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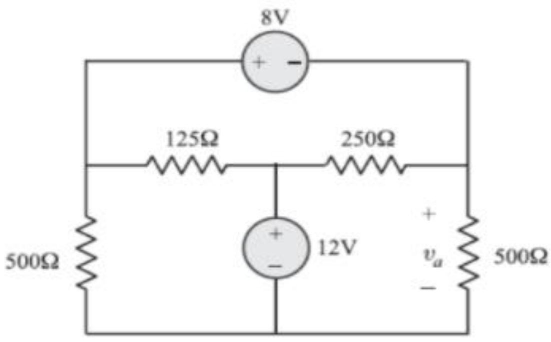
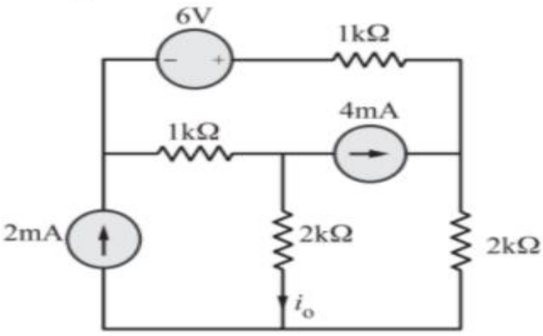
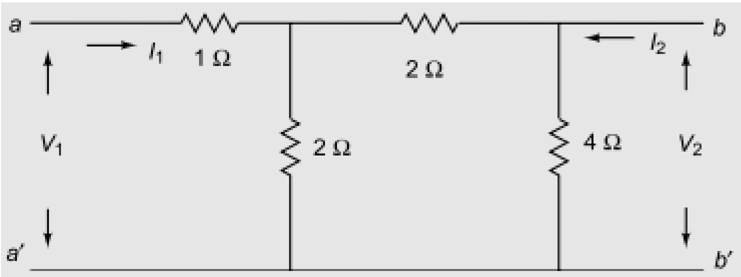
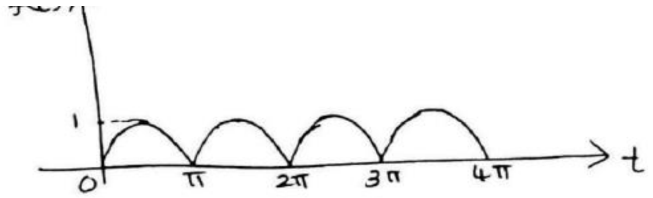
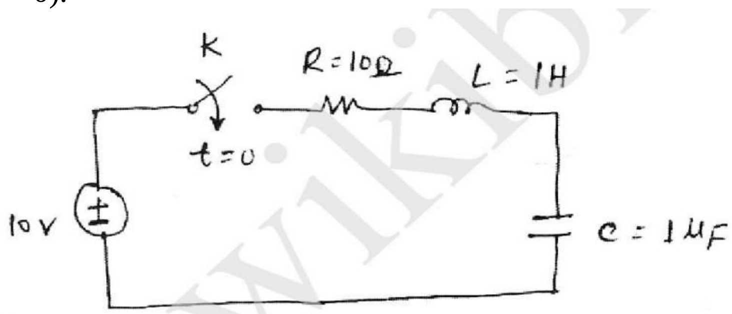
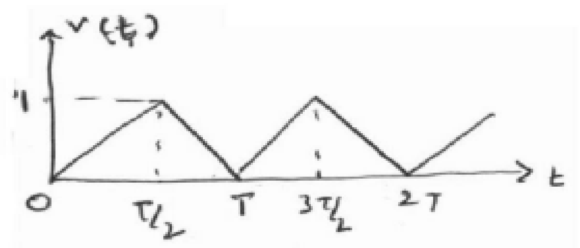
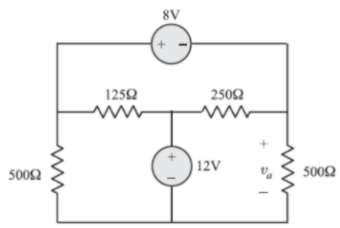
Internal Assessment Test II – Jan. 2022

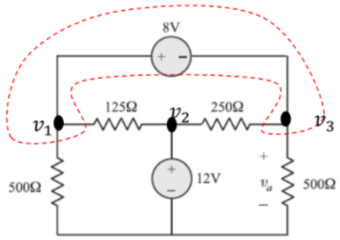
Sub:	**Network Theory**				Sub Code:	18EC32	Branch:	ECE
Date:	24-01-2022	Duration:	90 Minutes	Max Marks: 50	Sem / Sec:	**3/A,B,C,D**		**OBE**

	Answer any FIVE FULL Questions	MARKS	CO	RBT
1	Determine *Va* in the given circuit (Fig.1).  Fig.1	[10]	CO1	L3
2	Determine i_0 in the given circuit (Fig.2).  Fig.2	[10]	CO1	L3
3	For the network shown in Fig.3, obtain 'z' and 'y' parameters  Fig. 3	[10]	CO5	L3
4	(a).Obtain the relationship between 'y' and 'h' parameters	[5]	CO5	L2
	(b).Obtain the reciprocity condition for ABCD parameters.	[5]	CO5	L2

5	Derive the expression for the resonant frequency of the circuit where R_L resistance in inductor branch and R_C resistance in the capacitor branch. Also show that the circuit will resonate at all frequencies if $R_L = R_C$.	[10]	CO5	L2
6	a. Obtain the Laplace transform of (i). Unit step function, (ii). Unit impulse function. b. Find the Laplace transform of the periodic function as shown in the Fig.6  Fig.6	[4] [6]	CO4 CO4	L2 L3
7	Determine the loop current in the Fig.7 using Laplace transform when switch is closed (t = 0).  Fig. 7	[10]	CO4	L3
8	a. State and prove initial and final value theorem. b. Find the Laplace transform of the periodic function as shown in the Fig. 8.  Fig.8	[4] [6]	CO4 CO4	L2 L3

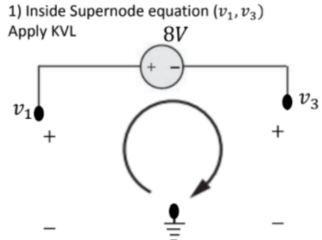
Ques 1. Find v_a in the circuit shown





$v_2 = 12V$

1) Inside Supernode equation (v_1, v_3)
Apply KVL



$v_1 - 8 - v_3 = 0$ (By KVL)

$v_1 - v_3 = 8$

2)Here v_1 and v_3 are forming super-node and $v_2 = 12V$ is forming is just an independent node

Supernode equation (v_1, v_3)

$$\left(\frac{v_1 - v_2}{125}\right) + \frac{v_1 - 0}{500} + \frac{v_3 - v_2}{250} + \frac{v_3 - 0}{500} = 0$$

$$\left(\frac{v_1 - 12}{5}\right) + \frac{v_1 - 0}{20} + \frac{v_3 - 12}{10} + \frac{v_3 - 0}{20} = 0$$

$$v_1\left(\frac{1}{5} + \frac{1}{20}\right) + v_3\left(\frac{1}{10} + \frac{1}{20}\right) = \frac{12}{5} + \frac{12}{10}$$

$$v_1\left(\frac{5}{20}\right) + v_3\left(\frac{3}{20}\right) = \frac{36}{10}$$

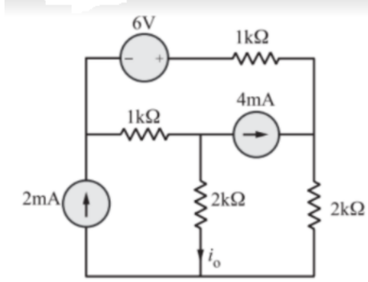
$$\Rightarrow v_1\,(0.25) + v_3(0.15) = 3.6$$

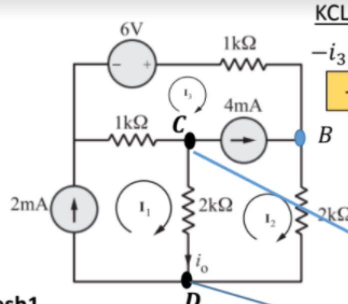
solving

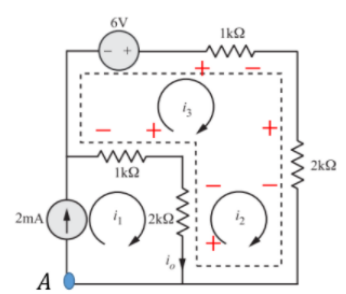
$v_1 = 12V$

$v_3 = v_a = 4V$

Ques 2. Find i_o in the circuit shown







KCL at node B

$-i_3 + i_2 - 4m = 0$

$-i_3 + i_2 = 4m$

$-i_3 + \frac{10}{3}m = 4m$

$i_3 = \frac{2}{3}mA$

KCL at node C

$4m + i_o - (i_1 - i_3) = 0$

$4m + i_o - (2m - \frac{2}{3}m) =$

$\Rightarrow i_o = \frac{-4}{3}m$

mesh1

KCL at node A

$2m - i_1 = 0$ $\Rightarrow i_1 = 2mA$

Supermesh(mesh2 and mesh3)

$6 - 1k(i_3) - 2ki_2 - 2k(i_2 - i_1) - 1k(i_3 - i_1) = 0$

$6 - 1k(i_3) - 2ki_2 - 2k(i_2 - 2m) - 1k(i_3 - 2m) = 0$

$12 - 4ki_2 - 2ki_3 = 0$ $\Rightarrow 2i_2 + i_3 = 6m$

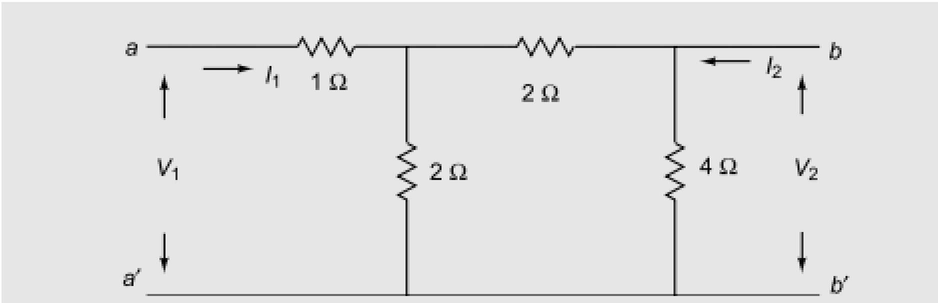
Solving

$i_2 = \frac{10}{3}mA$

KCL at node D

$-i_o + (i_1 - i_2) = 0 \Rightarrow -i_o = 2m - \frac{10}{3}m \Rightarrow i_o = \frac{-4}{3}m$

Q.NO3:



Solution $Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0}$

When b-b' is short circuited, $V_2 = 0$ and the network looks as shown in Fig. 16.8 (a).

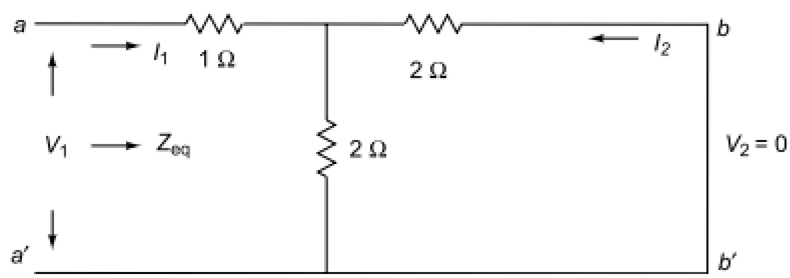


Fig. 16.8 (a)

$$V_1 = I_1 Z_{eq}$$

$$Z_{eq} = 2\ \Omega$$

$$\therefore \quad V_1 = I_1 2$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{2}\ \mho$$

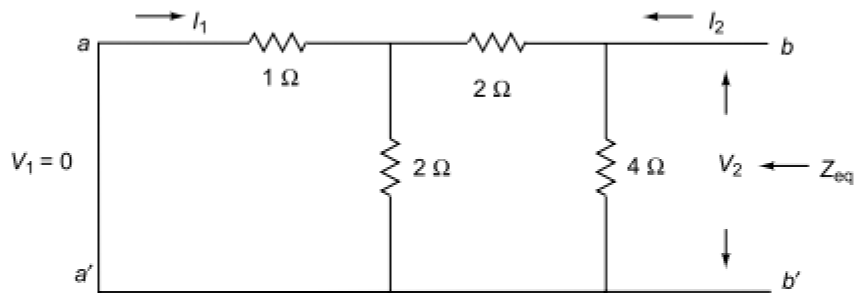
$$Y_{21} = \left.\frac{I_2}{V_2}\right|_{V_2=0}$$

With port b-b' short circuited, $-I_2 = I_1 \times \dfrac{2}{4} = \dfrac{I_1}{2}$

$$\therefore \quad -I_2 = \frac{V_1}{4}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{1}{4} \text{ ℧}$$

Similarly, when port a-a' is short circuited, $V_1 = 0$ and the network looks as shown in Fig. 16.8 (b).



$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$V_2 = I_2 Z_{eq}$$

where Z_{eq} is the equivalent impedance as viewed from b-b'.

$$Z_{eq} = \frac{8}{5} \ \Omega$$

$$V_2 = I_2 \times \frac{8}{5}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{5}{8} \text{ ℧}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

With a-a' short circuited, $-I_1 = \frac{2}{5} I_2$

Since $\quad I_2 = \frac{5V_2}{8}$

$$-I_1 = \frac{2}{5} \times \frac{5}{8} V_2 = \frac{V_2}{4}$$

$$\therefore \quad Y_{12} = \frac{I_1}{V_2} = -\frac{1}{4} \text{ ℧}$$

The describing equations in terms of the admittance parameters are

$$I_1 = 0.5\,V_1 - 0.25\,V_2$$

$$I_2 = -0.25\,V_1 + 0.625\,V_2$$

Ques 4: a)

Eqn for h parameter:

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

from (1)

$$I_1 = \left(\frac{1}{h_{11}}\right) V_1 + \left(\frac{-h_{12}}{h_{11}}\right) V_2 \quad \text{--- (3)}$$

using (3) in (2) we have

$$I_2 = h_{21}\left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2\right] + h_{22} V_2$$

$$= \left(\frac{h_{21}}{h_{11}}\right) V_1 + \left(h_{22} - \frac{h_{12}}{h_{11}}\right) V_2$$

$$I_2 = \left(\frac{h_{21}}{h_{11}}\right) V_1 + \left(\frac{h_{22} - h_{12}}{h_{11}}\right) V_2 \quad \text{--- (4)}$$

Comparing (3) & (4) with (A) & (B)

$$Y_{11} = \frac{1}{h_{11}} \qquad Y_{12} = \frac{-h_{12}}{h_{11}}$$

$$Y_{21} = \frac{h_{21}}{h_{11}} \qquad Y_{22} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}}$$

Ques 4B:

(4) Condition for reciprocity for ABCD parameter:

Eq^n: $V_1 = AV_2 + B(-I_2)$ — (1)

$I_1 = CV_2 + D(-I_2)$ — (2)

Consider fig (1), $V_1 = V_S$, $V_2 = 0$ and $I_2' = +I_2$

Eq (1): $V_S = BI_2'$ ⇒ $\frac{V_S}{I_2'} = B$ — (3)

Consider fig (2), $V_2 = V_S$, $V_1 = 0$ and $I_1' = -I_1$

Eq (1): $0 = AV_S - BI_2$ — (4) ⇒ $I_2 = \frac{A}{B}V_S$ — (4)

Eq (2): $-I_1' = CV_S - DI_2$ — (5)

using (4) in (5).

$-I_1' = CV_S - D\left(\frac{A}{B}\right)V_S$

CMR

$\therefore -I_1' = \frac{BC - AD}{B} V_S$

⇒ $\frac{V_S}{I_1'} = \frac{B}{AD - BC}$

The n/w is said to reciprocal if, $\frac{V_S}{I_1'} = \frac{V_S}{I_2'}$

⇒ $\frac{B}{AD - BC} = B$ ⇒ $\boxed{AD - BC = 1}$

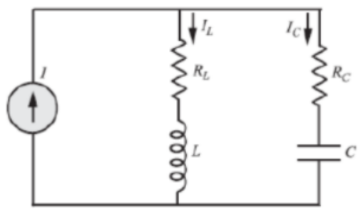
Conditions ...

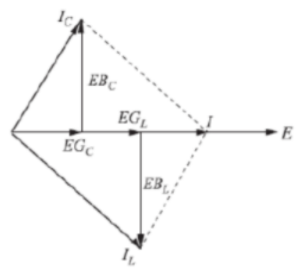
Ques 5:

Antiresonance Circuit: Case 2-> Practical Circuit for parallel resonance with RL & RC

Resonance in a Two Branch RL and RC Parallel Circuit

Consider the two branch parallel circuit shown in Fig. A Let E be the voltage across each of the parallel circuit shown in the figure. The vector diagram at resonance is shown in Figure B.





The admittance of the circuit is

$$Y = \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$Y = \frac{1}{R_L + j\omega L} + \frac{1}{R_C + \frac{1}{j\omega C}} \quad (1)$$

$$Y = \frac{1}{R_L + j\omega L}\left(\frac{R_L - j\omega L}{R_L - j\omega L}\right) + \frac{1}{R_C - \frac{j}{\omega C}}\left(\frac{R_C + \frac{j}{\omega C}}{R_C + \frac{j}{\omega C}}\right)$$

At resonance, $\omega = \omega_0$

$$Y = \frac{R_L}{R_L^2 + \omega_0^2 L^2} - \frac{j\omega_0 L}{R_L^2 + \omega_0^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega_0^2 C^2}} + \frac{\frac{j}{\omega_0 C}}{R_C^2 + \frac{1}{\omega_0^2 C^2}}$$

$$Y = \frac{R_L}{R_L^2 + \omega_0^2 L^2} + \frac{R_C}{R_L^2 + \frac{1}{\omega_0^2 C^2}} - \frac{j\omega_0 L}{R_C^2 + \omega_0^2 L^2} + \frac{\frac{j}{\omega_0 C}}{R_C^2 + \frac{1}{\omega_0^2 C^2}} \quad (2)$$

$$Y = G_L + G_C - j(B_L - B_C) \quad (3)$$

For resonance, $\omega = \omega_0$

$$B_L = B_C$$

$$\Rightarrow \frac{\omega_0 L}{R_L^2 + \omega_0^2 L^2} = \frac{\frac{1}{\omega_0 C}}{R_C^2 + \frac{1}{\omega_0^2 C^2}}$$

$$\Rightarrow \frac{\omega_0 L}{R_L^2 + \omega_0^2 L^2} = \frac{\omega_0 C}{\omega_0^2 C^2 R_C^2 + 1}$$

$$\Rightarrow L(\omega_0^2 C^2 R_C^2 + 1) = (R_L^2 + \omega_0^2 L^2)C$$

$$\Rightarrow \omega_0^2\,(C^2 R_C^2 L - L^2 C) = R_L^2 C - L$$

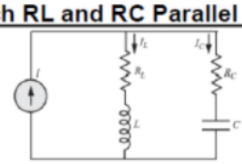
$$\Rightarrow \omega_0^2 = \frac{R_L^2 C - L}{C^2 R_C^2 L - L^2 C}$$

$$\Rightarrow \omega_0^2 = \frac{1}{LC}\frac{R_L^2 C - L}{R_C^2 C - L} \quad (4)$$

$$\Rightarrow \omega_0^2 = \frac{1}{LC}\frac{R_L^2 - \frac{L}{C}}{\left(R_C^2 - \frac{L}{C}\right)} \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{\left(R_C^2 - \frac{L}{C}\right)}}} \quad (5)$$

Resonance in a Two Branch RL and RC Parallel Circuit Cont…

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{\left(R_C^2 - \frac{L}{C}\right)}}$$



This is the expression for resonant frequency. It is to be noted that

1) resonance is not possible for certain combination of circuit elements unlike in a series circuit where resonance is always possible.

2) resonance is also possible by varying of R_L or R_C.

Consider the case where

$$R_L^2 < \frac{L}{C} < R_C^2$$

Or,

$$R_C^2 < \frac{L}{C} < R_L^2$$

In both the cases, the quantity under radical is negative and therefore resonance is not possible.

3) If, $\quad R_L = R_C \neq \sqrt{\frac{L}{C}}$

Then, $\quad \omega_0 = \frac{1}{\sqrt{LC}}$

as in R, L,C series circuit.

If, $\quad R_L = R_C = R = \sqrt{\frac{L}{C}}$

Or, $R_L^2 = R_C^2 = R^2 = \frac{L}{C} = X_L X_C \quad (6)$

$$now, Y = G_L + G_C - j(B_L - B_C)$$

$$Y = \frac{R_L}{R_L^2 + \omega_0^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega_0^2 C^2}} - \frac{j\omega_0 L}{R_L^2 + \omega_0^2 L^2} + \frac{\frac{j}{\omega_0 C}}{R_C^2 + \frac{1}{\omega_0^2 C^2}}$$

$$Y = \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} - \frac{jX_L}{X_L X_C + X_L^2} + \frac{jX_C}{X_L X_C + X_C^2}$$

$$Y = \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} - \frac{j}{X_C + X_L} + \frac{j}{X_C + X_L}$$

$$Y = \frac{R}{R^2 + X_L^2} + \frac{R}{R^2 + X_C^2} \quad (7)$$

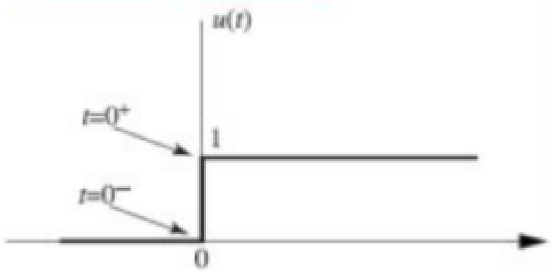
$$Y = \frac{R(R^2 + X_C^2) + R(R^2 + X_L^2)}{(R^2 + X_L^2)((R^2 + X_C^2))}$$

$$Y = \frac{1}{R} \quad (8) \qquad \because y = 1/z$$

$$\boxed{Z = R = \sqrt{\frac{L}{C}}} \quad (9) \qquad from\ (6)$$

Ques 6: a

1) Unit Step function



$$u(t-0) = u(t) = \begin{cases} 1; t \geq 0^{+} \\ 0; t \leq 0^{-} \end{cases}$$

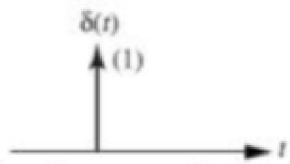
Laplace transform of $u(t)$

Given $x(t) = u(t)$

Then $L\{x(t)\} = \int_0^\infty x(t)e^{-st}dt = \int_0^\infty e^{-st}dt = \frac{1}{s}$

2) Impulse function

$$\frac{du(t)}{dt} = \delta(t) = \begin{cases} \infty; t = 0 \\ 0; t \neq 0 \end{cases}$$



Note: Impulse function is only **defined at a point** in time domain, not before and after that

Area under an impulse is **unity.**

$$\boxed{\int_0^\infty \delta(t)dt = 1}$$

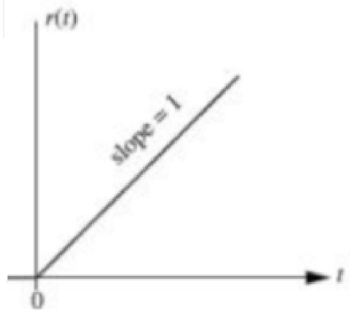
Laplace transform of $\delta(t)$

Given $x(t) = \delta(t)$

Then $L\{x(t)\} = \int_0^\infty x(t)e^{-st}dt = \int_0^\infty \delta(t)\, e^{-st}dt = \int_0^\infty \delta(t)\, dt = 1$

3) Ramp function

$$\int u(t)dt = r(t) = \begin{cases} t; t \geq 0 \\ 0; t \leq 0 \end{cases}$$



Given $x(t) = r(t)$

Then $L\{x(t)\} = \int_0^{\infty} x(t)e^{-st}dt$

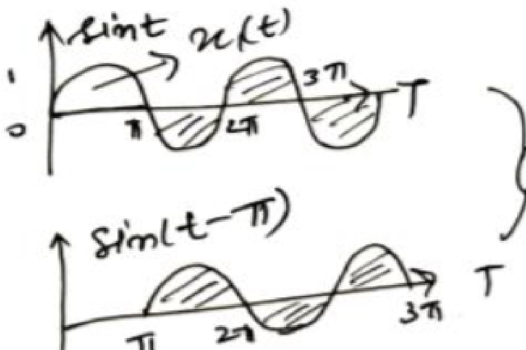
$$= \int_0^{\infty} te^{-st}dt$$

$$= \left[\frac{te^{-st}}{s}\right]_0^{\infty} + \frac{1}{s}\int_0^{\infty} e^{-st}dt$$

$$= \frac{1}{s^2}$$

Ques 6B:

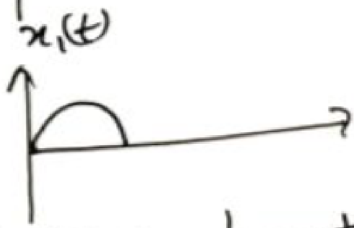
Solu: We know that



$x_1(t) = \sin t + \sin(t-\pi)$

OR

$\sin t \cdot u(t) + \sin(t-\pi) \cdot u(t)$



$LT(\sin(at)) \rightarrow \frac{a}{s^2+a^2}$

$$X_1(S) = \frac{1}{s^2+1} + \frac{e^{-\pi s}}{s^2+1} = \frac{1+e^{-\pi s}}{s^2+1}$$

$$X(S) = \frac{1}{1-e^{-Ts}} \cdot X_1(s)$$

$$= \frac{1}{1-e^{-Ts}}\left(\frac{1+e^{-\pi s}}{1+s^2}\right)$$

Ques 8A:

Initial-value theorem

The initial-value theorem allows us to find the initial value $x(0)$ of the function $x(t)$, directly from its Laplace transform $X(s)$

If $x(t)$, is a causal signal,
then $\boldsymbol{x(0) = \lim_{s\to\infty} sX(s)}$

Proof: To prove this theorem, we use time differentiation property

$$\Rightarrow L\left\{\frac{d}{dt}x(t)\right\} = sX(s) - x(0) = \int_0^\infty \frac{d}{dt}x(t)e^{-st}dt$$

Taking the limit $s \to \infty$ on both sides of above equation
If we let $s \to \infty$ then the integral on the right side of equation vanishes due to damping factor, e^{-st}

$$\lim_{s\to\infty}\left(sX(s) - x(0)\right) = 0$$

$$\Rightarrow x(0) = \lim_{s\to\infty} sX(s)$$

Final-value theorem

The final-value theorem allows us to find the final value $x(\infty)$ of the function $x(t)$, directly from its Laplace transform $X(s)$

If $x(t)$, is a causal signal,
then $\boldsymbol{x(\infty) = \lim_{s\to 0} sX(s)}$

Proof: To prove this theorem, we use time differentiation property

$$\Rightarrow L\left\{\frac{d}{dt}x(t)\right\} = sX(s) - x(0) = \int_0^\infty \frac{d}{dt}x(t)e^{-st}dt$$

Taking the limit $s \to 0$ on both sides of above equation
If we let $s \to 0$ then the integral on the right side of equation reduces to

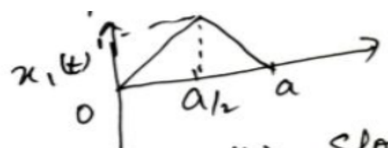
$$\lim_{s\to 0}\left(sX(s) - x(0)\right) = \int_0^\infty \frac{dx(t)}{dt}dt$$

$$\lim_{s\to 0} sX(s) - x(0) = x(t)\Big|_0^\infty$$

$$\lim_{s\to 0} sX(s) - x(0) = x(\infty) - x(0)$$

$$\Rightarrow x(\infty) = \lim_{s\to 0} sX(s)$$

Ques8 B



Slope $= y/x = \frac{1}{(a/2)} = \frac{2}{a}$

At $t=0 \to r(t)$

$t = a/2 \to$ 2 slopes $\to$ (1) Using $= \frac{-1}{a/2} = -\frac{2}{a}$

(2) Rising $= \frac{2}{a}$

change in slope $= -\frac{2}{a} - \frac{2}{a} = -\frac{4}{a}$

$x_1(t) = \frac{2}{a} r(t)$

& $r(t)$ is present at $a/2$

$\to -\frac{4}{a} r(t - a/2)$

At $t = a$, Change in Slope: $0 - \left(-\frac{2}{a}\right) = \frac{2}{a}$

$\to \frac{2}{a}[r(t-a)]$

$$x_1(t) = \frac{2}{a} r(t) - \frac{4}{a} r\left(t - \frac{a}{2}\right) + \frac{2}{a} r(t-a)$$

$$X_1(s) = \frac{2}{a}\cdot\frac{1}{s^2} - \frac{4}{a}\frac{e^{-a/2\,s}}{s^2} + \frac{2}{a}\cdot\frac{e^{-as}}{s^2}$$

$$X(s) = \frac{1}{1-e^{-as}}\left(\frac{2}{a}\cdot\frac{1}{s^2} - \frac{4}{a}\cdot\frac{e^{-a/2\,s}}{s^2} + \frac{2}{a}\cdot\frac{e^{-as}}{s^2}\right)$$