USN

Internal Assessment Test 1 – Dec. 2021

Sub:	Discrete Mathematical Structures				Sub Code:	18CS36	Branch:	CS & IS	
Date:	20/12/2021	Duration:	90 minutes	Max Marks: 50	Sem / Sec:	III A, B & C			OBE

Question 1 is compulsory and answer any six from Q.2 to Q.9

		MARKS	CO	RBT
1	Obtain an optimal prefix code for the letters of the message MISSION IMPOSSIBLE. Indicate the code for the message.	[08]	CO5	L3
2	Determine the order $\|V\|$ of the graph $G=(V,E)$ in the following cases: (i) G is a cubic graph of 9 edges. (ii) G is regular with 15 edges. (iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.	[07]	CO5	L3
3	Prove that a tree with n vertices has n-1 edges.	[07]	CO5	L3
4	Determine whether the graphs G and H shown in figure are isomorphic.	[07]	CO5	L3

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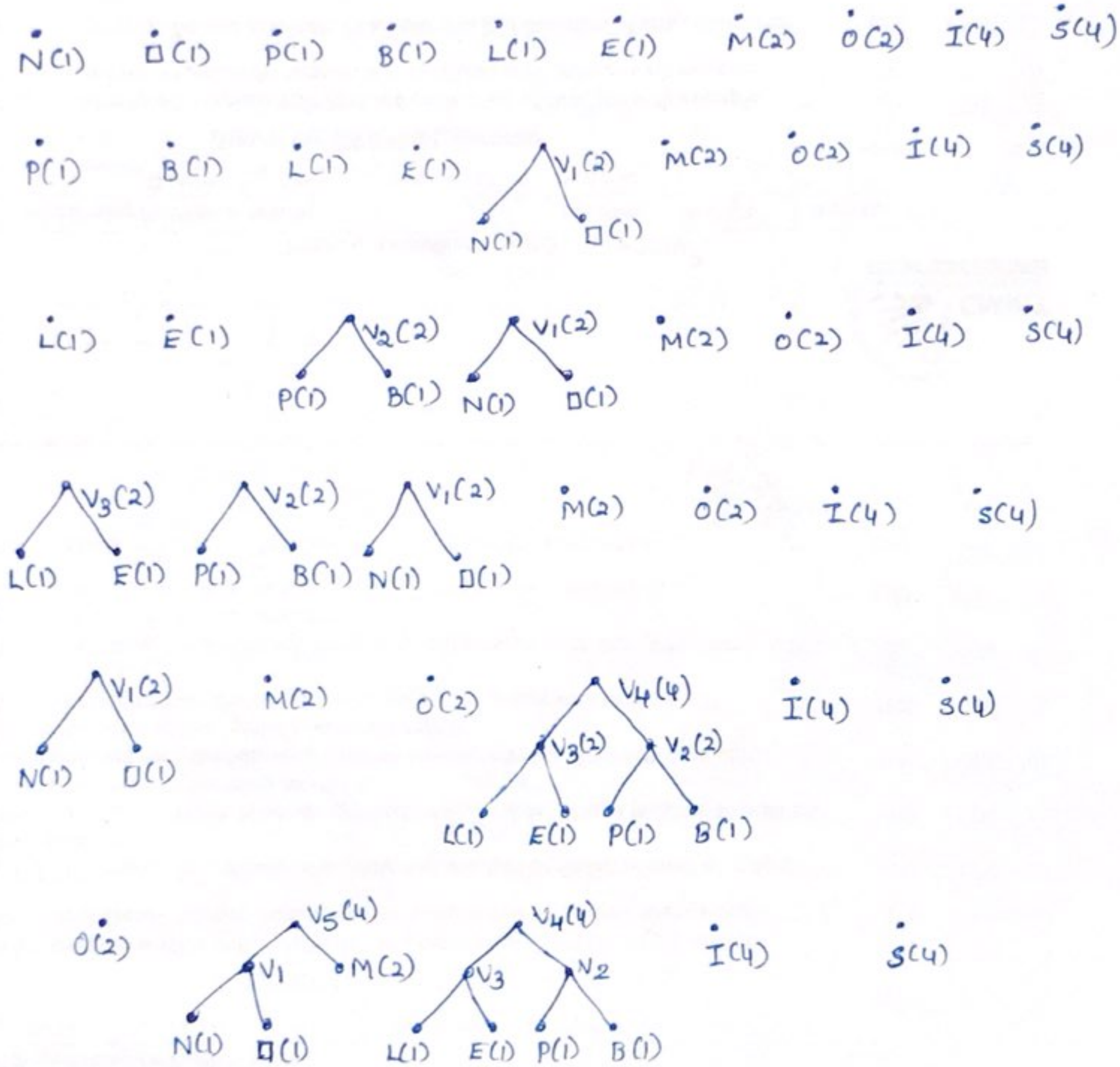
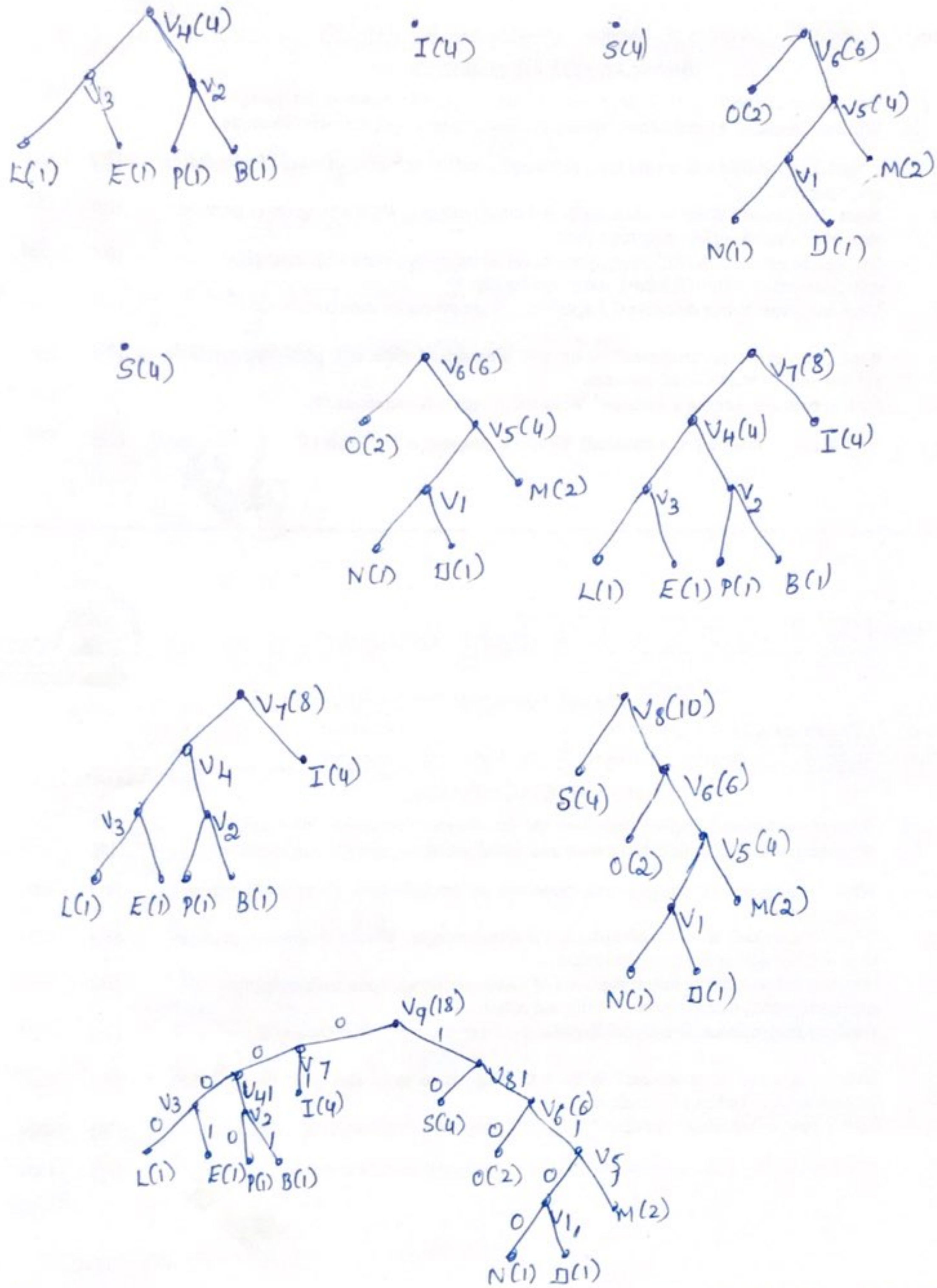
Q1. Message "MISSION IMPUSSIBLE" contains the letters M, I, S, O, N, □, P, B, L, E with the frequencies 2, 4, 4, 2, 1, 1, 1, 1, 1, 1 respectively.

Here, □ stands for a blank space.

Let's write these symbols with the non-decreasing order of their frequencies.





M: 1111, I: 01, S: 10, O: 110, N: 11100, □: 1101, P: 0010, B: 0011,

L: 0000, E: 0001

Code for MISSION IMPOSSIBLE : 1111 01 10 10 01 110 11100 1101 01 1111 0010 110 10 10 01
0011 0000 0001

2. Let the order of the graph be $|V|=n$.

(i) G is a cubic graph with $|E|=9$

$\Rightarrow$ Degree of every vertex is 3.

From Hand-shaking property, we have

$$\sum \deg(v_i) = 2|E|$$

$$3n = 2(9)$$

$$n = 6$$

(ii) Let G ~~be~~ a k-regular graph with $|E|=15$

$\Rightarrow$ Degree of every vertex is k.

From Hand-shaking property,

$$\sum \deg(v_i) = 2|E|$$

$$kn = 2(15)$$

$$k = \frac{30}{n}$$

$\Rightarrow n = 1, 2, 3, 5, 6, 10, 15, 30$

(iii) G has 10 edges with 2 vertices of deg 4 and all other vertices of degree 3.

From Hand-shaking property,

$$\sum \deg(v_i) = 2|E|$$

$$(2\times4) + (n-2)3 = 2(10) \Rightarrow 3n = 18$$

$$n = 6$$

3. Let us prove this by the Principle of Mathematical Induction.

$n=1$ $|E|=0$

$n=2$  $|E|=1$

$n=3$  $|E|=2$

$\therefore$ The result is true for $n=1, 2, 3$.

Assume that the result is true for $n=1, 2, 3 \dots k$.

Now consider a tree T with $k+1$ vertices. Let e be an edge in this graph. Let's delete the edge e. Tree is disconnected & has two components, T_1 and T_2 (say).

Let T_1 has k_1 number of vertices & T_2 has k_2 number of vert $\therefore$

$k_1, k_2 \leq k$.

But the result is true for $n = 1, 2, 3 \dots k$.

$\therefore$ T_1 has $k_1 - 1$ edges & T_2 has $k_2 - 1$ edges.

T_1 & T_2 together have $(k_1 - 1) + (k_2 - 1)$ edges

$\Rightarrow$ $T-e$ has $(k_1 + k_2) - 2$ edges.

$\Rightarrow$ $T-e$ has $(k+1) - 2$ edges

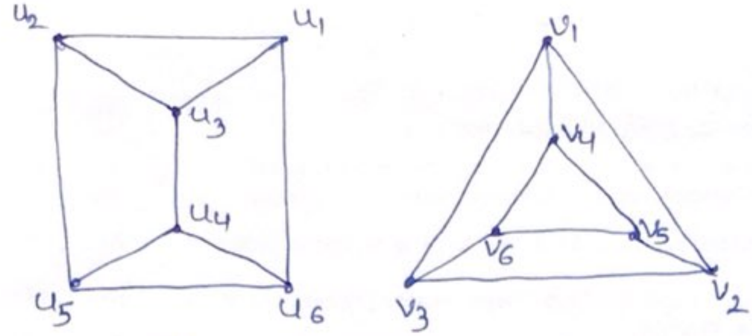
$\Rightarrow$ $T-e$ has $k-1$ edges.

$\Rightarrow$ T has k edges (keeping the edge e back in its place.)

$\therefore$ the result is true for $n = k+1$ also.

Hence by M.I., the result is true for all $+^{ve}$ int n.

4.



G H

Both G & H have 6 vertices.

Both G & H have 9 edges.

Both G & H are 3-regular graphs.

Mapping between the vertices:

$u_1 \longleftrightarrow v_5$

$u_2 \longleftrightarrow v_6$

$u_3 \longleftrightarrow v_4$

$u_6 \longleftrightarrow v_2$

$u_5 \longleftrightarrow v_3$

$u_4 \longleftrightarrow v_1$

Mapping between the edges:

$\{u_1, u_2\} \longleftrightarrow \{v_5, v_6\}$ $\{u_2, u_5\} \longleftrightarrow \{v_6, v_3\}$

$\{u_2, u_3\} \longleftrightarrow \{v_6, v_4\}$ $\{u_1, u_6\} \longleftrightarrow \{v_5, v_2\}$

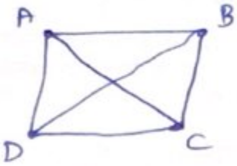
$\{u_3, u_1\} \longleftrightarrow \{v_4, v_5\}$ $\{u_3, u_4\} \longleftrightarrow \{v_4, v_1\}$

$\{u_4, u_6\} \longleftrightarrow \{v_1, v_2\}$, $\{u_6, u_5\} \longleftrightarrow \{v_2, v_3\}$, $\{u_5, u_4\} \longleftrightarrow \{v_3, v_1\}$

There exists a 1-1 correspondence b/w G and H.

$\therefore$ G & H are isomorphic.

5. (i) Complete Graph: Complete graph is a simple graph in which there is an edge b/w every pair of vertices.



(ii) Bipartite graph: Suppose a simple graph G is such that its vertex set V is the union of its mutually disjoint nonempty subsets V_1 & V_2 which are such that each edge in G joins a vertex in V_1 and a vertex in V_2. Then G is called a bipartite graph.

(iii) Induced subgraph: Given a graph $G = (V, E)$, suppose there is a subgraph $G_1 = (V_1, E_1)$ of G such that every edge $\{A, B\}$ of G, where $A, B \in V_1$ is an edge of G_1 also, then G_1 is called an induced subgraph of G.

6. Tautology: A compound statement is said to be a tautology if it is always true regardless of truth values of its components.

p	q	r	$p \to q$	$q \to r$	① $(p \to q) \vee (q \to r)$	$(q \vee r)$	② $p \to (q \vee r)$	① $\leftrightarrow$ ②
0	0	0	1	1	1	0	1	1
0	0	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	1	0	0	0
1	0	1	0	1	1	1	1	1
1	1	0	1	0	1	1	1	1
1	1	1	1	1	1	1	1	1

From the last column, we conclude that it's a contingency as the column contains both 0s & 1s

7 (i) LHS = $p \vee [p \wedge (p \vee q)]$

$\Leftrightarrow p \vee p$ using absorption law

$\Leftrightarrow p$ using idempotent law

$= RHS$

(ii) LHS = $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)]$

WKT $p \rightarrow q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow (\neg p \vee q) \wedge \neg q$ using absorption law.

$\Leftrightarrow (\neg p \wedge \neg q) \vee (q \wedge \neg q)$ using Distributive law

$\Leftrightarrow (\neg p \wedge \neg q) \vee F_0$ using Inverse law

$\Leftrightarrow \neg p \wedge \neg q$ using identity law

$\Leftrightarrow \neg (p \vee q)$ using De-Morgan's law

8. Test the validity of the argument

$$\begin{array}{l} p \rightarrow r \\ r \rightarrow s \\ t \vee \neg s \\ \neg t \vee u \\ \neg u \\ \hline \therefore \neg p \end{array} \quad \Rightarrow \quad \begin{array}{l} p \rightarrow s \text{ using rule of syllogism} \\ \neg t \rightarrow \neg s \\ t \rightarrow u \\ \neg u \\ \hline \therefore \neg p \end{array} \quad \Rightarrow \quad \begin{array}{l} p \rightarrow s \\ s \rightarrow t \quad \text{contra+ve} \\ \neg t \quad \text{modus Tollens} \\ \hline \therefore \neg p \end{array}$$

$$\Rightarrow \begin{array}{l} p \rightarrow t \quad \text{syllogism} \\ \neg t \\ \hline \therefore \neg p \end{array}$$

This is a valid argument in view of Modus Tollens.