume EA constant for all members  fig 2			
	Analyze the given continuous beam using stiffness matrix method of analysis 	[18]	CO1	L3
	Analyze the given frame shown in fig. by Stiffness matrix method. 	[25]	CO2	L3

Solutions:

1. ## Problem 1:

Fixed end moment:

Mfab= -100*2.67*$(2.67+2.67)^2$/8*8-100*(2.67+2.67)*2.67^2/8*8- 6*2*200*10^{-6}*80*10^6*10^{-6} 0.01/8*8 = -178.44kN.m

Mfba= 100*2.67^2*2.67+2.67/8*8+100*$(2.67+2.67)^2$*2.67/8*8- 6*2*200*10^{-6}*80*10^6*10^{-6} 0.01/8*8 = 178.44kN.m

Mfbc= -60*6*6/12-6*2*200*10^{-6}*80*10^6*10^{-6} (-0.005)/6*6= -180kN.m

Mfcb= 60*6*6/12-6*2*200*10^{-6}*80*10^6*10^{-6} (-0.005)/6*6=+180kN.m

Mfcd= -100*5/8-6*2*200*10^{-6}*80*10^6*10^{-6} (-0.005)/5*5= -62.4kN.m

Mfdc = 100*5/8-6*2*200*10^{-6}*80*10^6*10^{-6} (-0.005)/5*5= 62.4kN.m

Mfde= -(40*2+20*2*1)= -120kN.m

2. [Δ], [P], [P_L]

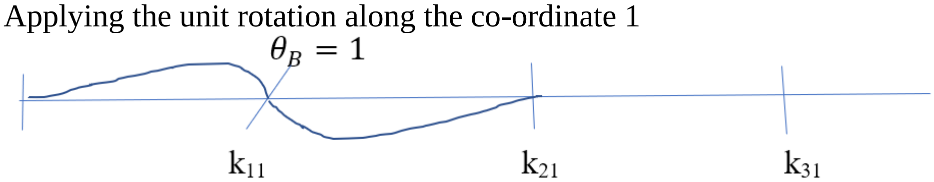
$$[\Delta] = unknown\ displacment\ matix = \begin{bmatrix}\theta_B\\ \theta_C\\ \theta_D\end{bmatrix}$$

[P]= moments acting -External $=\begin{bmatrix}0\\0\\-120\end{bmatrix}=$

[P_L]= joint moments $=\begin{bmatrix}M_{FBA}+M_{FBC}\\ M_{FCB}+M_{FCD}\\ M_{FDC}\end{bmatrix} = = \begin{bmatrix}178.44-180\\ 180-62.4\\ 62.4\end{bmatrix} = \begin{bmatrix}-1.56\\ 117.6\\ 62.4\end{bmatrix} =$

3. Stiffness matrix= to find the θ_B, θ_c

Applying the unit rotation along the co-ordinate 1



$k_{11}=\frac{4*2EI}{8}+\frac{4*2EI}{6}$= 2.33EI

$k_{21}=\frac{2*2EI}{6}$= 0.67EI

k_{31}= 0

Applying the unit rotation along the co-ordinate 2

$k_{21}=\frac{2*2EI}{6}$ =0.67EI

$k_{22}=\frac{4*2EI}{6}+\frac{4*EI}{5}$= 2.13EI

$k_{23}=\frac{2EI}{5}$= 0.4EI

Applying the unit rotation along the co-ordinate 3

k_{31}=0

$k_{32}=\frac{2*EI}{5}=0.4EI$

$k_{33}=\frac{4EI}{5}= 0.8EI$

$$\begin{bmatrix} k_{11} & k_{21} & k_{31} \\ k_{12} & k_{22} & k_{32} \\ k_{13} & k_{23} & k_{33} \end{bmatrix} = [EI]\begin{bmatrix} 2.33 & 0.67 & 0 \\ 0.67 & 2.13 & 0.4 \\ 0 & 0.4 & 0.8 \end{bmatrix}$$

$[\Delta] = [k]^{-1} [P-P_L]$

$$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix}=\frac{1}{EI}\begin{bmatrix} 2.33 & 0.67 & 0 \\ 0.67 & 2.13 & 0.4 \\ 0 & 0.4 & 0.8 \end{bmatrix}^{-1}\begin{bmatrix} 0+1.56 \\ 0+117.6 \\ -120-62.4 \end{bmatrix}$$

$\theta_B = \ , \theta_C = \ , \theta_D =$

4. Substitute the above values in the slope deflection equation

$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C - \frac{3\partial}{l})$

$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + B - \frac{3\partial}{l})$

M_{CD}= -128.47 kN.m

M_{DC} = -7.93 kN.m

Problem 2:

1. **Determine the static indeterminacy**

   Member+reaction-2*joint = DOSI= m+r-2j = 5+4-2*4= 1

2. **Selection of redundant: reaction at A is considered as redundant = V_A**

   Length of the members

   AB= 5.83m

   BC= 5m

   DB= 5.83m

   AC= 3m

   CD= 3m

3. **Computing the axial force- actual force (P): by method of joint**

Horizontal forces = 0, H_c+60= 0

H_c=-60kN

Since A is the redundant support A will be released

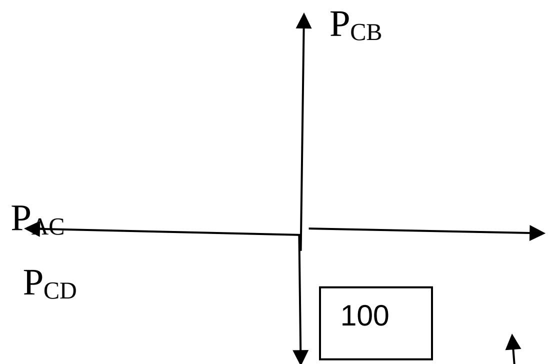
Take moment WRT A : -V_c*3-V_D*6+60*5=0 ,,,,,,,,,,,,1

Take moment WRT C: -V_D*3+60*5= 0

V_D= 60*5/3 = 100kN

From eq 1 ……… **V_c** = -100kN

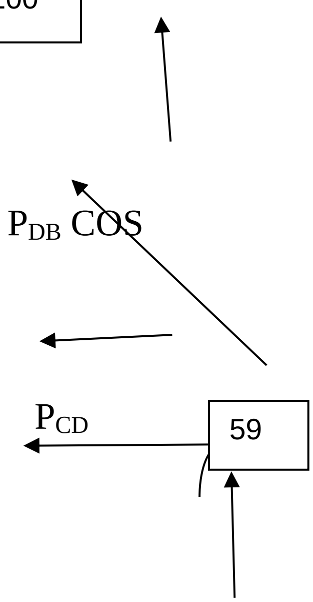
Consider joint C =



$\sum H$=0= -P_{AC}+P_{CD}-60=0

-P_{AC}=0kN

V=0 = P_{CB}=100Kn



$P_{DBsin59}$

59

Consider joint D

$\sum H$=0 = -P_{DB}COS59- P_{CD} =0

$\sum V$= 0= P_{DB}Sin59=-100

P_{DB}= -116.67 kN

-P_{DB}COS59 = P_{CD}

P_{CD}= 60.66kN

4. **Computing the axial force – Unit load application at redundant (K)**

Apply unit load at redundant I, e at V_A=1kN

Take moment wrt to D

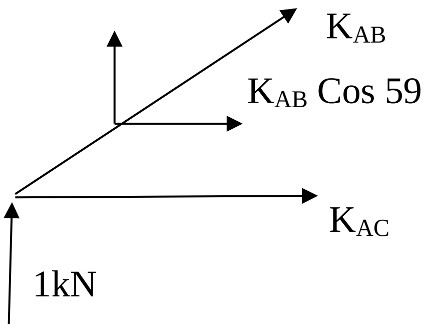
V_A* 6 + V_C*3=0

1*6+ V_C*3=0

V_C=-2kN

V_A+ V_C +V_D=0 = 1-2= 1kN

Consider the joint A =

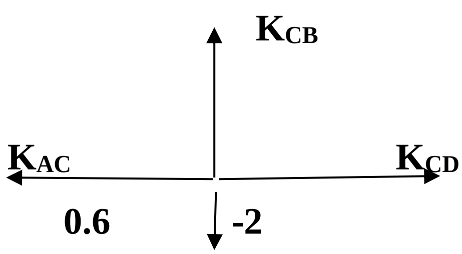


$\sum H$== 0 = K_{AC}+ K_{AB} Cos 59=0

$\sum V$=0= K_{AB} Sin 59 = -1

K_{AB}= -1.17kN

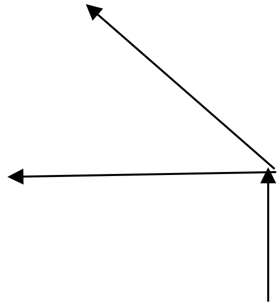
Consider joint C



$\sum H$= 0= 0.6= K_{CD}

$\sum V$=0= K_{CB}= 2

Consider joint D



$H=0= -K_{CD}- K_{DB}$ Cos 59=0
K_{DB}=-1.17kN
V= 0= 1=- K_{DB} Sin 59
K_{DB}= -1.16kN

5. Force in the member – actual load and unit load

Member	Length(L)	Area (mm^2)	E (kN/mm^2)	P (kN)	K(kN)	$\frac{PKL}{AE}$	$\frac{K^2L}{AE}$
AB	5.83	A	E	0	-1.17	0	7.98/AE
BC	5	A	E	100	2	1000/AE	20/AE
BD	5.83	A	E	-116.67	-1.17	795.81/AE	7.98/AE
AC	3	A	E	0	0.6	0	1.08/AE
CD	3	A	E	60.66	0.6	109.18/AE	1.08/AE
						1904.99/AE	38.12/AE

6. Calculation of deflection

$$\Delta_L = \sum \frac{PKL}{AE} = 1904/AE$$

7. Compatibility equation

$$F = \sum \frac{K^2L}{AE} = 38.12/AE$$

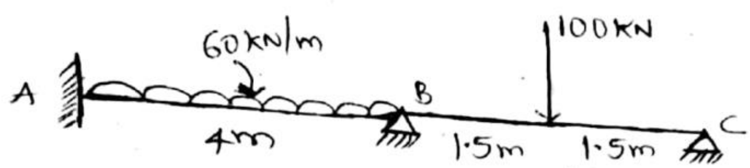
$\{\Delta - \Delta_L\}$= [F] {R}
{0-1904.99} *[38.12]$^{-1}$= {R}

$$\frac{-1904.99}{38.12} = -49.97$$

8. Final moment

Member	P (kN)	K(kN)	R(kN)	P_F=P+KR (kN)	Nature of force
AB	0	-1.17	−49.97	58.46	T
BC	100	2	−49.97	0.06	T
BD	-116.67	-1.17	−49.97	-58.20	C
AC	0	0.6	−49.97	-29.98	C
CD	60.66	0.6	−49.97	+30.67	T

Problem 3:
Analyze the given continuous beam using stiffness matrix method of analysis



Steps

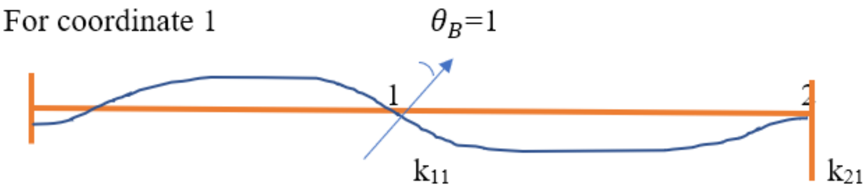
1. Fixed end moments
   M_{FAB}= -80
   Mfba= 80
   Mfbc= -37.5
   Mfcb= 37.5
2. [Δ], [P], [P_L]
   a. $[\Delta] = unknown\ displacment\ matix = \begin{bmatrix}\theta_B \\ \theta_c\end{bmatrix}$

      [P]= moments acting in unit directions =$\begin{bmatrix}0 \\ 0\end{bmatrix}$

      [P_L]= joint moments =$\begin{bmatrix}M_{FBA} + M_{FBC} & . \\ M_{FCB} & .\end{bmatrix}$= $\begin{bmatrix}80 - 37.5 & . \\ 37.5 & .\end{bmatrix}$=
3. Stiffness matrix= to find the θ_B, θ_c
   a. Applying the unit rotation along the 1 co-ordinate directions

      $[\Delta] = [k]^{-1}\ [P-P_L]$

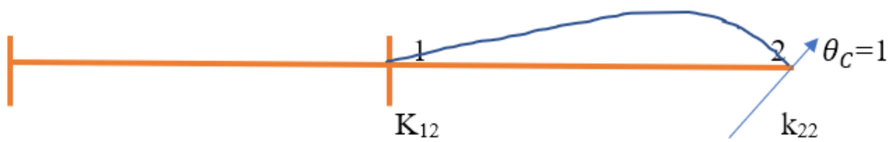
For coordinate 1



$$k_{11}= \frac{4EI}{4}+\frac{4EI}{3}= 2.33EI$$

$$k_{21}= \frac{2EI}{L}= 0.67EI$$

applying the unit rotation at co-ordinate 2



$$K_{12}= \frac{2EI}{L}= 0.67EI$$

$$K_{22}= \frac{4EI}{L}= 1.33EI$$

$$[k]= \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

$$[k]=EI \begin{bmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix}$$

$$[k]^{-1}=\frac{1}{EI}\begin{bmatrix} 2.33 & 0.66 \\ 0.67 & 1.33 \end{bmatrix}^{-1}$$

$$[\Delta] = [k]^{-1} [P-P_L]$$

$$\begin{bmatrix} \theta_B \\ \theta_c \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 2.33 & 0.66 \\ 0.67 & 1.33 \end{bmatrix}^{-1} \begin{bmatrix} 0-(80-37.5) & . \\ 0-(37.5) & . \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_c \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -11.89 & . \\ -22.36 & . \end{bmatrix}$$

θ_B= -11.89/EI, θ_C= -22.36/EI

4. Slope deflection equation:

$M_{AB} = M_{FAB}+ \frac{2EI}{l}(2\theta_A+\theta_B- \frac{3\partial}{l})$

= -80+2EI/4(0-11.89/EI-0)= -85.94KN.M

$M_{BA}= M_{FBA}+ \frac{2EI}{l}(2\theta_B+\theta_A- \frac{3\partial}{l})$

= 80+2EI/4(2/(-11.89/EI))= 68.11kN.m

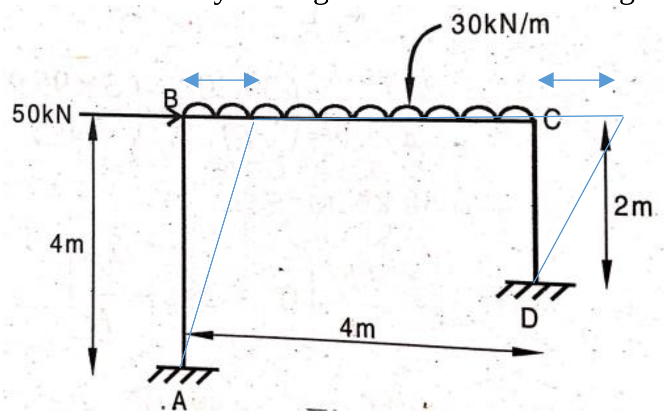
$M_{BC}= M_{FBC}+ \frac{2EI}{l}(2\theta_B+\theta_C- \frac{3\partial}{l})$

=-37.5+2EI/3(2*-11.89/EI-22.36/EI)= -68.16Kn.M

M_{CB}= M_{FCB}+ $\frac{2EI}{l}(2\theta_C+\theta_B-\frac{3\partial}{l})$

= 37.5+2EI/3(2*-22.36/EI-11.89/EI)= -0.05kN.m

Problem 4: Analyze the given frame shown in fig. by Stiffness matrix method.



Unknows = 3

1. Fixed end moment:
   Mfbc= -40
   Mfcb= 40

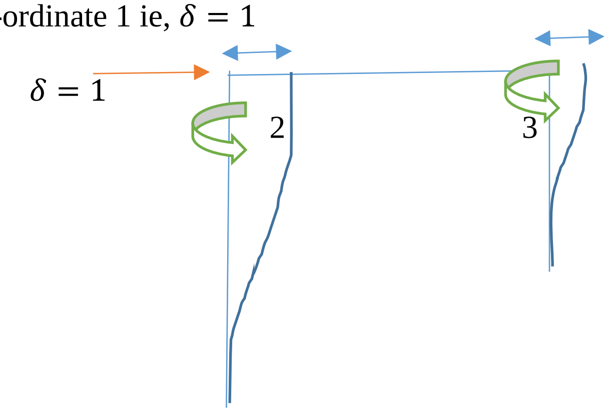
2. [Δ], [P], [P_L]

$$[\Delta] = unknown\ displacment\ matix = \begin{bmatrix}\delta\\ \theta_B\\ \theta_C\end{bmatrix}$$

$$[P]= \text{External forces} = \begin{bmatrix}50\\0\\0\end{bmatrix}$$

$$[P_L]= \text{joint force(moments)} = \begin{bmatrix}0\\ M_{FBA}+M_{FBC}\\ M_{FCB}+M_{FCD}\end{bmatrix} = = \begin{bmatrix}0\\-40\\40\end{bmatrix}$$

3. Stiffness matrix= to find the $\delta, \theta_B, \theta_c$
   Applying the unit rotation along the co-ordinate 1 ie, $\delta = 1$



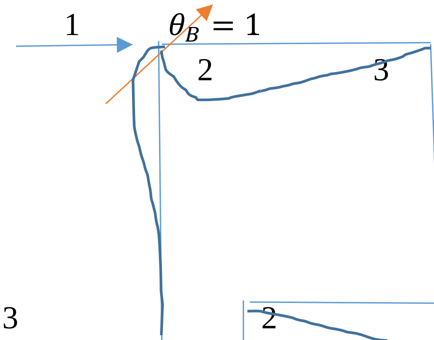
k_{11}=$\frac{12EI}{4^3}$+$\frac{12EI}{2^3}$=1.68EI

k_{21}= - $\frac{6EI}{4^2}$= -0.37EI

k_{31}= - $\frac{6EI}{2^2}$= -1.5 EI

Applying the unit rotation along the co-ordinate 2, ie =

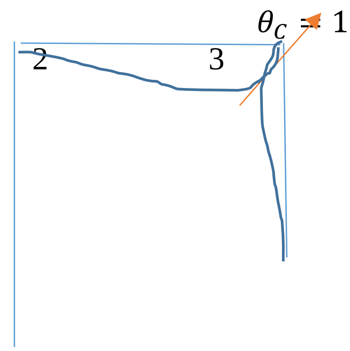
k_{12}= - $\frac{6EI}{L^2}$= -0.37EI



k_{22}= $\frac{4EI}{L}$+$\frac{4EI}{L}$= 2EI

k_{23} = $\frac{2EI}{L}$= 0.5EI

Applying the unit rotation along the co-ordinate 3



K_{13}= - $\frac{6EI}{L^2}$= -1.5EI

K_{23}= $\frac{2EI}{L}$= 0.5EI

k_{33}= $\frac{4EI}{L}$+$\frac{4EI}{L}$=3EI

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = [EI] \begin{bmatrix} 1.68 & -0.37 & -1.5 \\ -0.37 & 2 & 0.5 \\ -1.5 & 0.5 & 3 \end{bmatrix}$$

$[\Delta] = [k]^{-1} [P-P_L]$

$$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 1.68 & -0.37 & -1.5 \\ -0.37 & 2 & 0.5 \\ -1.5 & 0.5 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 50 \\ 40 \\ -40 \end{bmatrix}$$

$\delta = 35.68/EI, \theta_B = 26.6/EI \ , \theta_C = 0.1/EI$

4. Substitute the above values in the slope deflection equation

M_{AB} = M_{FAB}+ $\frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\partial}{l})$

M_{AB} = -0.3

M_{BA} = M_{FBA}+ $\frac{2EI}{l}(\theta_A + 2\theta_B - \frac{3\partial}{l})$

M_{BA} = 13.33

For Span BC , l=

M_{BC} = M_{FBC}+ $\frac{2EI}{l}(2\theta_B + \theta_c - \frac{3\partial}{l})$

M_{BC}= -13.33

M_{CB} = M_{FCB}+ $\frac{2EI}{l}(\theta_B + 2\theta_c - \frac{3\partial}{l})$

M_{CB}= 53.33

For Span CD , l=

M_{CD} = M_{FCD} + $\frac{2EI}{l}(2\theta_C + \theta_D - \frac{3\partial}{l})$

M_{CD}= -53.33

$M_{DC} = M_{FDC} + \frac{2EI}{l}(2\theta_D + \theta_C - \frac{3\partial}{l})$

M_{DC}= -53.33