USN

Internal Assessment Test III – March 2022

| Sub: | Discrete Mathematical Structures | | | | | Sub Code: | 18CS36 | Branch: | CS & IS | |
|---|---|---|---|---|---|---|---|---|---|---|
| Date: | 08/03/2022 | Duration: | 90 minutes | Max Marks: | 50 | Sem / Sec: | III A, B & C | | OBE | |

| | **Question 1 is compulsory and answer any six from Q.2 to Q.8** | MARKS | CO | RBT |
|---|---|---|---|---|
| 1 | Find the number of solutions of the equation $x_1+x_2+x_3+x_4 = 15$ in<br>a) Non – negative integers,<br>b) Integers satisfying $x_1 > 2$, $x_2 > -2$, $x_3 > 0$, $x_4 > -4$ | [08] | CO2 | L3 |
| 2 | How many integers between 1 and 300 (inclusive) are (i) divisible by at least one of 5, 6, 8? (ii) divisible by none of 5, 6, 8? | [07] | CO2 | L2 |
| 3 | There are 8 pairs of children's socks in a box. Each pair is of a different colour. Suppose the right socks are distributed at random to n children and thereafter left socks are also distributed to them at random, find the probability that (i) no child gets a matching pair (ii) every child gets a matching pair (iii) exactly one child gets a matching pair, and (iv) at least two children get matching pairs. | [07] | CO2 | L3 |
| 4 | Students S1, S2, S3, S4 are to be accommodated in desks D1, D2, D3, D4, D5, D6 such that no two of these students sit in the same desk. S1 doesn't want D1 and D2; S2 doesn't want D3; S3 doesn't want D3 and D4; and S4 doesn't want D4, D5 and D6. In how many ways can they be accommodated? | [07] | CO2 | L3 |

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| 5 | Let A= {1, 2, …… 20 } and let R be a relation on A defined by (x,y)R if and only if x-y is a multiple of 4. Find the partition of A induced by R. | [07] | CO3 | L3 |

| | | | | |
|---|---|---|---|---|
| 6 | The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. | [07] | CO6 | L3 |
| 7 | Draw the Hasse diagram representing the relation R defined by "aRb iff a divides b" on the set of all positive divisors of 72. | [07] | CO3 | L3 |
| 8 | Solve the recurrence relation $$a_{n+2}^2 - 5a_{n+1}^2 + 4a_n^2 = 0, \textit{for } n \geq 0, \textit{ given } a_0 = 4 \textit{ and } a_1 = 13.$$ | [07] | CO6 | L3 |

| | | | | |
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DMS – 10CS36

IAT III – March 2022

1. $x_1+x_2+x_3+x_4=15$

(a) $n=4,\ r=15$

No of non-negative int solutions $= {}^{n+r-1}C_r = {}^{18}C_{15}$

(b) $x_1>2,\ x_2>-2,\ x_3>0,\ x_4>-4$

$x_1\geq 3,\ x_2\geq -1,\ x_3\geq 1,\ x_4\geq -3$

Let $x_1-3=y_1,\ \ x_2+1=y_2,\ \ x_3-1=y_3,\ \ x_4+3=y_4$

Given eqn becomes

$(y_1+3)+(y_2-1)+(y_3+1)+(y_4-3)=15$

$y_1+y_2+y_3+y_4=15$ , $n=4,\ r=15$

No of non-negative solns $= {}^{n+r-1}C_r = {}^{18}C_{15}$.

2. $A=\{1,2,3,\dots 20\}$

$R=\{(1,1),(1,5),(1,9),\dots\dots\}$

$[1]=\{1,5,9,13,17\} = A_1$, say

$[2]=\{2,6,10,14,18\} = A_2$, say

$[3]=\{3,7,11,15,19\} = A_3$, say

$[4]=\{4,8,12,16,20\} = A_4$, say

The partition of A is :

$P=\{A_1,A_2,A_3,A_4\}$ as $A_1\cup A_2\cup A_3\cup A_4 = A$

& $A_i\cap A_j=\phi$ for $i,j=1,2,3,4$ for $i\neq j$

2. $|S| = 300$. Let $A_1, A_2, A_3$ be subsets of $S$ whose elements are divisible by $5, 6, 8$ resp$^{ly}$. Then

$|A_1| = \lfloor 300/5 \rfloor = 60$, $|A_2| = \lfloor 300/6 \rfloor = 50$

$|A_3| = \lfloor 300/8 \rfloor = 37$, $|A_1 \cap A_2| = \lfloor 300/30 \rfloor = 10$,

$|A_1 \cap A_3| = \lfloor 300/40 \rfloor = 7$, $|A_2 \cap A_3| = \lfloor 300/24 \rfloor = 12$

LCM of 6 & 8 is 24

$|A_1 \cap A_2 \cap A_3| = \lfloor 300/120 \rfloor = 2$ LCM of 5, 6, 8 is 120.

(4)

wkt

(i) ∴ $|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2|$ .

$= (60 + 50 + 37) - (10 + 7 + 12) + 2 = 120$

(ii) $|\bar{A_1} \cap \bar{A_2} \cap \bar{A_3}| = |S| - |A_1 \cup A_2 \cup A_3|$

$= 300 - 120 = 180$

3) There are $n$ pairs of children's gloves in a box. Each pair is of a different colour. Suppose the right gloves are distributed at random to $n$ children, and thereafter the left gloves are also distributed to them at random. Find the prob that (i) no child gets a matching pair, (ii) every child gets a matching pair. (iii) exactly one child gets a matching pair. (iv) atleast 2 children get matching pairs.

③

Any one distribution of n right gloves to n children determines a set of n places for the n pairs of glovel. Let us take these as the natural places for the pairs of gloves. The left gloves can be distributed to n children in $n!$ ways.

(i) The event of no child getting a matching pair occurs if the distribution of the left gloves is a derangement. The no of derangements is $d_n$. ∴ the required prob in this case, is

$$P_1 = \frac{d_n}{n!} = \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + - \cdots + (-1)^n \frac{1}{n!}\right)$$

(ii) The event of every child getting a matching pair occurs on only one distribution of the left glovel. ∴ the required probability, in this case is $P_2 = \frac{1}{n!}$

(iii) The event of exactly one child getting a matching pair occurs when only one left glove is in the natural place, and all others are in wrong places. The no of such distributions is $d_{n-1}$. The req prob, in this case, is $P_3 = \frac{d_{n-1}}{n!} = \frac{1}{n}\left\{1 - \frac{1}{1!} + \frac{1}{2!} - + \cdots + (-1)^{n-1}\frac{1}{(n-1)!}\right\}$

(iv) The event of atleast two children getting a matching pair occurs if the event of no child or one child getting a matching pair does not occur. The prob in this case is $P_4 = 1 - (P_1 + P_3)$.

4)

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $D_6$ |
|---|---|---|---|---|---|---|
| $S_1$ | 1 | 2 | | | | |
| $S_2$ | | | 3 | | | |
| $S_3$ | | | 4 | 5 | | |
| $S_4$ | | | | 6 | 7 | 8 |

$n=6$ , $m=4$

$C_1$

| | |
|---|---|
| 1 | 2 |

$C_2$

| | | | | |
|---|---|---|---|---|
| 3 | | | | |
| 4 | 5 | | | |
| | 6 | 7 | 8 | |

(3,5) (3,6), (3,7), (3,8)
(4,6), (4,7), (4,8),
(5,7), (5,8)
(3,5,7), (3,5,8)

$C_1(r;x) = 1+2x \qquad C_2(r,x) = 1+6x+9x^2+2x^3$

$$C(r,x) = C_1(r,x) \times C_2(r,x)$$
$$= (1+2x)(1+6x+9x^2+2x^3)$$
$$= 1+6x+9x^2+2x^3+2x+12x^2+18x^3+4x^4$$
$$= 1+8x+21x^2+20x^3+4x^4$$

$$\overline{N} = P(6,4) - 8 \cdot P(5,3) + 21\,P(4,2) - 20\,P(3,1) + 4\,P(2,0)$$

$$= \frac{6!}{2!} - 8 \cdot \frac{5!}{2!} + 21 \cdot \frac{4!}{2!} - 20 \cdot \frac{3!}{2!} + 4 \cdot \frac{2!}{2!}$$

$$= 360 - 480 + 252 - 60 + 4$$

$$= 616 - 540 = \underline{76}$$

5) The no of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the no of virus affected files in the system after one day.

Let $a_0 = 1000$. Let $a_n$ denote the no of virus affected files after $2n$ hours. Then the no increases by $a_n \times \frac{250}{100}$ in the next two hours. Thus, after $2n+2$ hours, the no is $a_{n+1} = a_n + a_n \times \frac{250}{100}$

$\Rightarrow a_{n+1} = a_n(1+2.5) = a_n(3.5)$ This is rec relⁿ for the no of virus affected files. Solving this we get, $a_n = (3.5)^n a_0 = (1000) \times (3.5)^n$

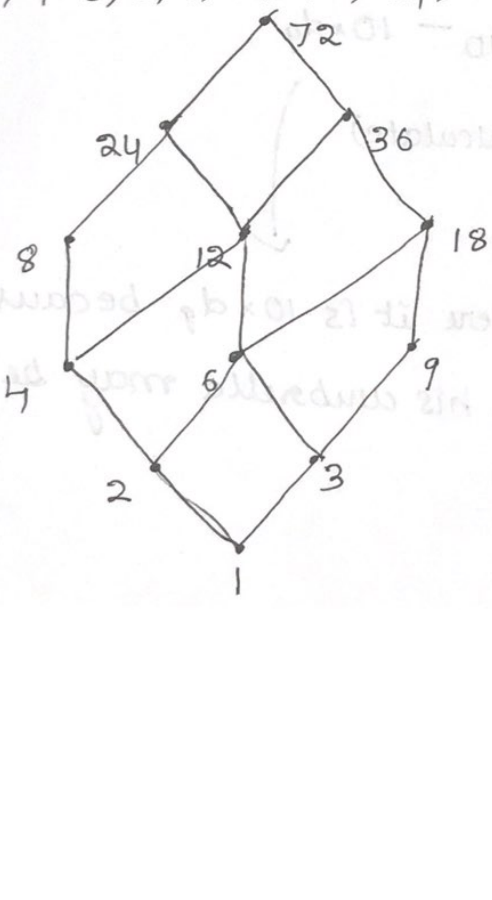
This is the no of virus affected files after $2n$ hours. From this, for $n=12$, $a_{12} = 1000 \times (3.5)^{12} = 3379220508$ is the no of virus affected files after one day.

7) ~~Let the~~ The set of all positive divisors of 72 be:

$A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$

Define a relation R on A by

$(x, y) \in R$ iff $x$ divides $y$.



Solⁿ:- Let $b_n = a_n^2$.

Then the given relation reads

$$b_{n+2} - 5b_{n+1} + 4b_n = 0, \quad n \geq 0$$

or equivalently,

$$b_n - 5b_{n-1} + 4b_{n-2} = 0, \quad n \geq 2$$

Characteristic eqⁿ is $k^2 - 5k + 4 = 0$

$\Rightarrow k_1 = 4$ & $k_2 = 1$

$\therefore$ Solⁿ for $b_n$ is $b_n = A \times 4^n + B \times 1^n$ — (1)

Given $a_0 = 4$ & $a_1 = 13$

$\Rightarrow b_0 = a_0^2 = 16$ & $b_1 = a_1^2 = 13^2 = 169$

(1) $\Rightarrow$ $16 = A + B$ & $169 = 4A + B$

$\Rightarrow A = 51$ & $B = -35$

$\Rightarrow b_n = 51 \times 4^n - 35$

$\Rightarrow a_n = \pm\sqrt{(51 \times 4^n - 35)}$

This is the required solⁿ for $a_n$.