CMR INSTITUTE OF TECHNOLOGY

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Internal Assessment Test-I									
Sub:	Electromagnetic Waves							Code:	18EC55
Date:	13/11 /2021	Duration:	90 mins	Max Marks:	50	Sem:	5th	Branch:	ECE(A,B,C,D)
Solutions									

	Marks	OBE CO	OBE RBT
1.(a) State and explain Coulomb's law in vector form.	[06]	CO1	L1

a) Coulomb's law and electric field intensity:

Static electric fields in vacuum or free space.

Greeks → electricity

Dr. Gilbert → physician to Her Majesty, the Queen of England → 1600 (worked this effect)

French Army Engineer officer → Colonel Charles Coulomb
↓
experiments (Torsion balance → force exerted b/w two objects having static charge of electricity)

i) Coulomb's law:

The force between two very small objects separated in vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the distance between them.

$$\boxed{F = k\frac{(Q_1 Q_2)}{R^2}}$$

$(Q_1 Q_2)$ → positive or negative quantities of charge

k → proportionality constant

R → separation

$$\boxed{k = \frac{1}{4\pi\varepsilon_0}}$$

ε_0 → Permittivity of free space.

$$\varepsilon_0 = \frac{10^{-9}}{36\pi} \text{ F/m}.$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$k = \frac{1}{4\pi \times \frac{10^{-9}}{36\pi}}$$

$$k = 9 \times 10^9$$

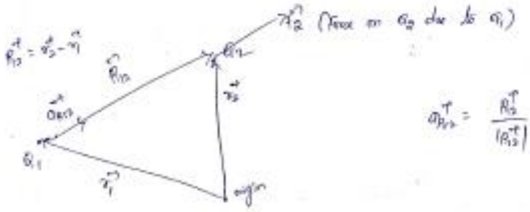
$$\boxed{F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}}$$

Vector form:

Force acts along line joining two charges.

Like charges → repulsive force

unlike charges → attractive force.



$$\boxed{\vec{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 |\vec{R_{12}}|^2} \cdot \vec{a_{R_{12}}}}$$

$$\vec{F_2} = \frac{Q_1 Q_2 \vec{R_{12}}}{4\pi\varepsilon_0 |\vec{R_{12}}|^3}$$

(b) A charge Q_A of -20μC is located at A (-6,4,7) and a second charge Q_B of 50 μC is located at B(5,8,-2). Find force on Q_B due to charge Q_A. Assume both the charges are placed in free space. [04] CO1 L3

A (-6,4,7)

$Q_A = -20\mu C$

$\vec{R_{AB}}$

B (5,8,-2)

$Q_B = 50\,\mu C$

$\vec{R_{AB}} = (11\vec{a_x} + 4\vec{a_y} - 9\vec{a_z})$ m

$|\vec{R_{AB}}| = \sqrt{11^2 + 4^2 + 9^2}$

$|\vec{R_{AB}}| = 14.76$ m

$\vec{R_{BA}} = -11\vec{a_x} - 4\vec{a_y} + 9\vec{a_z}$

$$\vec{F_{BA}} = \frac{1}{4\pi \times \frac{10^{-9}}{36\pi}} \times \frac{-20\times10^{-6} \times 50\times10^{-6}}{(14.76)^2} \times \frac{(11\vec{a_x} + 4\vec{a_y} - 9\vec{a_z})}{(14.76)}$$

$\vec{F_{BA}} = (-30.78\,\vec{a_x} - 11.195\,\vec{a_y} + 25.189\,\vec{a_z}) \times 10^{-3}$ N

$\vec{F_{BA}} = (-30.78\,\vec{a_x} - 11.195\,\vec{a_y} + 25.189\,\vec{a_z})$ mN

2. Define electric field intensity. Obtain an expression for electric field intensity due to an infinitely long uniform line charge distribution. [02+08] CO1 L1

Electric - field Intensity

- consider ~~the~~ one charge fixed in position say Q_1
- we move a charge slowly around

There exists a force everywhere on this 2nd charge. i.e. force field.

- Let the 2nd charge be Q_t.

Then the force on it is.

$$\vec{F}_t = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{1t}^2} \hat{a}_{1t}$$

$\therefore$ Force per unit charge:

$$\frac{F_t}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2} \hat{a}_{1t} .$$

R.H.S. f^n of Q_1 and ~~a~~ the directed line segment from Q_1 to the position of the test charge

- This describes a vector field and is called the electric field intensity.

Def^n Electric field intensity is the vector force on a unit +ve test charge when the charge placed in a electric field.

$\boxed{\text{Unit. N/C}}$

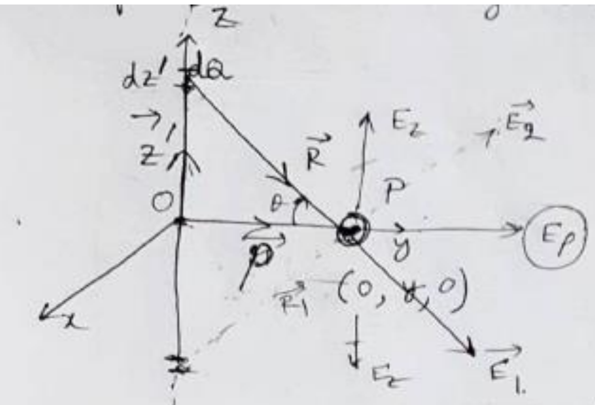
But $V = \frac{J}{C} = \frac{N.m}{C}$.

$$\Rightarrow \frac{V}{m} = \frac{N}{C}$$

$\therefore$ E practically expressed in $\frac{V}{m}$.

Finally, $\boxed{\vec{E} = \frac{\vec{F_t}}{Q_t}}$

$$\boxed{\vec{E} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2} \hat{a}_{1t}}$$



$\frac{z'}{\rho} = \tan\theta$

$z' = \rho \tan\theta$

ρ_L C/m

Find $\vec{E}$ at P (0, y, 0) because of the line charge along z-axis.

We consider incremental charge

$dQ = \rho_L dz'$ . . . . (1) [charge on length dz']

$\vec{R} = \vec{\rho} - \vec{z}'$ $\quad (\vec{z'} + \vec{R} = \vec{\rho})$

$\vec{R} = (\rho \hat{a}_\rho - z' \hat{a}_z)$ . . (2) $\quad |\vec{R}| = \sqrt{\rho^2 + z'^2}$

∴ The field at P, because of dQ,

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 |\vec{R}|^2} \hat{a}_R = \frac{\rho_L dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)} \cdot \frac{(\rho \hat{a}_\rho - z' \hat{a}_z)}{(\rho^2 + z'^2)^{1/2}}$$

$[\because |\vec{R}| = \sqrt{\rho^2 + z'^2}]$

$$= \frac{\rho_L dz' \cdot (\rho \hat{a}_\rho - z' \hat{a}_z)}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

We know from the symmetry of the problem — (3)

∴ The field at P

$$\vec{E} = \int d\vec{E} = \int_{z' \to -\infty}^{\infty} \frac{\rho_L \, dz' \rho \, \hat{a}_\rho}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dz'}{(\rho^2 + z'^2)^{3/2}} \hat{a}_\rho$$

Note: $\tan\theta = \frac{z'}{\rho}$

$z' = \rho \tan\theta$ | $z' \to \infty, \ \theta = \pi/2$

$dz' = \rho \sec^2\theta \, d\theta$ | $z' \to -\infty, \ \theta = -\pi/2$

$$\therefore \vec{E} = \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{(\rho^2 + \rho^2 \tan^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{(\rho^2)^{3/2} (1 + \tan^2\theta)^{3/2}} \hat{a}_\rho$$

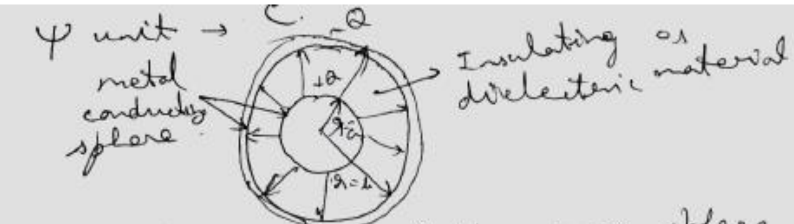
$$= \frac{\rho_L \cancel{\rho}}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\cancel{\rho} \sec^2\theta \, d\theta}{\cancel{\rho^3}\rho \, (\sec^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta \, d\theta}{\sec^3\theta} \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta} d\theta \, \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} \left[\sin\theta \right]_{-\pi/2}^{\pi/2} \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} [1 + 1] \hat{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho$$

3.(a) Define electric flux density. Derive the relation between electric flux density and electric field intensity. [04] CO2 L1



At the surface of the inner sphere, Ψ C of electric flux are produced by charge Q.

charge on inner sphere = Q

" " outer " = −Q.

Surface area of inner sphere = $4\pi a^2$ m².

∴ Density of the flux at this surface is

$$\frac{\Psi}{4\pi a^2} \text{ or } \frac{Q}{4\pi a^2}\ c/m^2.$$

Electric flux density = $\vec{D}$.

Also called displacement flux density or displacement density.

∴ The direction of D at a pt. is the direction of the flux lines at that point, and the mag. is given by the no. of flux lines crossing a surface normal to the lines divided by the surface area

$$\therefore \vec{D}\Big|_{r=a} = \frac{Q}{4\pi a^2}\,\hat{a}_r \quad \text{(inner sphere)}$$

$$\vec{D}\Big|_{r=b} = \frac{-Q}{4\pi b^2}\,\hat{a}_r \quad \text{(outer sphere)}.$$

$a \le r \le b$

$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r$$

Shrink inner sphere, smaller and smaller. We reach point charge.

$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r.$$

Q lines of flux are symmetrically directed outward from the pt. and pass through an imaginary spherical surface of area $4\pi r^2$.

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r.$$

$\therefore \boxed{D = \epsilon_0 E}$ → free space.

3. (b) i. A uniform line charge of infinite length with ρ_L = 40 nC/m, lies along the z-axis. Find **E** at (-2,2,8) in air. [06] CO2 L3

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a}_\rho \text{ V/m}$$

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$\rho = \sqrt{x^2+y^2}$

$$\rho = \sqrt{(-2)^2+2^2} = \sqrt{8} = 2\sqrt{2}$$

$$\therefore \vec{E} = \frac{40\times10^{-9}}{2\times3.14\times8.854\times10^{-12}\times2\sqrt{2}}\hat{a}_\rho$$

$$= 254.2\,\hat{a}_\rho \text{ V/m}$$

$$E = 254.2 \text{ V/m}$$

$$\therefore \boxed{\vec{E} = 254.2\,\hat{a}_\rho \text{ V/m}}$$

ii. Calculate **E** and **D** in rectangular coordinates at point P(2,−3, 6) produced by a point charge Q_A = 55 mC at A(−2, 3,−6).

Calculate $\vec{D}$ in rectangular co-ordinates at point P(2,−3,6) produced by a point charge $Q_A = 55mC$ at A(−2,3,−6)

$$\vec{R} = 4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z$$

55 mC A (−2,3,−6) P (2,−3,6)

$$|\vec{R}| = 14$$

$$\therefore \vec{D} = \frac{55\times10^{-3}}{4\pi\times14^2}\cdot\frac{4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z}{14}$$

$$= .0016\times10^{-3}(4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z)$$

$$= (.0064\,\hat{a}_x - .0096\,\hat{a}_y + .0192\,\hat{a}_z)\times10^{-3}$$

$$= (6.4\,\hat{a}_x - 9.6\,\hat{a}_y + 19.2\,\hat{a}_z)\,\mu C/m^2$$

$$= (6.4\,\hat{a}_x - 9.6\,\hat{a}_y + 19.2\,\hat{a}_z)\times10^{-6}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{1}{8.854\times10^{-12}}(6.4\,\hat{a}_x - 9.6\,\hat{a}_y + 19.2\,\hat{a}_z)\times10^{-6}$$

$$= (0.72\,\hat{a}_x - 1.08\,\hat{a}_y + 2.168\,\hat{a}_z)\times10^{6} \text{ V/m}$$

4.(a) Transform the vector $\mathbf{B} = y\,\mathbf{a}_x - x\,\mathbf{a}_y + z\,\mathbf{a}_z$ into cylindrical coordinates. [05] CO1 L3

$\vec{B} = y\,\vec{a_x} - x\,\vec{a_y} + z\,\vec{a_z}$,

$B_x = y = \rho \sin\phi$

$B_y = -x = -\rho\cos\phi$

$B_z = z = z$

$$\begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho\sin\phi \\ -\rho\cos\phi \\ z \end{bmatrix}$$

$\vec{B} = B_\rho\,\vec{a_\rho} + B_\phi\,\vec{a_\phi} + B_z\,\vec{a_z}$

$B_\rho = \cos\phi\,\rho\sin\phi - \sin\phi\,\rho\cos\phi + 0 = 0$

$B_\phi = -\rho\sin^2\phi - \rho\cos^2\phi + 0 = -\rho(\sin^2\phi + \cos^2\phi)$

$B_\phi = -\rho$

$B_z = 0 + 0 + z = z$

$\boxed{\vec{B} = -\rho\,\vec{a_\phi} + z\,\vec{a_z}}$

(b) i. Give the rectangular coordinates of the point $C(\rho = 4.4,\ \varphi = -115^\circ,\ z = 2)$. [05] CO1 L3

$C(\rho = 4.4,\ \phi = -115,\ z = 2)$

$C(x = ?,\ y = ?,\ z = ?)$

$x = \rho\cos\phi$; $y = \rho\sin\phi$; $z = z$

$x = -1.859$; $y = -3.987$; $z = 2$

Ans : $C(-1.859, -3.987, 2)$

ii. Give the cylindrical coordinates of the point $D(x = -3.1,\ y = 2.6,\ z = -3)$.

$\rho = \sqrt{x^2 + y^2} = \sqrt{(-3.1)^2 + (2.6)^2} = 4.05$

$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2.6}{-3.1}\right) = -39.98 + 180^\circ = 140^\circ$

z = -3

5.(a)	Derive the expression for the electric field intensity at a point due to n number of point charges.	[04]	CO1	L2

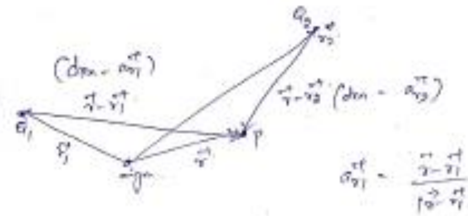
Since Coulomb's force is linear,

$\vec{E}$ due to two point charges, Q_1 at $\vec{r_1}$ and Q_2 at $\vec{r_2}$

$\Uparrow$

Sum of forces on Q_t caused by Q_1 and Q_2 acting alone

$$\vec{E}(\vec{r}) = \frac{Q_1}{4\pi\epsilon_0 |\vec{r}-\vec{r_1}|^2} \cdot \vec{a_{r1}} + \frac{Q_2}{4\pi\epsilon_0 |\vec{r}-\vec{r_2}|^2} \cdot \vec{a_{r2}}$$

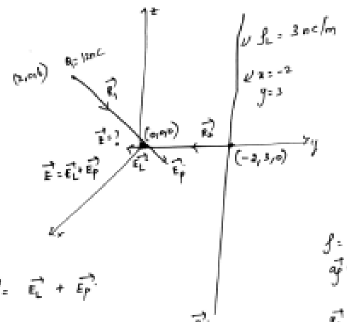


$$\vec{a_{r1}} = \frac{\vec{r}-\vec{r_1}}{|\vec{r}-\vec{r_1}|} \quad \& \quad \vec{a_{r2}} = \frac{\vec{r}-\vec{r_2}}{|\vec{r}-\vec{r_2}|}.$$

'n' number of charges

$$\boxed{\vec{E}(\vec{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0 |\vec{r}-\vec{r_m}|^2} \cdot \vec{a_{rm}}} \quad V/m \text{ or } N/C$$

(b)	Find **E** at origin due to a point charge 12nC at (2, 0, 6) and a uniform line charge 3nC/m at x = -2, y = 3.	[06]	CO1	L3



$\vec{R_1} = -2\vec{a_x} - 6\vec{a_z}$

$|\vec{R_1}| = \sqrt{4+36} = \sqrt{40}$

$\rho = |\vec{R_2}|$

$\vec{a_\rho} = \vec{a_{R_2}} = \frac{\vec{R_2}}{|\vec{R_2}|}$

$\vec{R_2} = 2\vec{a_x} - 3\vec{a_y}$

$|\vec{R_2}| = \sqrt{4+9} = \sqrt{13}$

$\vec{E} = \vec{E_L} + \vec{E_P}$

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho}\vec{a_\rho} + \frac{Q}{4\pi\epsilon_0 |\vec{R_1}|^2}\vec{R_1}$$

$$= \frac{3\times10^{-9} \times}{2\pi\times\epsilon_0\times\sqrt{13}} \frac{(2\vec{a_x} - 3\vec{a_y})}{\sqrt{13}} + \frac{12\times10^{-9}}{4\pi\times\epsilon_0} \times \frac{(-2\vec{a_x} - 6\vec{a_z})}{(\sqrt{40})^3}$$

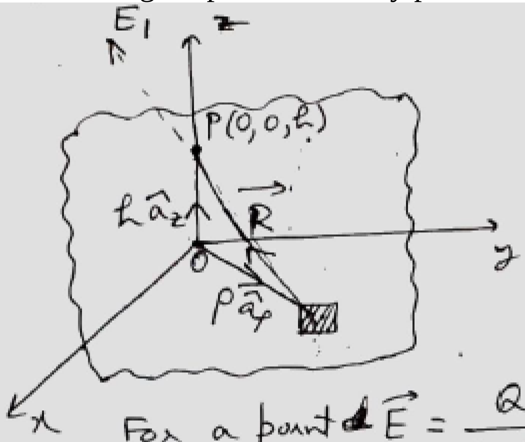
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{3\times10^{-9}\times 2\,(2\vec{a_x} - 3\vec{a_y})}{13} + \frac{12\times10^{-9}\,(-2\vec{a_x} - 6\vec{a_z})}{(\sqrt{40})^3}\right]$$

$$= 9\times10^{9} \times 6\times10^{-9}\left[\frac{2}{13}\vec{a_x} - \frac{3}{13}\vec{a_y} - \frac{4}{(\sqrt{40})^3}\vec{a_x} - \frac{12}{(\sqrt{40})^3}\vec{a_z}\right]$$

$$= 54\left[0.138\,\vec{a_x} - 0.230\,\vec{a_y} - 0.0474\,\vec{a_z}\right]$$

$$\boxed{\vec{E} = 7.452\,\vec{a_x} - 12.42\,\vec{a_y} - 2.5596\,\vec{a_z}} \quad V/m$$

6. Define surface charge density. Obtain an expression of electric field intensity due to an infinite sheet of charge with uniform surface charge distribution ρ_s C/m^2. Assume the charge is placed over x-y plane. [02+08] CO1 L2

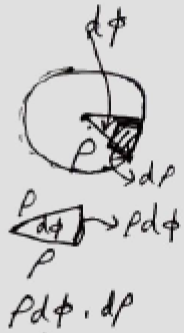


Surface charge density,

$$\rho_s = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s} \quad C/m^2$$

For a point charge, $\cancel{d}\vec{E} = \frac{Q}{4\pi\epsilon_0 |\vec{R}|^2}\hat{a}_R$

1) $dQ = ??$
2) $\vec{R} = ??$ $\longrightarrow$ $\vec{\rho} + \vec{R} = h\hat{a}_z$
   $\vec{R} = h\hat{a}_z - \rho\hat{a}_\rho$
3) $\hat{a}_R = ??$
4) ~~Variable~~ Symmetry of problem
5) Integration.



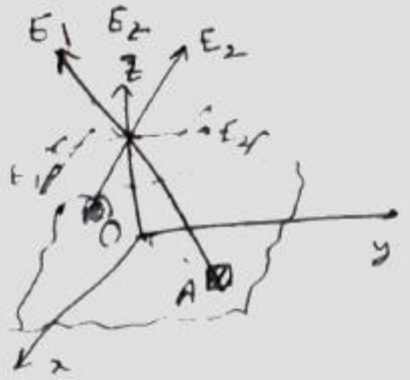
uniform charge density ρ_s C/m^2

$$dQ = \rho_s . |\vec{dA}| = \rho_s . (\rho\, d\rho\, d\phi)$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{(h\hat{a}_z - \rho\hat{a}_\rho)}{\sqrt{h^2+\rho^2}}$$

∴ The intensity at P due to the chosen infinitesimal surface area,

$$\vec{dE} = \frac{\rho_s . (\rho\, d\rho\, d\phi)}{4\pi\epsilon_0 (h^2+\rho^2)} \frac{(h\hat{a}_z - \rho\hat{a}_\rho)}{\sqrt{h^2+\rho^2}} \quad \text{--- } (1)$$



From symmetry of the problem,

$$\overrightarrow{dE} = \frac{dQ}{4\pi\epsilon_0(\rho^2+h^2)} \cdot \frac{h\,\hat{a}_z}{\sqrt{\rho^2+h^2}}$$

$$\therefore \overrightarrow{dE} = \frac{\rho_s.\,(\rho\,d\rho\,d\phi)\,(h\,\hat{a}_z)}{4\pi\epsilon_0\left(\rho^2+h^2\right)^{3/2}} \quad \text{--- (4)}$$

$\therefore$ The total electric field intensity at P,

$$\vec{E} = \iint \overrightarrow{dE} = \frac{\rho_s.\,h}{4\pi\epsilon_0} \int_{\rho=0}^{\infty}\int_{\phi=0}^{2\pi} \frac{\rho.\,d\rho.\,d\phi}{(\rho^2+h^2)^{3/2}}\,\hat{a}_z$$

$$= \frac{\rho_s.\,h}{4\pi\epsilon_0} \int_{\rho=0}^{\infty} \frac{\rho\,d\rho}{(\rho^2+h^2)^{3/2}} \int_{\phi=0}^{2\pi} d\phi \quad \hat{a}_z$$

$$= \frac{\rho_s.\,h}{2\epsilon_0} \int_{\rho=0}^{\infty} \frac{\rho\,d\rho}{(\rho^2+h^2)^{3/2}} \quad \hat{a}_z$$

Let, $\rho = h\tan\theta$ | when $\rho = 0$, $\theta = 0$

$d\rho = h\sec^2\theta\,d\theta$ | when $\rho \to \infty$, $\theta = \pi/2$

$$\vec{E} = \frac{\rho_s.\,h}{2\epsilon_0} \int_0^{\pi/2} \frac{(h\tan\theta).\,h\sec^2\theta\,d\theta}{(h^2+h^2\tan^2\theta)^{3/2}} \quad \hat{a}_z$$

$$= \frac{\rho_s.\,h}{2\epsilon_0} \int_0^{\pi/2} \frac{h^2\tan\theta\,\cancel{\sec^2\theta}\,d\theta}{h^3.\,\cancel{\sec^2\theta}\,\sec\theta} \quad \hat{a}_z$$

$$= \frac{\rho_s}{2\epsilon_0} \int_0^{\pi/2} \tan\theta \cdot \frac{1}{\sec\theta} \, d\theta \, \hat{a}_z$$

$$= \frac{\rho_s}{2\epsilon_0} \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta} \cdot \cos\theta \, d\theta \, \hat{a}_z$$

$$= \frac{\rho_s}{2\epsilon_0} \int_0^{\pi/2} \sin\theta \, d\theta \, \hat{a}_z$$

$$= \frac{\rho_s}{2\epsilon_0} \left[-\cos\theta\right]_0^{\pi/2} \hat{a}_z = \frac{\rho_s}{2\epsilon_0} \hat{a}_z$$

i.e. $\boxed{\vec{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_z}$ V/m.

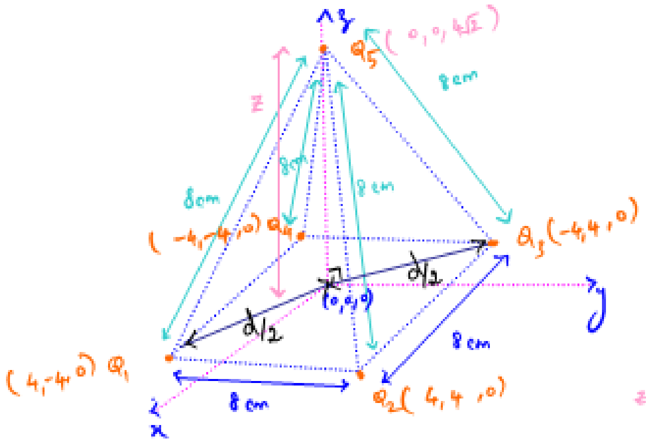
Unit vector along a normal to a surface, is given as, $\hat{a}_n$

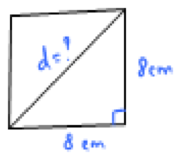
$\therefore \vec{E}$ in general can be given as

$\boxed{\vec{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_n}$ V/m.

E_2

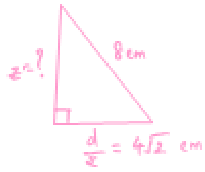
7. Four 10 nC positive charges are located in z=0 plane at the corners of a square 8 cm. on a side. A fifth 10 nC charge is located at a point 8 cm. distant from other charges. Calculate the magnitude of total force on this fifth charge for $\varepsilon = \varepsilon_0$. [10] CO1 L3





$d = \sqrt{8^2 + 8^2}$

$d = 8\sqrt{2}$ cm



$8^2 = (4\sqrt{2})^2 + z^2$

$z = \sqrt{8^2 - (4\sqrt{2})^2}$

$z = 4\sqrt{2}$ cm

$\vec{R_1}$ from q_1 to q_5

$\vec{R_1} = (-4\vec{a_x} + 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z})$ cm

$|\vec{R_1}| = 8$ cm

$\vec{R_2}$ from q_2 to q_5

$\vec{R_2} = (-4\vec{a_x} - 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z})$ cm

$|\vec{R_2}| = 8$ cm

$\vec{R_3}$ from q_3 to q_5

$\vec{R_3} = (4\vec{a_x} - 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z})$ cm

$|\vec{R_3}| = 8$ cm

$\vec{R_4}$ from Q_4 to Q_5

$$\vec{R_4} = (4\vec{a_x} + 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z})\,\text{cm}$$

$$|\vec{R_4}| = 8\text{ cm}$$

Force on Q_5 due to Q_1, Q_2, Q_3 and Q_4

$$\vec{F} = \frac{Q_1 Q_5}{4\pi\varepsilon|\vec{R_1}|^3}\cdot\vec{R_1} + \frac{Q_2 Q_5}{4\pi\varepsilon|\vec{R_2}|^3}\cdot\vec{R_2} + \frac{Q_3 Q_5}{4\pi\varepsilon|\vec{R_3}|^3}\cdot\vec{R_3} + \frac{Q_4 Q_5}{4\pi\varepsilon|\vec{R_4}|^3}\cdot\vec{R_4}$$

$$\varepsilon = \varepsilon_0$$

$$\vec{F} = \frac{10\times10^{-9}\times10\times10^{-9}\times9\times10^{9}}{(8\times10^{-2})^3}\begin{bmatrix} -4\vec{a_x} + 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z} \\ -4\vec{a_x} - 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z} \\ +4\vec{a_x} - 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z} \\ +4\vec{a_x} + 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z} \end{bmatrix}\times10^{-2}$$

$$\vec{F} = \frac{100\times10^{-9}\times9}{8^3\times10^{-6}}\times4\times4\sqrt{2}\,\vec{a_z}\times10^{-2}$$

$$\vec{F} = 9\times\frac{4\times4\sqrt{2}}{8^3}\times10^{-3}\,\vec{a_z}$$

$$\vec{F} = 0.3977\times10^{-3}\,\vec{a_z}$$

Magnitude of force,

$$\vec{F} = 3.977\times10^{-4}\,\vec{a_z}\text{ N} \qquad |\vec{F}| = 4\times10^{-4}\text{ N}$$