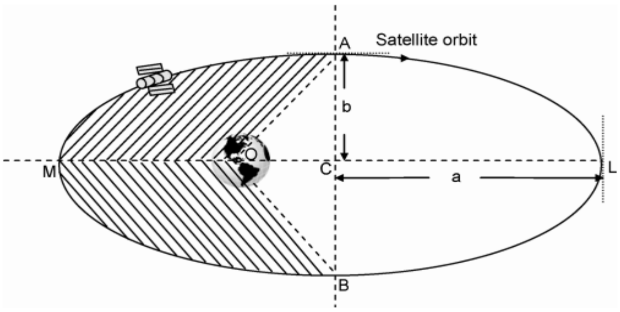
CMR INSTITUTE OF TECHNOLOGY	USN										

Internal Assesment Test - I

Sub:	Satellite Communication							Code:	15EC755
Date:	12.11.21	Duration:	90 mins	Max Marks:	50	Sem:	VII	Branch:	ECE-A,B,C,D

Answer Any FIVE FULL Questions

		Marks	OBE	
			CO	RBT
1.	With neat figures explain Injection velocity and satellite Trajectories.	[10]	CO1	L2
2.	An earth station is located at 30° W longitude and 60° N latitude. Determine the Earth station's azimuth and elevation angles with respect to a geostationary satellite located at 50°W longitude. The orbital radius is 42164 Km. (Assume Earth's radius is 6378 Km)	[10]	CO1	L3
3.	A satellite is launched with an injection velocity v1 from a point above the surface of the earth at a distance P from the centre of the earth attains an elliptical orbit with an apogee distance A1. The same satellite when launched with an injection velocity v2 from the same perigee distance attains an elliptical orbit with an apogee distance A2. Derive the relationship between v1 and v2 in terms of P, A1 and A2.	[10]	CO1	L3
4.	Define the following Orbital Parameters with relevant diagrams. – Right Ascension of Ascending Node – Apogee , Perigee and Eccentricity	[10]	CO1	L1
5.	Explain Kepler's laws of Planetary motion with necessary equations.	[10]	CO1	L2
6.	Mention functions carried by different subsystems of a typical satellite.	[10]	CO1	L1
7.	a) Satellite A is orbiting Earth in a circular orbit of radius 7000 km. Satellite B is orbiting Earth in an elliptical orbit with its apogee and perigee distances of 47000 km and 7000 km respectively. Determine velocities of two satellites at point X. Take $\mu=39.8\times10^{13}$ Nm^2/kg. b) The apogee and perigee distances of a satellite orbiting in an elliptical orbit are , respectively 45000 km and 7000 km .Determine the following- a) semi major axis of the elliptical orbit b) Orbit eccentricity	[5+5]	CO1	L3
8.	The satellite is moving in an elliptical orbit with its semi-major and semi-minor axes as a and b respectively and an eccentricity of 0.5. The satellite takes 2 hours to move from point B to point A. How much time will it take to move from point A to point B? 	10	CO1	L3

1) Injection velocity and satellite Trajectory

$$v = \sqrt{\frac{2\mu}{P} - \frac{2\mu}{A+P}}$$

It is found with 3 critical velocity.

- when the orbit is a ~~~~ circular with the apogee distance equal to the perigee distance, the the first ~~injection~~ critical velocity becomes

$$v_1 = \sqrt{\frac{\mu}{P}}$$

- when the injection velocity is less than the 1st critical velocity

$$v < \sqrt{\frac{\mu}{P}}$$

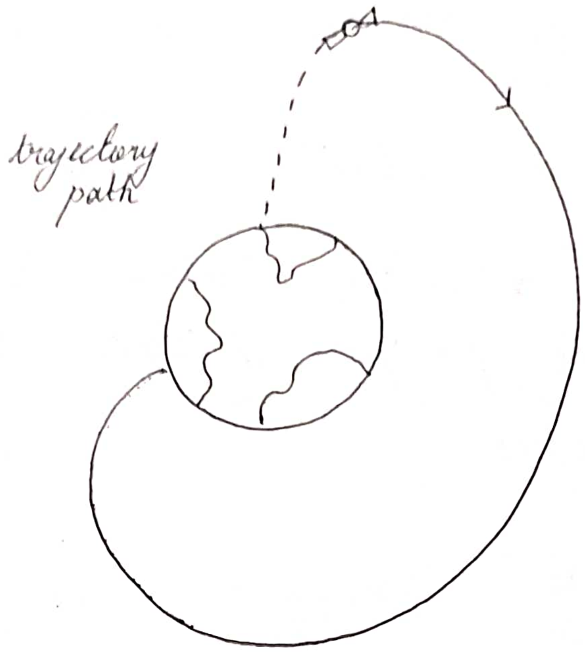
then the satellite follows a ballistic trajectory path and it falls back to the surface of Earth

- when the injection velocity is greater than first critical velocity and less than second critical velocity.

$$v > \sqrt{\frac{\mu}{P}} \quad \text{and} \quad v < \sqrt{\frac{2\mu}{P}}$$

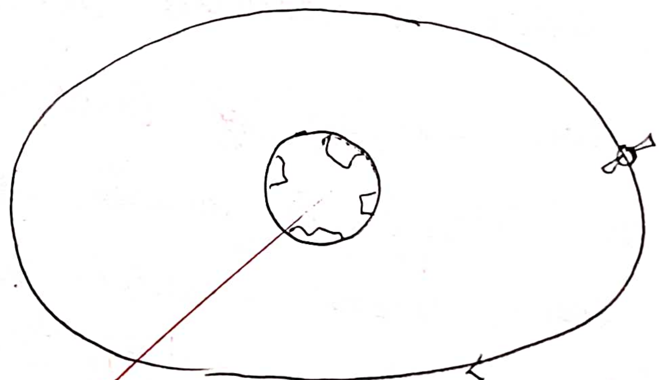
then the orbit is elliptical and eccentric

- when injection velocity is equal to the second critical velocity, then the satellite follows a Parabolic path and escapes the solar system when $v < \sqrt{\frac{\mu}{p}}$



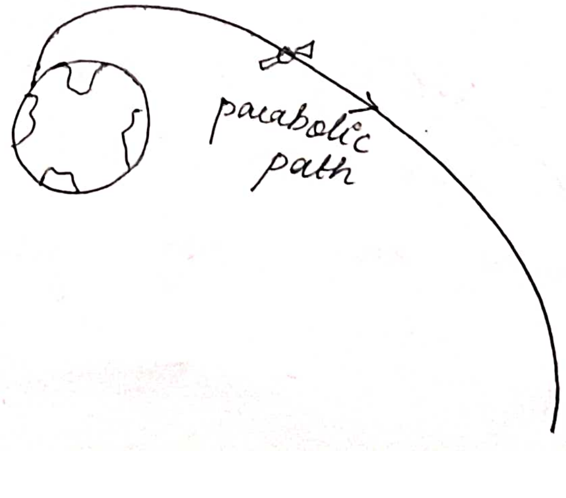
The satellite falls back to the surface of the Earth.

when $\sqrt{\frac{\mu}{p}} < v < \sqrt{\frac{2\mu}{p}}$



The orbit is eccentric and elleptical and the satellite moves around the Earth

when $v < \sqrt{\frac{2\mu}{p}}$



The satellite escapes the solar system.

2)

$\theta_l = 60^\circ N$

$\theta_L = 30^\circ W$

$\theta_S = 50^\circ$

$r = 42164 \text{ km}$

$R = 6378 \text{ km}$

Azimuth angle $= 180^\circ + A'$

$$A' = \tan^{-1}\left(\frac{\tan|\theta_S - \theta_L|}{\sin\theta_l}\right)$$

$$= \tan^{-1}\left(\frac{\tan 20^\circ}{\sin 60^\circ}\right)$$

$$= \tan^{-1}(0.4202)$$

$$= 22.795^\circ$$

$\therefore$ Azimuth angle $= 180 + 22.795$

$$= \underline{202.795^\circ}$$

$$E = \tan^{-1}\left(\frac{r - R\cos\theta_e \cos|\theta_s - \theta_L|}{R\sin\left(\cos^{-1}\left(\cos\theta_e \cos|\theta_s - \theta_L|\right)\right)}\right) - \cos^{-1}\left(\cos\theta_e \cos|\theta_s - \theta_L|\right)$$

$$= \tan^{-1}\left(\frac{42164 - 6378\cos 60 \cos 20}{6378 \sin\left(\cos^{-1}\left(\cos 60 \cos 20\right)\right)}\right) - \cos^{-1}\left(\cos 60 \cos 20\right)$$

$$= \tan^{-1}\left(\frac{42164 - 6378(0.4698)}{6378 \sin\left(\cos^{-1}(0.4698)\right)}\right) - \cos^{-1}(0.4698)$$

$$= \tan^{-1} \frac{42164 - 2996.384}{6378 \sin(61.9786)} - 61.9786$$

$$= \tan^{-1}\left(\frac{39167.616}{5630.32}\right) - 61.9786$$

$$= 81.82088 - 61.9786$$

$$= \underline{19.8422^{\circ}}$$

3.)

$$V1 = \sqrt{\frac{2\mu}{P} - \frac{2\mu}{A_1 + P}}$$

$$V1 = \sqrt{2\mu\left(\frac{1}{P} - \frac{1}{A_1 + P}\right)}$$

$$V2 = \sqrt{\frac{2\mu}{P} - \frac{2\mu}{A2 + P}}$$

$$= \sqrt{2\mu\left(\frac{1}{P} - \frac{1}{A2 + P}\right)}$$

Taking ratio $\frac{V2}{V1}$ and squaring both sides.

$$V1^2 = 2\mu\left(\frac{1}{P} - \frac{1}{A_1 + P}\right)$$

$$V_2^2 = 2\mu\left(\frac{1}{P} - \frac{1}{A2 + P}\right)$$

$$\left(\frac{V_2}{V_1}\right)^2 = \frac{1/P - \frac{1}{A2 + P}}{1/P - \frac{1}{A1 + P}}$$

$$= \frac{\dfrac{A_2 + P - P}{P(A_2+P)}}{\dfrac{A_1 + P - P}{P(A_1+P}}$$

$$= \frac{\dfrac{A_2}{P(A_2+P)}}{\dfrac{A_1}{P(A_1+P)}}$$

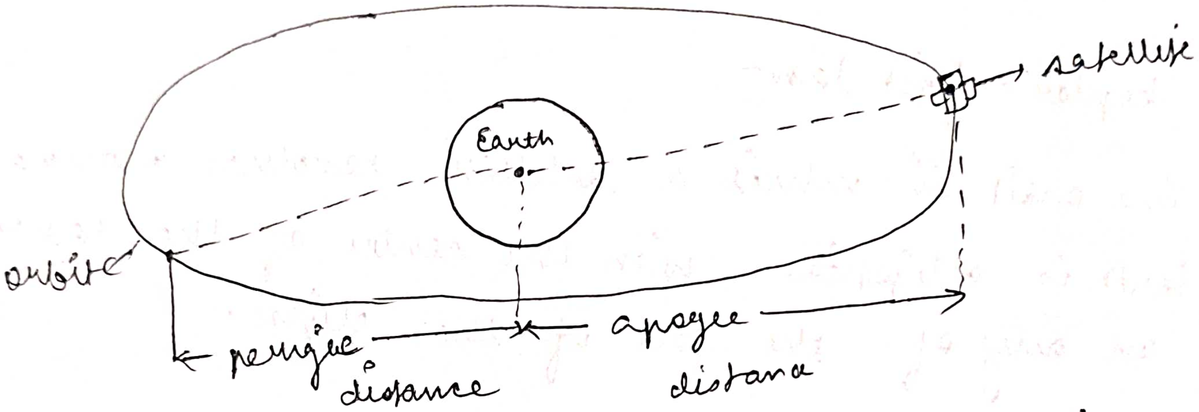
$$= \frac{A_2}{P(A_2+P)} \times \frac{P(A_1+P)}{A_1}$$

$$\left(\frac{V_2}{V_1}\right)^2 = \frac{PA_1A_2 + A_2P^2}{PA_1A_2 + A_1P^2}$$ divide by PA_1A_2.

$$\boxed{\left(\frac{V_2}{V_1}\right)^2 = \frac{1 + P/A_1}{1 + P/A_2}}$$

(4). Apogee – The point on the orbit by the satellite, which is farthest from the centre of the earth is called apogee. It can be calculated by the known values of perigee distance and velocity at the perigee.

Apogee distance, $A = a(1-e)$



perigee – The point on the orbit by the satellite which is nearest from the centre of the earth is called perigee.

Perigee distance, $P = a(1+e)$.

Eccentricity – Eccentricity (e) is defined as ratio of distance between centre of earth and centre of ellipse ~~and~~ to semi major axis of the ellipse is called eccentricity. It can be given by any one of the formulae –

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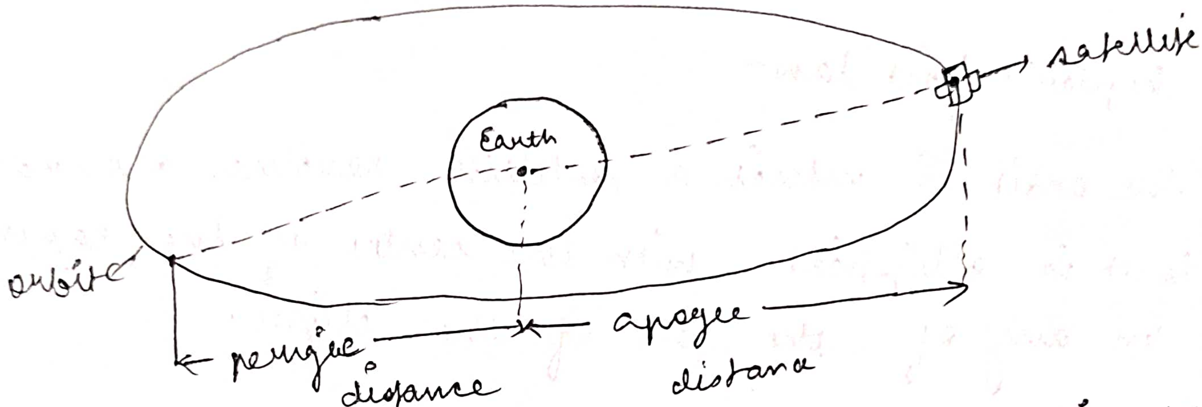

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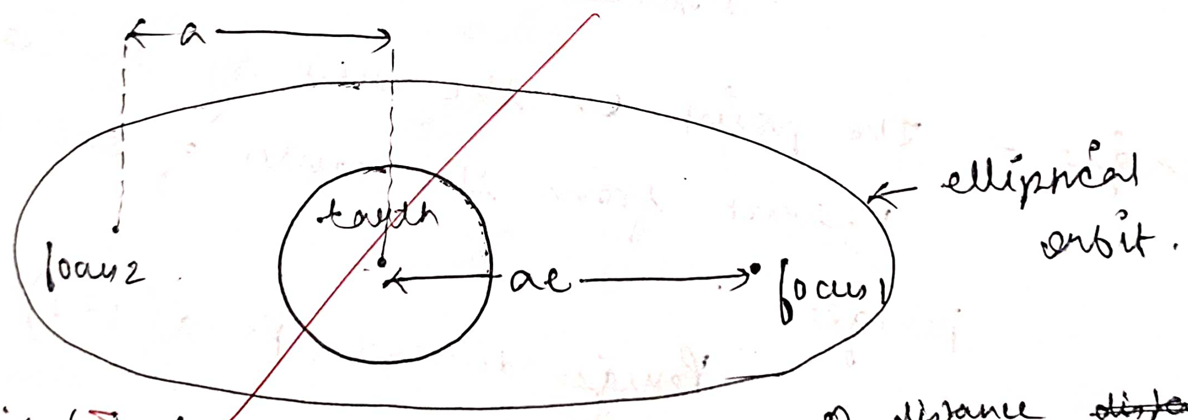
eccentricity, $e = \dfrac{\text{apogee} - \text{perigee}}{\text{apogee} + \text{perigee}}$

$$e = \frac{\text{apogee} - \text{perigee}}{2a}$$

$$e = \frac{\sqrt{a^2 - b^2}}{a}.$$

5). kepler's first law—

- The orbit in which a satellite revolves around the Earth is elliptical, with the centre of the earth lying on any of the foci of the ellipse.



eccentricity(e), is defined as the ratio of distance ~~distance~~ between centre of ellipse and any one its foci to semi-major axis of the ellipse.

The law of conservation of energy remains conserved on all points of the orbit.

In the satellite motion, energies such as kinetic energy $\left(\frac{1}{2}mv^2\right)$ and potential energy $\left(-\frac{Gm_1m_2}{r}\right)$ remains constant, which is equal to ~~$-Gm_1Gm$~~ $-\frac{Gm_1m_2}{2a}$.

$$\frac{1}{2}m_2v^2 - \frac{Gm_1m_2}{r} = \frac{-Gm_1m_2}{2a}$$

$$v^2 = Gm_1\left[\frac{2}{r} - \frac{1}{a}\right].$$

$$V = \sqrt{Gm_1\left[\frac{2}{r} - \frac{1}{a}\right]}$$

$\therefore m_1$ = mass of earth

$\therefore m_2$ = mass of satellite

$\therefore G = 6.67 \times 10^{-11}\ m^2/kg\ s^2$

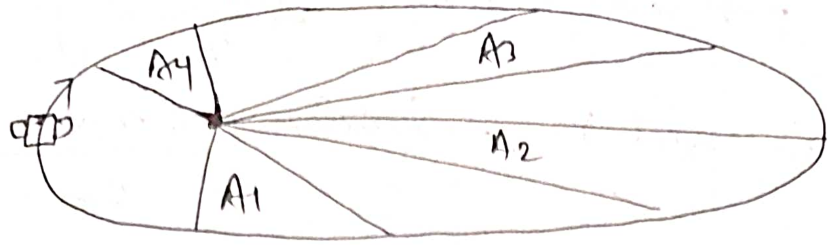
$$\boxed{V = \sqrt{\mu\left[\frac{2}{r} - \frac{1}{a}\right]}}$$

$\therefore$ $\boxed{\text{time period} = T = \frac{2\pi}{\sqrt{\mu}} r^{3/2}}$

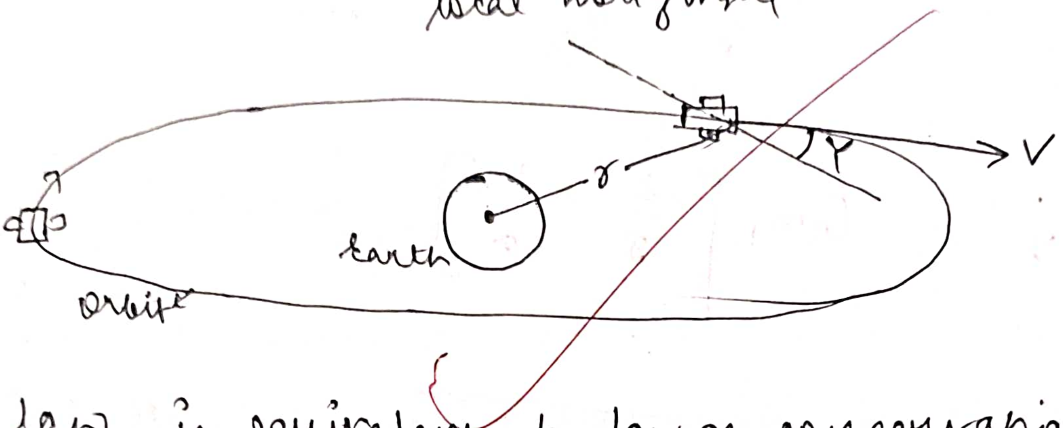
Kepler's second law

The line joining the centre of the earth and the satellite sweeps out equal areas in the plane of orbit, in equal intervals of time. So, the area swept out is given by -

$$\therefore \frac{dA}{dt} = \frac{\text{angular momentum of satellite}}{2m}.$$



Swept out area.



This law is equivalent to law of conservation of momentum which dictates that angular moment is equal to the product of radius vector and component of linear momentum of ~~total horizonta~~ perpendicular to radius vector.

$$\boxed{V_p r_p = V_a r_a = V r \cos\gamma}$$

where,

V_p = velocity at perigee

r_p = ~~dis~~ perigee distance

V_a = velocity at apogee

r_a = apogee distance.

r = ~~radii~~ distance b/w centre of earth and the satellite.

V = velocity of satellite.

Kepler's third law-

→ The square of the time-period of satellite is the cube of semi-major axis of its ellipse.

→ ~~Centrifug~~ Circular orbit with radius r is assumed.

→ Circular orbit is the special case of an elliptical orbit, where semi-major axis and semi-minor axis is equal to the semi-major axis radius.

equating gravitational and centrifugal force,

$$\frac{Gm_1 m_2}{r^2} = \frac{m_2 v^2}{r}$$

$$\Rightarrow \frac{Gm_1}{r} = v^2$$

replacing $v \to \omega r$

$$\frac{Gm_1}{r} = \omega^2 r^2$$

$$\therefore Gm_1 = \omega^2 r^3.$$

replacing $\omega \to \frac{2\pi}{T}$.

$$\frac{Gm_1}{r^3} = \frac{4\pi^2}{T^2}$$

$$\therefore T^2 = \frac{4\pi^2 r^3}{Gm_1}$$

$$\therefore \boxed{T = \frac{2\pi}{\sqrt{\mu}} r^{3/2}}$$

where, $\mu = GM$

~~$= 39.8 \times 10^{11}$~~

$= 3.98 \times 10^{11}\ Nm^2/kg$

The above equation holds good when radius is replaced with semi-major axis a -

$$\therefore \boxed{T = \frac{2\pi}{\sqrt{\mu}} a^{3/2}}$$

7

a). given,

radius of earth ~~satellite A~~ = 7000 km = 7000 000 m.

apogee, $A_{SB} = 47000\,km = 47000000\,m$.

$P_{SB} = 7000000\,m$.

$\mu = 39.8 \times 10^{13}\ N\,m^2/kg$

semi-major axis of satellite B = $a_B = \frac{\text{perigee} + \text{apogee}}{2}$

$$= \frac{47000000 + 7000000}{2}$$

$a_B = 27000000\,m$

velocity of satellite B = $V_B = \sqrt{\frac{\mu}{R}}$

$$= \sqrt{\frac{39.8 \times 10^{13}}{7000000}} = 7540.36\ m/s$$

(7)

5). Apogee distance., $A = 45000$ km

perigee distance, $P = 7000$ km.

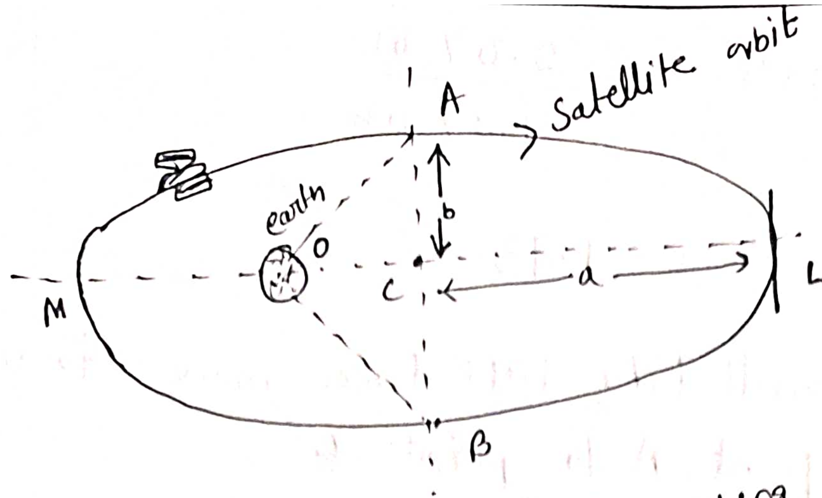
a) semi-major axis = $\frac{\text{apogee} + \text{perigee}}{2}$

$= 26000$ km

b). orbit eccentricity = $\frac{\text{apogee} - \text{perigee}}{2a}$

$\therefore \boxed{e = 0.73}$

Q8)



This question can be solved by using keplers law of area.

area covered by the sattelite while moving from point B to A is given by .

$$Area = \frac{1}{2} \times (\text{area of ellipse}) - \text{area of } \Delta\, AOB.$$

$$= \frac{1}{2}\pi ab - bae$$

$$= 1.57ab - 0.5ab$$

$$= 1.07ab$$

area covered by the satellite while moving from point A to B is given by

$$Area = \frac{1}{2} \times (\text{area of ellipse}) + \text{area of } \Delta\, AOB$$

$$= \frac{1}{2}\pi ab + bae$$

$$= 1.57ab + 0.5ab$$

$$= 2.07ab$$

ratio of Area's $= \dfrac{2.07\,ab}{1.07\,ab}$

$= 1.93$

∴ satellite will take 1.93 times more to move from point A to point B

∴ time taken $= 1.93 \times 2$

∴ $\boxed{\text{time taken} = 3.86 \text{ hours}}$