USN:									
Internal Assessment Test 3 –Jan. 2022									
Sub:	Control Engineering				Sub Code:	18ME71/17ME73/ 15ME73	Branch:	ME	
Date:	24.01.22	Duration:	90 mins	Max Marks: 50	Sem/Sec:	VII/A&B		OBE	

	<u>Answer All the Questions</u>	MARKS	CO	RBT
1	(i) Determine the unit step response for a first order system and explain its features. (6 M) (ii) Define: (i) Time Constant (ii) Damped frequency of oscillation. (4 M)	10	CO3	L3/L2
2	Derive the expression for unit step response of underdamped second order system.	10	CO3	L3
3	A unity feedback system is characterized by an open loop transfer function $G(s) = \frac{10}{s^2 + 2s + 6}$ If the system is subjected to unit step input, determine: (i) Undamped natural frequency (ii) Damping Ratio (iii) Peak overshoot (iv) Peak time (v) Settling time.	10	CO3	L3
4	Sketch the Bode plot for a unity feedback system whose open loop transfer function is given by $G(s) = \frac{10}{s(1+s)(1+0.02s)}$ Determine: (i) Gain and Phase crossover frequencies (ii) Gain and Phase Margin (iii) Stability of the control system.	20	CO5	L3

Faculty Signature CCI Signature HOD Signature

1)

- Consider a simple system shown in the Fig. 7.11.1(a).

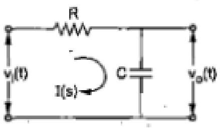

Fig. 7.11.1 (a)

- Find $v_o(t)$ i.e. response if it is excited by unit step input.

$$v_i(t) = 1, \quad t \geq 0$$
$$= 0, \quad t < 0$$

$$\therefore \quad V_i(s) = 1/s$$

- Now first calculate system T.F. The Laplace network is shown in the Fig. 7.11.1 (b).

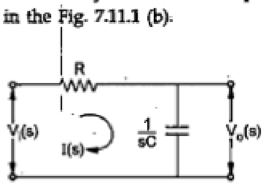

Fig. 7.11.1 (b)

$$V_i(s) = I(s)\,R + \frac{1}{sC} I(s) \quad \text{... (7.11.1)}$$

$$V_o(s) = \frac{1}{sC} I(s) \quad \text{... (7.11.2)}$$

$$\therefore \quad \boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{1+sRC} = \frac{1}{1+Ts}} \quad \text{where } T = RC$$

• Let input applied $v_i(t)$ is unit step voltage.

Substituting

$$V_i(s) = 1/s \text{ in the transfer function}$$

$$V_o(s) = \frac{1}{s(1+sRC)} = \frac{A'}{s} + \frac{B'}{1+sRC}$$

$$A' = 1 \text{ and } B' = -RC$$

$$\therefore \quad V_o(s) = \frac{1}{s} - \frac{RC}{1+sRC} = \frac{1}{s} - \frac{1}{s+(1/RC)}$$

Taking Laplace inverse,

$$v_o(t) = 1 - e^{-t/RC} \Rightarrow C_{ss} + c_t(t) \text{ form}$$

So $\boxed{C_{ss} = 1 \text{ and } c_t(t) = e^{-t/RC}}$

7.11.3 Time Constant

• The time constant of a system is defined as the time required by the system output to reach 63.2 % of its final steady state value during first attempt. It is denoted as τ or T.

This response is oscillatory, with oscillating frequency $\omega_n \sqrt{1-\xi^2}$ but decreasing amplitude as it is associated with exponential term with negative index $e^{-\xi \omega_n t}$. Such oscillations are called **damped oscillations** and frequency of such oscillations is called **damped frequency of oscillations** ω_d which is nothing but $\omega_n \sqrt{1-\xi^2}$.

2)

For underdamped systems, $\xi < 1$.

$$\therefore \quad s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \text{ has two roots,}$$

$$s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

Now let $\xi\omega_n = \alpha$

and $\omega_n\sqrt{1-\xi^2} = \omega_d$ (as discussed earlier)

$$\therefore \quad s_{1,2} = -\alpha \pm j\omega_d$$

For unit step input $R(s) = 1/s$ and

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Substituting R(s), $C(s) = \dfrac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$

The partial fraction can be calculated for the Laplace inverse as below,

$$C(s) = \frac{a_1}{s} + \frac{a_2 s + a_3}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\therefore \quad \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)} = \frac{a_1(s^2+2\xi\omega_n s+\omega_n^2)+s(a_2 s+a_3)}{s(s^2+2\xi\omega_n s+\omega_n^2)}$$

equating numerators on both sides, $\omega_n^2 = s^2(a_1+a_2)+s(a_1 2\xi\omega_n+a_3)+a_1\omega_n^2$

$\therefore$ $a_1\omega_n^2 = \omega_n^2$ equating constant

$a_1+a_2 = 0$ equating coefficients of s^2

$a_1 2\xi\omega_n + a_3 = 0$ equating coefficients of s

$\therefore$ $a_1 = 1,\ a_2 = -1,\ a_3 = -2\xi\omega_n$

As $\xi\omega_n = \alpha$ assumed earlier for ease of computations.

$\therefore$ $a_1 = 1,\ a_2 = -1,\ a_3 = -2\alpha$

$$\therefore \quad C(s) = \frac{1}{s}+\frac{-s-2\alpha}{s^2+2\alpha s+\omega_n^2}$$

$$\therefore \quad C(s) = \frac{1}{s}-\left\{\frac{s+2\alpha}{s^2+2\alpha s+\omega_n^2}\right\}$$ .... (Taking negative sign outside)

So adjusting denominator as, $s^2+2\alpha s+\alpha^2+\omega_n^2-\alpha^2 = (s+\alpha)^2+\omega_n^2-\alpha^2$

but $\alpha = \xi\omega_n \quad \therefore \ \alpha^2 = \xi^2\omega_n^2$

Substituting in above we get, $(s+\alpha)^2+\omega_n^2-\xi^2\omega_n^2 = (s+\alpha)^2+\omega_n^2\left(1-\xi^2\right)$

Now $\omega_d = \omega_n\sqrt{1-\xi^2}$ i.e. $\omega_d^2 = \omega_n^2(1-\xi^2)$

Substituting this in the expression of C(s) we get,

$$\therefore \quad C(s) = \frac{1}{s}-\left\{\frac{s+2\alpha}{(s+\alpha)^2+\omega_d^2}\right\}$$

$$L^{-1}\left\{\frac{(s+a)}{(s+a)^2+\omega^2}\right\} = e^{-at}\cos\omega t \quad \text{and} \quad L^{-1}\left\{\frac{\omega}{(s+a)^2+\omega^2}\right\} = e^{-at}\sin\omega t$$

Adjusting C(s) as,

$$C(s) = \frac{1}{s}-\left\{\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}+\frac{\alpha}{(s+\alpha)^2+\omega_d^2}\right\}$$

Multiplying and dividing by ω_d to the last term,

$$C(s) = \frac{1}{s}-\left\{\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}+\frac{\alpha}{\omega_d}\cdot\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}\right\}$$

Taking Laplace inverse,

$$c(t) = 1-e^{-\alpha t}\cos\omega_d t-\frac{\alpha}{\omega_d}e^{-\alpha t}\sin\omega_d t$$

Using $\alpha = \xi\omega_n$, $\boxed{\omega_d = \omega_n\sqrt{1-\xi^2}}$

$$c(t) = 1-e^{-\xi\omega_n t}\left[\cos\omega_d t+\frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right]$$

$$= 1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{1-\xi^2}\cos\omega_d t+\xi\sin\omega_d t\right]$$

- Now, $\sin(\omega_d t+\theta) = \sin(\omega_d t)\cos\theta+\cos(\omega_d t)\sin\theta$

Comparing this with the expression in bracket we can write $\sin\theta = \sqrt{1-\xi^2}$ and $\cos\theta = \xi$.

Hence $\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$

$$\therefore \quad \boxed{\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} \text{ radians}}$$

- Hence using this in the expression.

$$\therefore \quad \boxed{c(t) = 1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\theta)}$$ ... Required expression

where $\omega_d = \omega_n\sqrt{1-\xi^2}$

and $\theta = \tan^{-1}\left\{\frac{\sqrt{1-\xi^2}}{\xi}\right\}$ radians

3)

Sol. : From given G(s), the closed loop T.F. is,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{10}{s^2+2s+16}$$

Comparing denominator with $s^2 + 2\xi\omega_n s + \omega_n^2$,

$$\omega_n^2 = 16,\ \omega_n = 4,\ 2\xi\omega_n = 2,\ \xi = 0.25$$

i) $\omega_n = 4$ rad/sec

ii) ξ = Damping ratio = 0.25

iii) $\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 44.43\ \%$

iv) $\omega_d = \omega_n\sqrt{1-\xi^2} = 3.873$ rad/sec

$\therefore$ $T_p = \frac{\pi}{\omega_d} = 0.811$ sec

v) $T_s = \frac{4}{\xi\omega_n} = 4$ sec

4)

Ex. 11.7.2 *For a unity feedback system with open loop transfer function :*

$$G(s) = \frac{40(s+5)}{s(s+10)(s+2)}$$

Draw the Bode plot. Determine gain margin, phase margin, ω_{gc}, ω_{pc}. Comment on the stability of the system.

Sol. : Step 1 : Obtain time constant form of G(s)H(s).

$$G(s)H(s) = \frac{40\times5\left(1+\frac{s}{5}\right)}{s\times10\times2\times\left(1+\frac{s}{10}\right)\left(1+\frac{s}{2}\right)} = \frac{10(1+0.2s)}{s(1+0.1s)(1+0.5s)}$$

Step 2 : Factors

1) K = 10, 20 Log K = 20 Log 10 = 20 dB

2) $\frac{1}{s}$, one pole at origin.

Straight line of slope –20 dB/dec passing through intersection of $\omega = 1$ and 0 dB.

3) $\frac{1}{(1+0.5s)}$, simple pole, $T_1 = 0.5$,

$\omega_{C1} = \frac{1}{T_1} = 2$ rad/sec

Straight line of slope –20 dB/dec for $\omega > 2$.

4) (1 + 0.2s), simple zero, $T_2 = 0.2$,

$\omega_{C2} = \frac{1}{T_2} = 5$ rad/sec.

Straight line of slope +20 dB/dec for $\omega > 5$.

5) $\frac{1}{(1+0.1s)}$, simple pole, $T_3 = 0.1$,

$\omega_{C3} = \frac{1}{T_3} = 10$ rad/sec.

Straight line of slope –20 dB/dec for $\omega > 10$.

Resultant slope table :

Range of ω	Resultant slope
$0 < \omega < \omega_{C1}(2)$	-20 dB/dec (Pole at origin)
$2 < \omega < \omega_{C2}(5)$	$-20 - 20 = -40$ dB/dec
$5 < \omega < \omega_{C3}(10)$	$-40 + 20 = -20$ dB/dec
$10 < \omega < \infty$	$-20 - 20 = -40$ dB/dec

Step 3 : Phase angle table

$$G(j\omega)H(j\omega) = \frac{10(1+0.2\,j\omega)}{j\omega(1+0.1j\omega)(1+0.5\,j\omega)}$$

ω	$\frac{1}{j\omega}$	$-\tan^{-1}0.5\omega$	$+\tan^{-1}0.2\omega$	$-\tan^{-1}0.1\omega$	ϕ_R
0.2	−90°	−5.71°	+2.29°	−1.14°	−94.56°
5	−90°	−68.19°	+45°	−26.56°	−139.75°
10	−90°	−78.69°	+63.43°	−45°	−150.25°
50	−90°	−87.71°	+84.29°	−78.69°	−172.11°
∞	−90°	−90°	+90°	−90°	−180°

Step 4 : The Bode plot is shown in the Fig. 11.7.2. From the plot,

ω_{gc} = **4.4 rad/sec**, $\omega_{pc} = \infty$, G.M. = $+\infty$ dB P.M. = **+42°**

As G.M. = $+\infty$ dB, the system is **absolutely stable** in nature.