CMR INSTITUTE OF TECHNOLOGY

USN | | | | | | | | | |

Internal Assessment Test-I									
Sub:	Electromagnetic Waves							Code:	18EC55
Date:	13/11 /2021	Duration:	90 mins	Max Marks:	50	Sem:	5th	Branch:	ECE(A,B,C,D)
Solutions									

	Marks	OBE CO	OBE RBT
1.(a) State and explain Coulomb's law in vector form.	[06]	CO1	L1

a) Coulomb's law and electric field intensity:

Static electric fields in vacuum or free space

Greeks → electricity

Dr. Gilbert → physician to Her Majesty, the Queen of England → 1600 (worked this effect)

French Army Engineer officer → Colonel Charles Coulomb
↓
experiments (Torsion balance → force exerted b/w two objects having static charge of electricity)

i) Coulomb's law:

$\boxed{\text{The force between two very small objects separated in vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the distance between them.}}$

$$\boxed{F = k\frac{Q_1 Q_2}{R^2}}$$

$Q_1 Q_2$ → positive or negative quantities of charge

k → Proportionality constant; R → separation

$$\boxed{k = \frac{1}{4\pi\epsilon_0}}$$

ϵ_0 → Permittivity of free space.

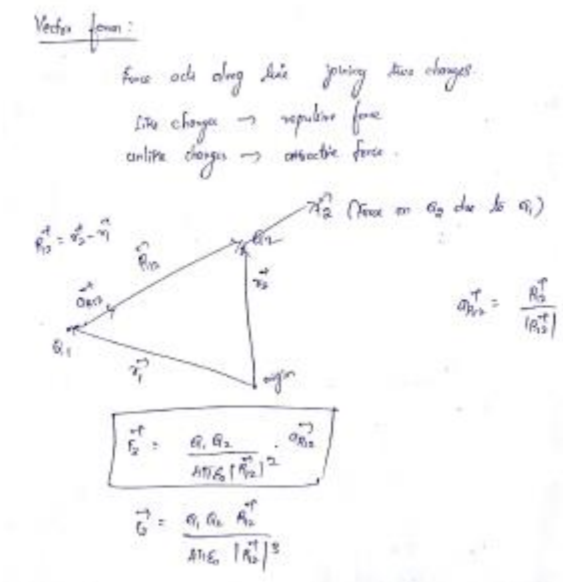
$$\epsilon_0 = \frac{10^{-9}}{36\pi} \text{ F/m.}$$

$$\epsilon_0 = 8.854\times10^{-12} \text{ F/m}$$

$$k = \frac{1}{\frac{4\pi\times10^{-9}}{36\pi}}$$

$$k = 9\times10^{9}$$

$$\boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}}$$



(b) A charge Q_A of -20μC is located at A (-6,4,7) and a second charge Q_B of 50 μC is located at B(5,8,-2). Find force on Q_B due to charge Q_A. Assume both the charges are placed in free space. [04] CO1 L3

A (−6, 4, 7)

$Q_A = -20\mu C$

B (5, 8, −2)

$Q_B = 50\,\mu C$

$\vec{R}_{AB} = (11\vec{a}_x + 4\vec{a}_y - 9\vec{a}_z)$ m

$|\vec{R}_{AB}| = \sqrt{11^2 + 4^2 + 9^2}$

$|\vec{R}_{AB}| = 14.76$ m

$\vec{R}_{BA} = -11\vec{a}_x - 4\vec{a}_y + 9\vec{a}_z$

$$\vec{F}_{BA} = \frac{1}{4\pi \times \frac{10^{-9}}{36\pi}} \times \frac{-20\times10^{-6} \times 50\times10^{-6}}{(14.76)^2} \times \frac{(11\vec{a}_x + 4\vec{a}_y - 9\vec{a}_z)}{(14.76)}$$

$$\vec{F}_{BA} = (-30.78\,\vec{a}_x - 11.195\,\vec{a}_y + 25.189\,\vec{a}_z) \times 10^{-3}\ \text{N}$$

$$\vec{F}_{BA} = (-30.78\,\vec{a}_x - 11.195\,\vec{a}_y + 25.189\,\vec{a}_z)\ \text{mN}$$

2. Define electric field intensity. Obtain an expression for electric field intensity due to an infinitely long uniform line charge distribution. [02+08] CO1 L1

Electric - field Intensity

- consider ~~the~~ one charge fixed in position say Q_1
- we move a charge slowly around

There exists a force everywhere on this 2nd charge. i.e. force field.

- let the 2nd charge be Q_t.

Then the force on it is.

$$\vec{F}_t = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{1t}^2}\,\hat{a}_{1t}$$

$\therefore$ Force per unit charge:

$$\frac{F_t}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2}\,\hat{a}_{1t}.$$

R.H.S. f^n of Q_1 and the directed line segment from Q_1 to the position of the test charge.

- This describes a vector field and is called the electric field intensity.

Def^n: Electric field intensity is the vector force on a unit +ve test charge when the charge placed in a electric field.

$\boxed{\text{Unit: } N/C}$

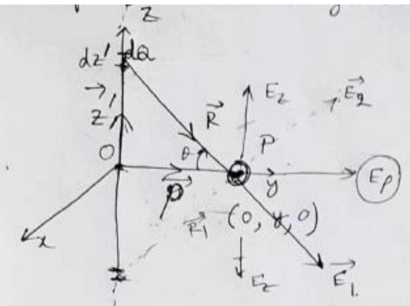
But $V = \frac{J}{C} = \frac{N.m}{C}$.

$$\Rightarrow \frac{V}{m} = \frac{N}{C}$$

$\therefore$ E practically expressed in $\frac{V}{m}$.

Finally, $\boxed{\vec{E} = \frac{F_t}{Q_t}}$

$$\boxed{\vec{E} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2}\,\hat{a}_{1t}}$$



$\frac{z'}{\rho} = \tan\theta$

$z' = \rho\tan\theta$

ρ_L C/m

Find $\vec{E}$ at $P(0, y, 0)$ because of the line charge along z-axis.

We consider incremental charge

$dQ = \rho_L\, dz'$ . . . ① [charge on length dz']

$\vec{R} = \vec{\rho} - \vec{z}'$ $\quad (\vec{z}' + \vec{R} = \vec{\rho})$

$\vec{R} = (\rho\hat{a}_\rho - z'\hat{a}_z)$ . . ② $\quad |\vec{R}| = \sqrt{\rho^2 + z'^2}$

∴ The field at P, because of dQ,

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 |\vec{R}|^2}\hat{a}_R = \frac{\rho_L\, dz'}{4\pi\epsilon_0(\rho^2 + z'^2)} \cdot \frac{\rho\hat{a}_\rho - z'\hat{a}_z}{(\rho^2 + z'^2)^{1/2}}$$

$[\because |\vec{R}| = \sqrt{\rho^2 + z'^2}]$

$$= \frac{\rho_L\, dz' \cdot (\rho\hat{a}_\rho - z'\hat{a}_z)}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$$

We know from the symmetry of the problem - ③

∴ The field at P

$$\vec{E} = \int d\vec{E} = \int_{z' \to -\infty}^{\infty} \frac{\rho_L\, dz' \rho \hat{a}_\rho}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}\hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0}\int_{-\infty}^{\infty} \frac{dz'}{(\rho^2 + z'^2)^{3/2}}\hat{a}_\rho$$

Note $\tan\theta = \frac{z'}{\rho}$

$z' = \rho\tan\theta$ $\quad$ $z' \to \infty,\ \theta = \pi/2$

$dz' = \rho\sec^2\theta\, d\theta$ $\quad$ $z' \to -\infty,\ \theta = -\pi/2$

$$\therefore \vec{E} = \frac{\rho_L \rho}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2} \frac{\rho\sec^2\theta\, d\theta}{(\rho^2 + \rho^2\tan^2\theta)^{3/2}}\hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2} \frac{\rho\sec^2\theta\, d\theta}{(\rho^2)^{3/2}(1 + \tan^2\theta)^{3/2}}\hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2} \frac{\rho\sec^2\theta\, d\theta}{\rho^3(\sec^2\theta)^{3/2}}\hat{a}_\rho$$

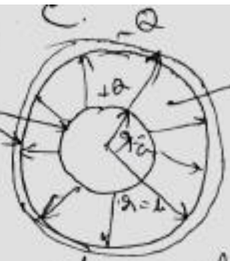
$$= \frac{\rho_L}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta\, d\theta}{\sec^3\theta}\hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta}\, d\theta\, \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta\, \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0}[\sin\theta]_{-\pi/2}^{\pi/2}\hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0 \rho}[1 + 1]\hat{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho}\hat{a}_\rho$$

3.(a) Define electric flux density. Derive the relation between electric flux density and electric field intensity. [04] CO2 L1

Ψ unit $\rightarrow$ C.



At the surface of the inner sphere, Ψ C of electric flux are produced by charge Q.

charge on inner sphere = Q

" " outer " = $-Q$.

Surface area of inner sphere = $4\pi a^2$ m^2.

$\therefore$ Density of the flux at this surface is

$$\frac{\Psi}{4\pi a^2} \text{ or } \frac{Q}{4\pi a^2} \; C/m^2.$$

Electric flux density = $\vec{D}$.

Also called displacement flux density or displacement density.

The direction of D at a pt. is the direction of the flux lines at that point, and the mag. is given by the no. of flux lines crossing a surface normal to the lines divided by the surface area

$$\therefore \vec{D}\Big|_{r=a} = \frac{Q}{4\pi a^2}\,\hat{a}_r \quad \text{(inner sphere)}$$

$$\vec{D}\Big|_{r=b} = \frac{-Q}{4\pi b^2}\,\hat{a}_r \quad \text{(outer sphere)}.$$

$a \le r \le b$

$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r$$

Shrink inner sphere, smaller and smaller. we reach point charge Q

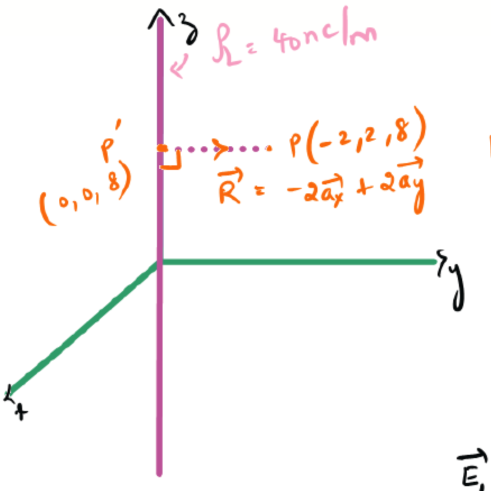
$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r .$$

Q lines of flux are symmetrically directed outward from the pt. and pass through an imaginary spherical surface of area $4\pi r^2$.

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r .$$

$\therefore$ $\boxed{D = \epsilon_0 E}$ $\rightarrow$ free space.

3. (b) i. A uniform line charge of infinite length with ρ_L = 40 nC/m, lies along the z-axis. Find **E** at (-2,2,8) in air. [06] CO2 L3



$$|\vec{R}| = \sqrt{4+4} = \sqrt{8}$$

$$\vec{E_L} = \frac{\rho_L}{2\pi\epsilon\rho}\,\vec{a_\rho}$$

$$\vec{E_L} = \frac{40\times10^{-9} \times 18\times10^{9} \times (-2\vec{a_x} + 2\vec{a_y})}{(\sqrt{8})^2}$$

$$\vec{E_L} = \frac{40\times18\times2}{8}(-\vec{a_x} + \vec{a_y})$$

$$\boxed{\vec{E_L} = -180\,\vec{a_x} + 180\,\vec{a_y} \ \text{V/m}}$$

ii. Calculate **E** and **D** in rectangular coordinates at point P(2,−3, 6) produced by a point charge Q_A = 55 mC at A(−2, 3,−6).

Calculate $\vec{D}$ in rectangular co-ordinates at point P(2,−3, 6) produced by a point charge $Q_A = 55\,mC$ at A (−2, 3, −6)

55 mC A (−2, 3, −6) → P (2, −3, 6)

$$\vec{R} = 4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z$$

$$|\vec{R}| = 14$$

$$\therefore \vec{D} = \frac{55 \times 10^{-3}}{4\pi \times 14^2} \cdot \frac{4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z}{14}$$

$$= .0016 \times 10^{-3}(4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z)$$

$$= (.0064\hat{a}_x - .0096\hat{a}_y + .0192\hat{a}_z) \times 10^{-3}$$

$$= (6.4\hat{a}_x - 9.6\hat{a}_y + 19.2\hat{a}_z)\,\mu C/m^2$$

$$= (6.4\hat{a}_x - 9.6\hat{a}_y + 19.2\hat{a}_z) \times 10^{-6}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{1}{8.854 \times 10^{-12}}(6.4\hat{a}_x - 9.6\hat{a}_y + 19.2\hat{a}_z) \times 10^{-6}$$

$$= (0.72\hat{a}_x - 1.08\hat{a}_y + 2.168\hat{a}_z) \times 10^{6}\ V/m$$

4.(a) Transform the vector **B** = y $\mathbf{a}_x$ – x $\mathbf{a}_y$ + z $\mathbf{a}_z$ into cylindrical coordinates. [05] CO1 L3

$$\vec{B} = y\vec{a_x} - x\vec{a_y} + z\vec{a_z},$$

$$B_x = y = \rho\sin\phi$$

$$B_y = -x = -\rho\cos\phi$$

$$B_z = z = z$$

$$\begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho\sin\phi \\ -\rho\cos\phi \\ z \end{bmatrix}$$

$$\vec{B} = B_\rho\vec{a_\rho} + B_\phi\vec{a_\phi} + B_z\vec{a_z}$$

$$B_\rho = \cos\phi\,\rho\sin\phi - \sin\phi\,\rho\cos\phi + 0 = 0$$

$$B_\phi = -\rho\sin^2\phi - \rho\cos^2\phi + 0 = -\rho(\sin^2\phi + \cos^2\phi)$$

$$B_\phi = -\rho$$

$$B_z = 0 + 0 + z = z$$

$$\boxed{\vec{B} = -\rho\,\vec{a_\phi} + z\,\vec{a_z}}$$

(b) i. Give the rectangular coordinates of the point $C(\rho = 4.4, \varphi = -115°, z = 2)$. [05] CO1 L3

$C(\rho = 4.4,\ \phi = -115,\ z = 2)$

$C(x = ?,\ y = ?,\ z = ?)$

$x = \rho\cos\phi\ ;\quad y = \rho\sin\phi\ ;\quad z = z$

$x = -1.859\ ;\quad y = -3.987\ ;\quad z = 2$

Ans : $C(-1.859,\ -3.987,\ 2)$

ii. Give the cylindrical coordinates of the point $D\ (x = -3.1, y = 2.6, z = -3)$.

$$\rho = \sqrt{x^2+y^2} = \sqrt{(-3.1)^2+(2.6)^2} = 4.05$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2.6}{-3.1}\right) = -39.98 + 180° = 140°$$

z = -3

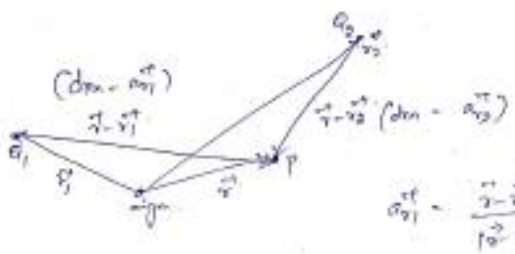
5.(a) Derive the expression for the electric field intensity at a point due to n number of point charges. [04] CO1 L2

since Coulomb's force is linear,

$\vec{E}$ due to two point charges, Q_1 at $\vec{r_1}$ and Q_2 at $\vec{r_2}$

Sum of forces on Q_t caused by Q_1 and Q_2 acting alone

$$\vec{E}(r) = \frac{Q_1}{4\pi\epsilon_0 |\vec{r}-\vec{r_1}|^2} \cdot \vec{a_{r1}} + \frac{Q_2}{4\pi\epsilon_0 |\vec{r}-\vec{r_2}|^2} \cdot \vec{a_{r2}}$$

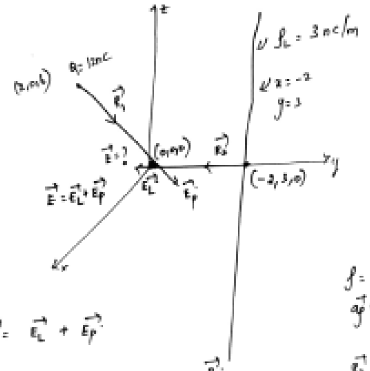


$$\vec{a_{r1}} = \frac{\vec{r}-\vec{r_1}}{|\vec{r}-\vec{r_1}|} \quad \& \quad \vec{a_{r2}} = \frac{\vec{r}-\vec{r_2}}{|\vec{r}-\vec{r_2}|}.$$

'n' number of charges

$$\boxed{\vec{E}(r) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0 |\vec{r}-\vec{r_m}|^2} \cdot \vec{a_{rm}}}\ \ \mathrm{V/m\ or\ N/C}$$

(b) Find **E** at origin due to a point charge 12nC at (2, 0, 6) and a uniform line charge 3nC/m at x = -2, y = 3. [06] CO1 L3



$\vec{R_1} = -2\vec{a_x} - 6\vec{a_z}$

$|\vec{R_1}| = \sqrt{4+36} = \sqrt{40}$

$\rho = |\vec{R_2}|$

$\vec{a_\rho} = \vec{a_{R_2}} = \frac{\vec{R_2}}{|\vec{R_2}|}$

$\vec{R_2} = 2\vec{a_x} - 3\vec{a_y}$

$|\vec{R_2}| = \sqrt{4+9} = \sqrt{13}$

$\vec{E} = \vec{E_L} + \vec{E_P}$

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho}\vec{a_\rho} + \frac{Q}{4\pi\epsilon_0 |\vec{R_1}|^2}\vec{a_{R_1}}$$

$$= \frac{3\times10^{-9}\times(2\vec{a_x} - 3\vec{a_y})}{2\pi\times\epsilon_0\times\sqrt{13}\times\sqrt{13}} + \frac{12\times10^{-9}}{4\pi\times\epsilon_0}\times\frac{(-2\vec{a_x} - 6\vec{a_z})}{(\sqrt{40})^3}$$

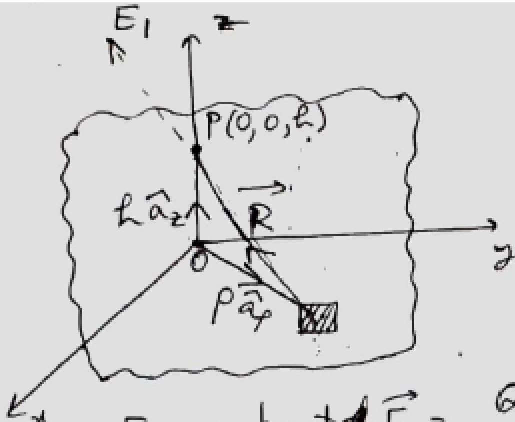
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{3\times10^{-9}\times2\,(2\vec{a_x} - 3\vec{a_y})}{13} + \frac{12\times10^{-9}\,(-2\vec{a_x} - 6\vec{a_z})}{(\sqrt{40})^3}\right]$$

$$= 9\times10^{9}\times6\times10^{-9}\left[\frac{2}{13}\vec{a_x} - \frac{3}{13}\vec{a_y} - \frac{4}{(\sqrt{40})^3}\vec{a_x} - \frac{12}{(\sqrt{40})^3}\vec{a_z}\right]$$

$$= 54\left[0.138\,\vec{a_x} - 0.230\,\vec{a_y} - 0.0474\,\vec{a_z}\right]$$

$$\boxed{\vec{E} = 7.452\,\vec{a_x} - 12.42\,\vec{a_y} - 2.5596\,\vec{a_z}} \text{ V/m}$$

6. Define surface charge density. Obtain an expression of electric field intensity due to an infinite sheet of charge with uniform surface charge distribution ρ_s C/m^2. Assume the charge is placed over x-y plane. [02+08] CO1 L2



Surface charge density,

$$\rho_s = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s} \quad C/m^2$$

For a point charge, $\vec{E} = \dfrac{Q}{4\pi\epsilon_0 |\vec{R}|^2}\hat{a}_R$

1) $dQ = ??$

2) $\vec{R} = ??$ $\longrightarrow$ $\vec{\rho} + \vec{R} = h\hat{a}_z$

$$\vec{R} = h\hat{a}_z - \rho\hat{a}_\rho$$

3) $\hat{a}_R = ??$

4) ~~Varied~~ Symmetry of problem

5) Integration

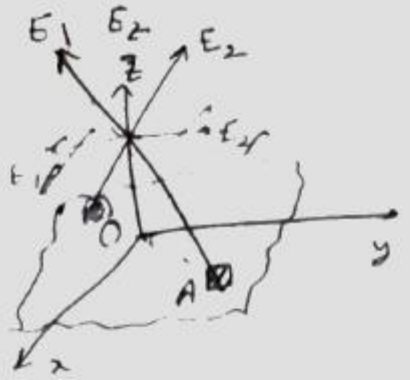
uniform charge density ρ_s C/m^2

$$dQ = \rho_s . |\vec{dA}| = \rho_s . (\rho\, d\rho\, d\phi)$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{(h\hat{a}_z - \rho\hat{a}_\rho)}{\sqrt{h^2+\rho^2}}$$

∴ The intensity at P due to the chosen infinitesimal surface area,

$$\vec{dE} = \frac{\rho_s . (\rho\, d\rho\, d\phi)}{4\pi\epsilon_0 (h^2+\rho^2)} \frac{(h\hat{a}_z - \rho\hat{a}_\rho)}{\sqrt{h^2+\rho^2}} \quad \text{(1)}$$



From symmetry of the problem,

$$\vec{dE} = \frac{dQ}{4\pi\epsilon_0(\rho^2+h^2)} \cdot \frac{h\,\hat{a}_z}{\sqrt{\rho^2+h^2}}$$

$$\therefore \vec{dE} = \frac{\rho_s.\,(\rho\, d\rho\, d\phi)\,(h\,\hat{a}_z)}{4\pi\epsilon_0(\rho^2+h^2)^{3/2}} \quad \text{--- (4)}$$

$\therefore$ The total electric field intensity at P,

$$\vec{E} = \iint \vec{dE} = \frac{\rho_s.h}{4\pi\epsilon_0}\int_{\rho=0}^{\infty}\int_{\phi=0}^{2\pi}\frac{\rho.d\rho.d\phi}{(\rho^2+h^2)^{3/2}}\,\hat{a}_z$$

$$= \frac{\rho_s.h}{4\pi\epsilon_0}\int_{\rho=0}^{\infty}\frac{\rho\, d\rho}{(\rho^2+h^2)^{3/2}}\int_{\phi=0}^{2\pi} d\phi \;\; \hat{a}_z$$

$$= \frac{\rho_s.h}{2\epsilon_0}\int_{\rho=0}^{\infty}\frac{\rho\, d\rho}{(\rho^2+h^2)^{3/2}}\;\hat{a}_z$$

Let, $\rho = h\tan\theta$ | when $\rho=0,\ \theta=0$

$d\rho = h\sec^2\theta\, d\theta$ | when $\rho\to\infty,\ \theta=\pi/2$

$$\vec{E} = \frac{\rho_s.h}{2\epsilon_0}\int_0^{\pi/2}\frac{(h\tan\theta).\,h\sec^2\theta\, d\theta}{(h^2+h^2\tan^2\theta)^{3/2}}\;\hat{a}_z$$

$$= \frac{\rho_s.h}{2\epsilon_0}\int_0^{\pi/2}\frac{h^2\tan\theta\,\sec^2\theta\, d\theta}{h^3.\sec^2\theta\,\sec\theta}\;\hat{a}_z$$

$$= \frac{\rho_s}{2\epsilon_0} \int_0^{\pi/2} \tan\theta \cdot \frac{1}{\sec\theta} \, d\theta \, \hat{a}_z$$

$$= \frac{\rho_s}{2\epsilon_0} \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta} \cdot \cos\theta \, d\theta \, \hat{a}_z$$

$$= \frac{\rho_s}{2\epsilon_0} \int_0^{\pi/2} \sin\theta \, d\theta \, \hat{a}_z$$

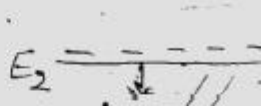
$$= \frac{\rho_s}{2\epsilon_0} \left[-\cos\theta\right]_0^{\pi/2} \hat{a}_z = \frac{\rho_s}{2\epsilon_0} \hat{a}_z$$

i.e. $\boxed{\vec{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_z}$ V/m.

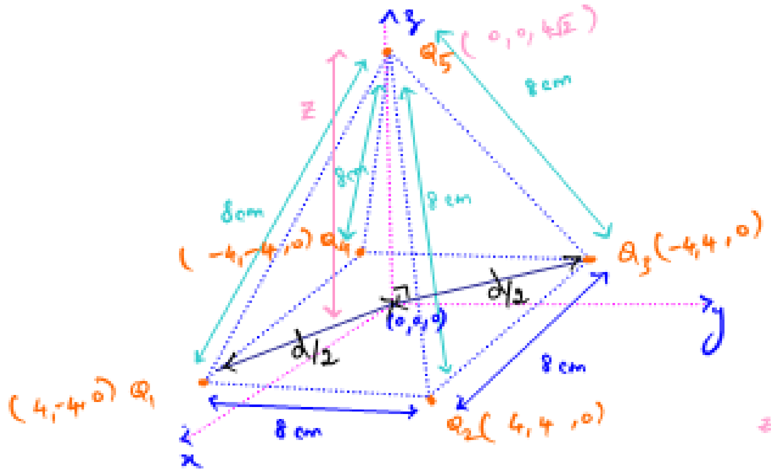
Unit vector along a normal to a surface, is given as, $\hat{a}_n$

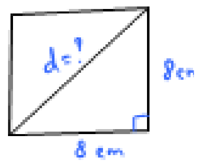
$\therefore \vec{E}$ in general can be given as

$$\boxed{\vec{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_n} \text{ V/m.}$$



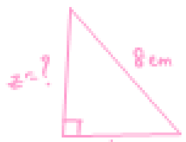
7. Four 10 nC positive charges are located in z=0 plane at the corners of a square 8 cm. on a side. A fifth 10 nC charge is located at a point 8 cm. distant from other charges. Calculate the magnitude of total force on this fifth charge for $\varepsilon = \varepsilon_0$. [10] CO1 L3





$d = \sqrt{8^2 + 8^2}$

$d = 8\sqrt{2}$ cm



$\frac{d}{2} = 4\sqrt{2}$ cm

$8^2 = (4\sqrt{2})^2 + z^2$

$z = \sqrt{8^2 - (4\sqrt{2})^2}$

$z = 4\sqrt{2}$ cm

$\vec{R_1}$ from q_1 to Q_5

$\vec{R_1} = (-4\vec{a_x} + 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z})$ cm

$|\vec{R_1}| = 8$ cm

$\vec{R_2}$ from Q_2 to Q_5

$\vec{R_2} = (-4\vec{a_x} - 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z})$ cm

$|\vec{R_2}| = 8$ cm

$\vec{R_3}$ from Q_3 to Q_5

$\vec{R_3} = (4\vec{a_x} - 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z})$ cm

$|\vec{R_3}| = 8$ cm

$\vec{R_4}$ from Q_4 to Q_5

$$\vec{R_4} = (4\vec{a_x} + 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z})\,\text{cm}$$

$$|\vec{R_4}| = 8\,\text{cm}$$

Force on Q_5 due to Q_1, Q_2, Q_3 and Q_4

$$\vec{F} = \frac{Q_1 Q_5}{4\pi\varepsilon|\vec{R_1}|^3}\cdot\vec{R_1} + \frac{Q_2 Q_5}{4\pi\varepsilon|\vec{R_2}|^3}\cdot\vec{R_2} + \frac{Q_3 Q_5}{4\pi\varepsilon|\vec{R_3}|^3}\cdot\vec{R_3} + \frac{Q_4 Q_5}{4\pi\varepsilon|\vec{R_4}|^3}\cdot\vec{R_4}$$

$\varepsilon = \varepsilon_0$

$$\vec{F} = \frac{10\times10^{-9}\times10\times10^{-9}\times9\times10^{9}}{(8\times10^{-2})^3}\begin{bmatrix} -4\vec{a_x} + 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z} \\ -4\vec{a_x} - 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z} \\ +4\vec{a_x} - 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z} \\ +4\vec{a_x} + 4\vec{a_y} + 4\sqrt{2}\,\vec{a_z} \end{bmatrix}\times10^{-2}$$

$$\vec{F} = \frac{100\times10^{-9}\times9}{8^3\times10^{-6}}\times4\times4\sqrt{2}\,\vec{a_z}\times10^{-2}$$

$$\vec{F} = 9\times\frac{4\times4\sqrt{2}}{8^3}\times10^{-3}\,\vec{a_z}$$

$$\vec{F} = 0.3977\times10^{-3}\,\vec{a_z}$$

$$\vec{F} = 3.977\times10^{-4}\,\vec{a_z}\ \text{N}$$

Magnitude of force,

$$|\vec{F}| = 4\times10^{-4}\ \text{N}$$