USN | | | | | | | | | |

Internal Assessment Test II – Jan. 2022

Sub:	Discrete Mathematical Structures				Sub Code:	18CS36	Branch:	CS & IS
Date:	27/01/2022	Duration:	90 minutes	Max Marks: 50	Sem / Sec:	III A, B & C		OBE

Question 1 is compulsory and answer any six from Q.2 to Q.8

		MARKS	CO	RBT
1	Find the negations of the following: (i) If all the triangles are right angled, then no triangle is equiangular. (ii) All integers are rational numbers and some rational numbers are not integers.	[08]	CO1	L3
2	Determine the coefficient of (i) x^0 in the expansion of $[3x^2 – (2/x)]^{15}$. (ii) xyz^{-2} in the expansion of $(x-2y+3z^{-1})^4$.	[07]	CO2	L2
3	Find whether the following argument is valid: No engineering student of I and II sem. studies logic. Anil is an engineering student who studies logic. ——— ∴ Anil is not in II sem.	[07]	CO1	L3
4	Prove "If n is an even integer, then n+3 is an odd integer." by direct method, indirect method and the method of contradiction.	[07]	CO4	L3

USN | | | | | | | | | |

Internal Assessment Test II – Jan. 2022

Sub:	Discrete Mathematical Structures				Sub Code:	18CS36	Branch:	CS & IS
Date:	27/01/2022	Duration:	90 minutes	Max Marks: 50	Sem / Sec:	III A, B & C		OBE

Question 1 is compulsory and answer any six from Q.2 to Q.8

		MARKS	CO	RBT
1	Find the negations of the following: (i) If all the triangles are right angled, then no triangle is equiangular. (ii) All integers are rational numbers and some rational numbers are not integers.	[08]	CO1	L3
2	Determine the coefficient of (i) x^0 in the expansion of $[3x^2 – (2/x)]^{15}$ (ii) xyz^{-2} in the expansion of $(x-2y+3z^{-1})^4$	[07]	CO2	L3
3	Find whether the following argument is valid: No engineering student of I and II sem. studies logic. Anil is an engineering student who studies logic. ——— ∴ Anil is not in II sem.	[07]	CO1	L3
4	Prove "If n is an even integer, then n+3 is an odd integer." by direct method, indirect method and the method of contradiction.		CO4	L3

5	Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and/or 7's.	[07]	CO4	L3
6	For the Lucas numbers $L_0, L_1, L_2 \ldots$ prove that $L_n = \left[\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$; $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n > 1$.	[07]	CO4	L3
7	A certain question paper contains three parts A, B, C with four questions in part A, five in B, six in C. It is required to answer 7 questions selecting at least two from each part. In how many different ways can a student select his seven questions for answering?	[07]	CO2	L3
8	Let p(x): $x^2 - 8x+15 = 0$, q(x): x is odd, r(x): $x > 0$ with the set of all integers as the universe. Find the truth values of the following statements. If a statement is false, give a counter example. (i) $\forall x,[\{p(x) \vee q(x)\} \rightarrow r(x)]$, (ii) $\forall x,[q(x) \rightarrow p(x)]$, (iii) $\exists x,[p(x) \rightarrow \{q(x) \wedge r(x)]$, (iv) $\forall x,[\neg q(x) \rightarrow \neg r(x)]$, (v) $\exists x,[q(x) \rightarrow p(x)]$, (vi) $\exists x,[p(x) \rightarrow q(x)]$	[07]	CO1	L3

5	Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and/or 7's.	[07]	CO4	L3
6	For the Lucas numbers $L_0, L_1, L_2 \ldots$ prove that $L_n = \left[\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$; $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n > 1$.	[07]	CO4	L3
7	A certain question paper contains three parts A, B, C with four questions in part A, five in B, six in C. It is required to answer 7 questions selecting at least two from each part. In how many different ways can a student select his seven questions for answering?	[07]	CO2	L3
8	Let p(x): $x^2 - 8x+15 = 0$, q(x): x is odd, r(x): $x > 0$ with the set of all integers as the universe. Find the truth values of the following statements. If a statement is false, give a counter example. (i) $\forall x,[\{p(x) \vee q(x)\} \rightarrow r(x)]$, (ii) $\forall x,[q(x) \rightarrow p(x)]$, (iii) $\exists x,[p(x) \rightarrow \{q(x) \wedge r(x)]$, (iv) $\forall x,[\neg q(x) \rightarrow \neg r(x)]$, (v) $\exists x,[q(x) \rightarrow p(x)]$, (vi) $\exists x,[p(x) \rightarrow q(x)]$	[07]	CO1	L3

1. (ii) * "All integers are rational nos. and some rational nos. are not integers." (True)

$p(x)$: x is an int. ; $q(x)$: x is rational no.

$\mathbb{Z}$: Set of Integers ; $\mathbb{Q}$: Set of rational no.

$$[\forall x \in \mathbb{Z}, q(x)] \wedge [\exists x \in \mathbb{Q}, \neg p(x)]$$

Negation

$$\{\exists x \in \mathbb{Z}, \neg q(x)\} \vee \{\forall x \in \mathbb{Q}, p(x)\}$$

Some integers are not rational no. or all rational nos. are integers. (F)

(i) * If all triangles are right angled then no triangle is equiangular.

T: Set of Δs.

$p(x)$: x is right angled Δ.

$q(x)$: x is equi. Δ.

$$\forall x \in T, p(x) \longrightarrow \forall x \in T, \neg q(x)$$

$$\equiv \exists x \in T, \neg p(x) \vee \forall x \in T, \neg q(x)$$

<u>Neg.</u>

$$[\forall x \in T, p(x)] \wedge [\exists x \in T, q(x)]$$

All Δs are right angled and some Δs are equiangular.

2 (i) x^0 in the exp. of $\left(3x^2 - \frac{2}{x}\right)^{15}$

$$\left(3x^2 - \frac{2}{x}\right)^{15} = \sum_{0}^{15} \binom{15}{r} (3x^2)^r \left(-\frac{2}{x}\right)^{15-r}$$

$$= \sum_{r=0}^{15} \binom{15}{r} (3)^r x^{2r} (-2)^{15-r} (x)^{-15+r}$$

$$= \sum_{0}^{15} \binom{15}{r} (3)^r (-2)^{15-r} (x)^{2r+15+r}$$

$$\sum_{0}^{15} \binom{15}{r} (3)^r (-2)^{15-r} (x)^{3r-15}$$

The coeff of x^0 which corresponds to $3r-15=0$

$r = 5$

$\therefore$ coeff $\binom{15}{5} (3)^5 (-2)^{10}$

2(ii) xyz^{-2} in the exp. of $(x-2y+3z^{-1})^4$

By multinomial theorem

$$(x-2y+3z^{-1})^4 = \sum \binom{4}{n_1\ n_2\ n_3} (x)^{n_1}(-2y)^{n_2}(3z^{-1})^{n_3}$$

$$= \sum \binom{4}{n_1\ n_2\ n_3} x^{n_1}(-2)^{n_2}(y)^{n_2}(3)^{n_3}(z)^{-n_3}$$

To get the coeff of xyz^{-2}, we put $n_1 = 1$, $n_2 = 1$, $n_3 = 2$

$\therefore$ coeff. is $\binom{4}{1\ 1\ 2}(-2)^1(3)^2$

$$= \frac{4!}{2!} \times -2 \times 9 = -216$$

3. E: Set of engg. students

$p(x)$: x is in I sem

$q(x)$: x is in II sem

$r(x)$: x studies logic

a : Anil

The given argument in the symbolic form:

$$\forall x,\ p(x) \vee q(x) \rightarrow \neg r(x)$$
$$\frac{r(a)}{\therefore \neg q(a)}$$

<u>Premises</u>: $p(a) \vee q(a) \rightarrow \neg r(a)$ by universal specification

$r(a)$

$\Rightarrow \neg(p(a) \vee q(a))$ by modus Tollens

$\Rightarrow \neg p(a) \wedge \neg q(a)$ by De-morgan's law

$\therefore \neg q(a)$ by rule of conjunc. simplification

$\therefore$ The argument is valid.

4. p: n is an even integer

q: $n+3$ is an odd integer

<u>Direct method</u>: Let p be true.

i.e. $n = 2k$

$\Rightarrow n+3 = 2k+3 = 2(k+1)+1 = \text{odd}$

$\therefore q$ is true

$\Rightarrow p \rightarrow q$ is true.

Indirect method: $\neg p$: n is an odd int.

$\neg q$: $n+3$ is an even int.

Let $\neg q$ be true

~~$n+3 = 2k+1 \Rightarrow n = 2k-2 = 2(k-1)$ = even no.~~

i.e. $n+3 = 2k \Rightarrow n = 2k-3$

$\Rightarrow n = 2(k-1)-1$

= odd no.

$\therefore$ $\neg p$ is true.

Hence $\neg q \to \neg p$ is true.

Method of contradiction – Let $p \to q$ be false.

It's possible when p is T and q is F

Let q be F.

i.e. q : $n+3$ is an even int.

$n+3 = 2k$

$n = 2k-3 = 2(k-1)-1$ = odd no. i.e. p is F.

which is contradiction to the assumption.

$\therefore$ $p \to q$ must be true.

5. ▶ Here, we have to prove that the statement

$S(n)$: n can be written as a sum of 5′s and/or 7′s

is true for all integers $n \geq 24$.

Basis step: We note that

$$24 = (7 + 7) + (5 + 5).$$

This shows that $S(24)$ is true.

Induction step: We assume that $S(n)$ is true for $n = k$ where $k \geq 24$. Then

$$k = (7 + 7 + \cdots) + (5 + 5 + \cdots).$$

Suppose this representation of k has r number of 7's and s number of 5's. Since $k \geq 24$, we should have $r \geq 2$ and $s \geq 2$.

Using this representation of k, we find that

$$\begin{aligned} k + 1 &= \{\underbrace{(7 + 7 + \cdots)}_{r} + \underbrace{(5 + 5 + \cdots)}_{s}\} + 1 \\ &= \underbrace{(7 + 7 + \cdots)}_{(r-2)} + (7 + 7) + \underbrace{(5 + 5 + \cdots)}_{s} + 1 \\ &= \underbrace{(7 + 7 + \cdots)}_{(r-2)} + \underbrace{(5 + 5 + \cdots)}_{s+3} \end{aligned}$$

This shows that $(k + 1)$ is a sum of 7's and 5's. Thus, $S(k + 1)$ is true.

Hence, by mathematical induction, $S(n)$ is true for all positive integers $n \geq 24$.

Aliter: The above result can also be proved with the use of th

7. ▶ The different possible ways in which a student can make a selection are

(I) 2 questions from Part A, 2 from Part B and 3 from Part C.

(II) 2 questions from Part A, 3 from Part B and 2 from Part C.

(III) 3 questions from Part A, 2 from Part B and 2 from Part C.

Now, selection (I) can be made in

$$C(4,2) \times C(5,2) \times C(6,3) = 6 \times 10 \times 20 = 1200 \quad \text{ways},$$

the selection (II) can be made in

$$C(4,2) \times C(5,3) \times C(6,2) = 6 \times 10 \times 15 = 900 \quad \text{ways},$$

and the selection (III) can be made in

$$C(4,3) \times C(5,2) \times C(6,2) = 4 \times 10 \times 15 = 600 \quad \text{ways}.$$

Consequently, the total number of possible selections is

$$1200 + 900 + 600 = 2700.$$

8.

(i) $\forall x, \{p(x) \vee q(x)\} \rightarrow s(x)$ F for $x = -1$

(ii) $\forall x, q(x) \rightarrow p(x)$ F for $x = \pm 1, -3, -5, \pm 7, \ldots$

(iii) $\exists x, p(x) \rightarrow (q(x) \wedge s(x))$ T

(iv) $\forall x, [\neg q(x) \rightarrow \neg s(x)]$ F for $x = 2$

(v) $\exists x, q(x) \rightarrow p(x)$ T

(vi) $\exists x, p(x) \rightarrow q(x)$ T

6.

$$S(n): \quad L_n = \left[\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Basis step – For $n = 0$

LHS. $L_0 = 2$

RHS. $\left(\frac{1+\sqrt{5}}{2}\right)^0 + \left(\frac{1-\sqrt{5}}{2}\right)^0 = 1 + 1 = 2$

$\therefore$ $S(1)$ is true.

Assumption – Let $S(K)$ be true.

i.e. $$L_K = \left(\frac{1+\sqrt{5}}{2}\right)^K + \left(\frac{1-\sqrt{5}}{2}\right)^K \quad \text{———} (1)$$

Inductive step – Now we'll $S(K+1)$ is true i.e. $L_{k+1} = \left(\frac{1+\sqrt{5}}{2}\right)^{k+1} + \left(\frac{1-\sqrt{5}}{2}\right)^{k+1}$

LHS.

$L_{k+1} = L_k + L_{k-1}$ by recursive def. of Lucas nos.

$= \left(\frac{1+\sqrt{5}}{2}\right)^{k} + \left(\frac{1-\sqrt{5}}{2}\right)^{k} + \left(\frac{1+\sqrt{5}}{2}\right)^{k-1} + \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}$ by (1)

$= \left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left\{\frac{1+\sqrt{5}}{2}+1\right\} + \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\left\{\frac{1-\sqrt{5}}{2}+1\right\}\right] \times \frac{2}{2}$

$= \left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left(\frac{1+\sqrt{5}}{2}\right)^{2} + \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\left(\frac{1-\sqrt{5}}{2}\right)^{2}$

$= \left(\frac{1+\sqrt{5}}{2}\right)^{k+1} + \left(\frac{1-\sqrt{5}}{2}\right)^{k+1}$

$\therefore$ $S(k+1)$ is true.

Hence $S(n)$ is true for $n \geq 0$.