CMR INSTITUTE OF TECHNOLOGY

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Internal Assesment Test – 2

Sub:	Information Theory and Coding			Sec	*ALL*			Code:	18EC54
Date:	16 / 11 / 2021	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE

Answer Any FIVE FULL Questions	**Marks**	**OBE**	
		CO	**RBT**

1 Consider a discrete memory less source $S = (X, Y, Z)$ Given its second order extension source symbols and probabilities, compute the code words, efficiency and redundancy using Shannon's encoding algorithm. — 10 — CO2 — L3

Symbol	Probability
XX	0.25
XY	0.15
XZ	0.10
YX	0.15
YY	0.09
YZ	0.06
ZX	0.10
ZY	0.06
ZZ	0.04

2 i) An information source produces sequences of independent symbols having the following probabilities. — 5 — CO2 — L3

A	B	C	D	E	F	G
1/3	1/27	1/3	1/9	1/9	1/27	1/27

Construct binary codes using Shannon-Fano procedure and find its efficiency and redundancy.

ii) Which of the sets of the length requirements shown in table can be considered for the construction of the binary prefix codes? Also suggest suitable codes and draw the decision tree. (5M)

Code Set A	Code Set B	Code Set C	Code Set D
2	1	1	1
1	1	1	2
2	2	1	3
2	1	2	4

3 Find the minimum variance code words for the given source A={A,B,C,D,E,F,G,H} with probabilities P={7/39, 8/39, 6/39,10/39,3/39,1/39,2/39,2/39}. Also find the efficiency and redundancy . — 10 — CO2 — L3

4 Channel matrix for a channel is given as follows. Find H(X), H(Y), H(X,Y), H(X/Y), H(Y/X) and I(X;Y) in terms of probabilities and verify your answers. — 10 — CO3 — L3

$$P(Y|X) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.2 & 0.5 \\ 0.1 & 0.4 & 0.5 \end{bmatrix} \end{array}$$

And $P[X] = \left[\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right]$

Q. No.	Question	Marks	CO	Level
5	List the properties of Mutual Information. Prove that Mutual Information is symmetric. With the help of mutual information as applied to a binary symmetric channel calculate the channel capacity if the input probabilities are $p(x_1) = \frac{2}{3}$ *and* $p(x_2) = \frac{1}{3}$ and P(Y/X)= $\begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}$. if r_s=1000 symbol/sec	10	CO3	L3
6	A message source produces two independent symbols A and B with probabilities $p(a) = 0.4$ and $p(b) = 0.6$. If the symbols are received in average with 4 in every 100 symbols in error, find the average rate of information transmitted.	10	CO3	L3

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ITC IAT-2 solutions

(1)

Symbol	probability	n_i	F_i	Binary expansion of F_i upto n_i bits	C_i upto n_i
XX	0.25	2	0	0.00	00
XY	0.15	3	0.25	0.010	010
YX	0.15	3	0.40	0.011	011
XZ	0.10	4	0.55	0.1000	1000
ZX	0.10	4	0.65	0.1010	1010
YY	0.09	4	0.75	0.1100	1100
YZ	0.06	5	0.84	0.11010	11010
ZY	0.06	5	0.90	0.11100	11100
ZZ	0.04	5	0.96	0.11110	11110

Efficiency $\eta = \dfrac{H(S^2)}{\hat{H_N}} \times 100$

$H(S^2) = 2.9709$ bits/sym

$\hat{H_N} = \sum_{i=1}^{q} n_i p_i = 3.36$ bits/sym

$$\boxed{\begin{aligned} \eta &= \frac{2.9709}{3.36} \times 100 = 88.41\% \\ \text{Redundancy} &= 1-\eta = 11.59\% \end{aligned}}$$

(2) i)

Symbol	probability						Ci	n_i
A	$\frac{1}{3}$	0.33	0	0			00	2
C	$\frac{1}{3}$	0.33	0	1			01	2
D	$\frac{1}{9}$	0.11	1	0	0		100	3
E	$\frac{1}{9}$	0.11	1	0	1		101	3
B	$\frac{1}{27}$	0.037	1	1	0		110	3
F	$\frac{1}{27}$	0.037	1	1	1	0	1110	4
G	$\frac{1}{27}$	0.037	1	1	1	1	1111	4

$$\eta = \frac{H(S)}{\hat{H_N}} = \frac{2.281}{2.403}$$

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ii)

Codeset A	Codeset B	Codeset C	Codeset D
2	1	1	1
1	1	1	2
2	2	1	3
2	1	2	4

set A) $2^{-2} + 2^{-1} + 2^{-2} + 2^{-2}$

set B) $2^{-1} + 2^{-1} + 2^{-2} + 2^{-1}$

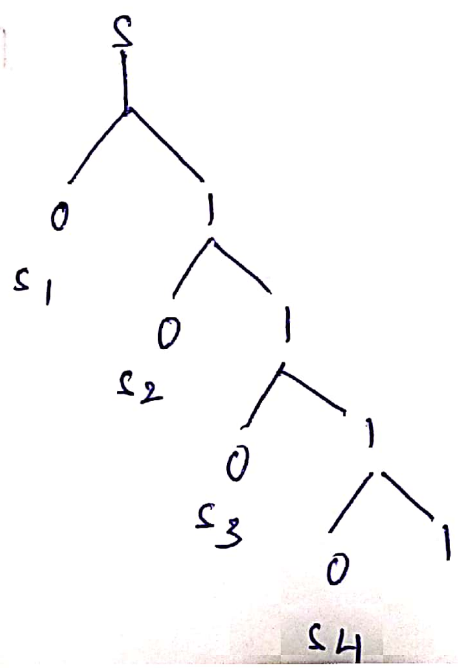
set C) $2^{-1} + 2^{-1} + 2^{-1} + 2^{-2}$

set D) $\sum_{i=1}^{q} 2^{-n_i} \leq 1$

$2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} =$

<u>set D</u> can be considered.

S_1	1	(or)	0
S_2	01		10
S_3	001		110
S_4	0000		1110



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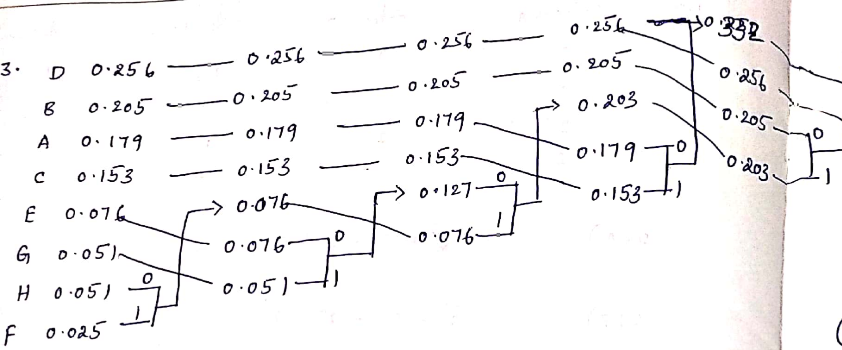
3.



D —	01	2
B —	10	2
A —	000	3
C —	001	3
E —	1100	4
G —	1101	4
H —	1110	4
F —	1111	4

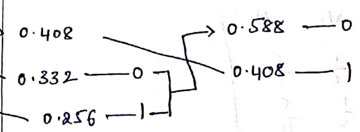
$H(s) = 2.468$ b/sym

$\hat{HN} = 2.698$

efficiency $= \frac{2.468}{2.698}$

$= 91.47\%$.

$R = 8.53\%$.



4

$$P\left(\frac{Y}{X}\right) = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ x_1 & 0.4 & 0.3 & 0.3 \\ x_2 & 0.3 & 0.2 & 0.5 \\ x_3 & 0.1 & 0.4 & 0.5 \end{array}$$

$\boxed{H(x) = 1.5 \text{ bits/sym}}$

$$P(XY) = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ x_1 & 0.2 & 0.15 & 0.15 \\ x_2 & 0.075 & 0.05 & 0.125 \\ x_3 & 0.025 & 0.1 & 0.125 \end{array}$$

$P(y_1) = 0.3$ $\quad P(y_2) = 0.3$ $\quad P(y_3) =$

$\boxed{H(y) = 1.57 \text{ b/sym}}$

$\boxed{H(XY) = 2.997 \text{ bits/sym}}$

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$$P\left(\frac{X}{Y}\right) = \begin{bmatrix} \frac{2}{3} & 1/2 & 3/8 \\ \frac{1}{4} & \frac{1}{6} & \frac{5}{16} \\ \frac{1}{12} & \frac{1}{3} & \frac{5}{16} \end{bmatrix}$$

$H\left(\frac{X}{Y}\right) = 1.427$ bits/sym

$H\left(\frac{Y}{X}\right) = 1.496$ bits/sym

$I(XY) = 0.074$ bits/sym

5).

$$P\left(\frac{Y}{X}\right) = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 3/4 & 1/4 \\ x_2 & 1/4 & 3/4 \end{array}$$

Channel Capacity $= \log_2 M - H\left(\frac{Y}{X}\right)$

$= 1 - \left[\frac{3}{4}\log\frac{4}{3} + \frac{1}{4}\log 4\right]$

$= 0.1887$ bits/symbol

$C = 0.1887 \times 1000$

$$\boxed{C = 188.7 \text{ bps}}$$

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6. $p(a) = 0.4$ $p(b) = 0.6$

$H(x) = 0.9709$ bits/sym

$$p\left(\frac{y}{x}\right) = \begin{array}{c|cc} & y_1 & y_2 \\ x_1 & 0.96 & 0.04 \\ x_2 & 0.04 & 0.96 \end{array}$$

$R_t = I(xy) \cdot r_s$

$= 0.7331 \times 100$

$$\boxed{R_t = 73.31 \text{ bps}}$$