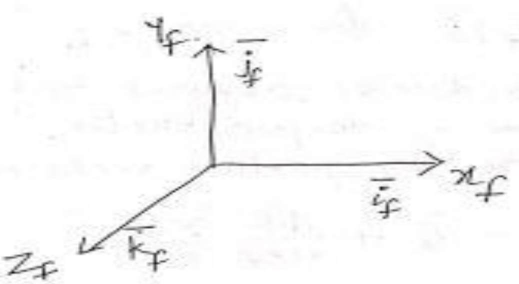
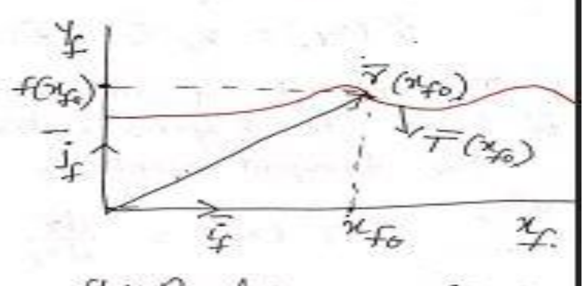
1 a)

Roadway fundamentals:—



a) fixed Coordinate s/y



(b) Roadway on fixed Coordinate s/y.

- A vehicle moves on a level road and also up & down the slope of roadway.
- Consider a straight roadway, Since the horizontal maneuvering has minimal impact on f & P requirement of propulsion unit.

Derivation:— The fixed coordinate s/y is attached to Earth such that force of gravity is $\perp^{lar}$ to unit vector $\bar{i}_f$.

→ Let us Consider a st. road way, i.e., the steering wheel is locked st along the x_f - direction. The road way is then on $x_f y_f$ plane of the fixed Coordinate s/y.

The two dimensional roadway can be described as $y_f = f(x_f)$.

The road way position vector $\bar{r}(x_f)$ b/w two points a & b along horizontal direction is

$$\bar{r}(x_f) = x_f\,\bar{i}_f + f(x_f)\,\bar{j}_f \quad \text{for } a \le x_f \le b$$

The direction of motion & distance traversed by vehicle is Easier to Express in terms of tangent vector,

The tangent vector of roadway position vector is given as,

$$\bar{T}(x_f) = \frac{d\bar{r}}{dx_f} = \bar{i}_f + \frac{df}{dx_f}\,\bar{j}_f$$

The distance norm of tangent vector $\|\bar{T}(x_f)\|$ is

$$\|\bar{T}(x_f)\| = \sqrt{1+\left[\frac{df}{dx_f}\right]^2} \qquad [\text{i.e., } \bar{i}_f = \bar{j}_f = 1]$$

The tangential roadway length is 'S' is the distance traversed along the roadway. Mathematically, S is arc length of $y_f = f(x_f)$ over $a \le x_f \le b$. Therefore,

$$S = \int_a^b \|\bar{T}(x_f)\|\,dx_f.$$

The roadway % grade can be described as a fn. of roadway as

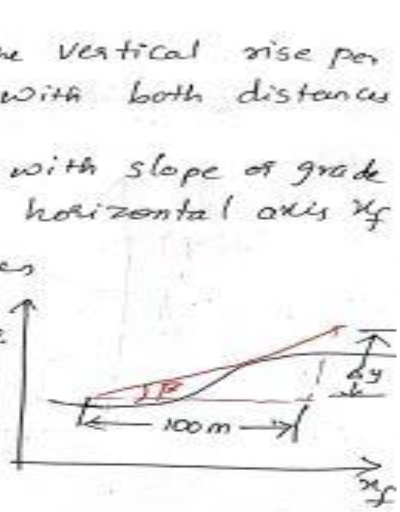
$$\beta(x_f) = \tan^{-1}\left[\frac{df(x_f)}{dx_f}\right].$$

The average roadway % grade is the vertical rise per 100 horizontal distance of roadway with both distances expressed in same unit.

→ Angle 'β' of roadway associated with slope of grade is angle b/w tangent vector & horizontal axis x_f

→ If Δy is vertical rise in 'm', then

$$\% \text{ grade} = \frac{\Delta y}{100m} \cdot 100\%$$
$$= \Delta y \ \%$$



The tangent of slope angle is

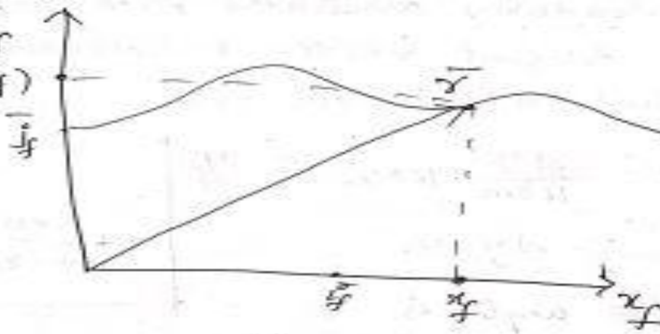
$$\tan\beta = \frac{\Delta y}{100m}.$$

→ The % grade of $\beta > 0$, when vehicle is on an upward slope & $\beta < 0$, when vehicle is going downhill.

1 b)

Sol :— Given, $f(x_f) = 3.9\sqrt{x_f}$ $\quad 0 \le x_f \le 2\,mi$

a)



b) $$\beta(x_f) = \tan^{-1}\left[\frac{df(x_f)}{dx_f}\right]$$

$$= \tan^{-1}\left[\frac{d}{dx_f}\left(3.9\sqrt{x_f}\right)\right]$$

$$= \tan^{-1}\left[3.9\,\frac{d}{dx_f}(x_f)^{1/2}\right]$$

$$= \tan^{-1}\left[3.9 \times \frac{1}{2}\,\frac{1}{\sqrt{x_f}}\right]$$

$$\beta(x_f) = \tan^{-1}\left[\frac{1.95}{\sqrt{x_f}}\right]$$

1 mile = 5280 feets.

~~β 5280~~ % $\beta = \tan^{-1}\left[\frac{1.95}{\sqrt{5280}}\right] \times 100$

$$\beta = 2.68\ \%$$

d) $f = 3.9\sqrt{x_f}$.

$$\frac{df}{dx_f} = \frac{1.95}{\sqrt{x_f}}$$

$$S = \int_0^{x_f} \|T\|\, dx_f = \int_0^{2\,\text{mile}} \sqrt{1+\left(\frac{1.95}{\sqrt{x_f}}\right)^2}\, dx_f.$$

1 mile = 5280 feets.

2 mile → ?

= 10,560 feets

$$S = \int_0^{10,560} \sqrt{1+\left(\frac{1.95}{\sqrt{x_f}}\right)^2}\, dx_f.$$

If $\sqrt{x_f} = u$

$x_f = u^2$

$dx_f = 2u\,du$

$x_f = 0, u = 0$

$x_f = 10,560$

$u = \sqrt{10,560}$

$$= \int_0^{\sqrt{10,560}} \sqrt{1+\left(\frac{1.95}{u}\right)^2}\, 2u\,du$$

$$= 2\int_0^{\sqrt{10,560}} \sqrt{\frac{u^2+(1.95)^2}{u^2}}\, u\,du$$

$$= 2\int_0^{\sqrt{10,560}} \frac{\sqrt{u^2+(1.95)^2}}{\cancel{u}}\, \cancel{u}\,du$$

$$= 2\int_0^{\sqrt{10,560}} \sqrt{u^2+(1.95)^2}\, du$$

$$\left[\because \int\sqrt{a^2+x^2} = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)\right]$$

$$= 2\left[\frac{u}{2}\sqrt{u^2+(1.95)^2} + \frac{(1.95)^2}{2}\sinh^{-1}\left(\frac{u}{1.95}\right)\right]_0^{\sqrt{10,560}}$$

$$= \left[u\sqrt{u^2+(1.95)^2} + (1.95)^2\sinh^{-1}\left(\frac{u}{1.95}\right)\right]_0^{\sqrt{10,560}}$$

$$= \left[\sqrt{10,560}\cdot\sqrt{10,560+(1.95)^2} + (1.95)^2\sinh^{-1}\left(\frac{\sqrt{10,560}}{1.95}\right)\right] - \left[(1.95)^2\sinh^{-1}\left(\frac{0}{1.95}\right)\right]$$

$$= 10,561.9011 + (3.8025)(4.6575)$$

$$S = 10,579.6113$$

$$S \simeq 10,580 \text{ feets}$$

2 a)

→ Newton's Second law of motion can be applied to cg of a vehicle in tangential Coordinate system as

$$\sum \bar{F}_T = m\bar{a}_T = m\frac{d\bar{v}_T}{dt}.$$

where, m = total vehicle mass.

In terms of Components of Coordinate system,

$$\sum \bar{F}_{xT} = m\frac{d\bar{v}_{xT}}{dt} \quad \text{(Component tangent to road)}$$

$$\sum \bar{F}_{yT} = m\frac{d\bar{v}_{yT}}{dt} \quad \text{(Component normal to road)}$$

$$\sum \bar{F}_{zT} = m\frac{d\bar{v}_{zT}}{dt} = 0 \quad \text{[Since motion is assumed Confined to xy plane]}$$

Here, v_{xT} is vehicle tangential velocity.

→ The gravitational force in the normal direction is balanced by the road reaction force, and hence, there will be no motion in the y_T normal direction. In the sense, the tires always remains in Contact with road.

∴ The normal velocity v_{yT} is zero.

→ As we have assumed $x_f y_f$ or $x_T y_T$ plane, there is neither force nor velocity is acting in z-direction.

→ with these justifications, we can use a 1-dimensional analysis for vehicle propulsion in x_T-direction.

→ All tractive forces and opposing forces are in x_T direction, so we will not be using — sign in symbols.

The propulsion unit of the vehicle Exerts the tractive force F_{TR} to propel the vehicle forward at a desired velocity.

→ The tractive force must overcome the opposing forces, which are summed together and labelled as road load force F_{RL}.

$$\therefore F_{RL} = F_{gxT} + F_{roll} + F_{AD} \longrightarrow (1)$$

where, F_{gxT} = Gravitational force.

F_{roll} = rolling resistance of tires.

F_{AD} = Aerodynamic drag force.

x_T = Tangential direction along the roadway.

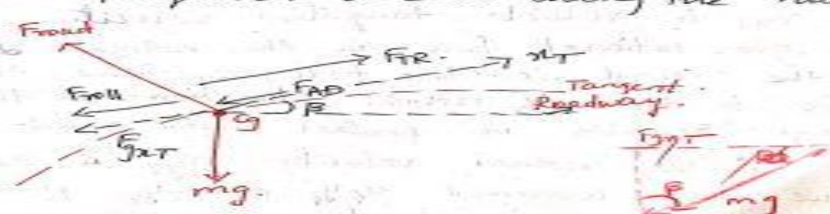


fig:- Forces acting on a vehicle.

→ Always, gravitational force depends on ~~angle~~ slope of roadway.

→ The gravitational force to overcome by the vehicle moving forward is

$$F_{gxT} = mg\sin\beta \longrightarrow (2)$$

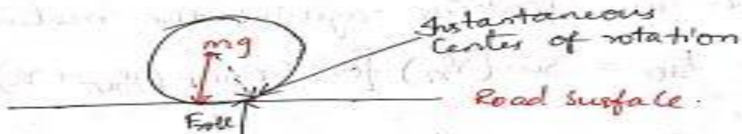
where, m = Total mass of vehicle

g = gravitational acceleration Constant.

β = grade angle w.r.to horizon.

→ The rolling Resistance is produced by the flattening of tire at Contact Surface with roadway.

→ when the tire, flattens, the instantaneous Center of rotation at the wheel moves forward from beneath the axle toward the direction of motion of vehicle.



→ The weight on wheel and the road normal force are misaligned due to the flattening of tire and form a Couple that Exerts a retardation torque on wheel

→ F_{roll}, the rolling resistance force is the force due to couple, which opposes the motion of wheel.

→ The rolling Resistance Can be ~~kept~~ minimized by Keeping the tires as much inflated as possible.

The rolling resistance force on roadway of slope β is

$$F_{roll} = \begin{cases} Sgn(v_{xT})\, mg\cos\beta\,(C_0 + C_1 v_{xT}^2) & \text{if } v_{xT} \neq 0 \\ F_{TR} - F_{gxT} & \text{if } v_{xT}=0 \ \& \ |F_{TR}-F_{gxT}| \le C_0 mg\cos\beta \\ Sin(F_{TR}-F_{gxT})\,(C_0 mg\cos\beta) & \text{if } v_{xT}=0 \ \& \ |F_{TR}-F_{gxT}| > C_0 mg\cos\beta \end{cases} \quad \rightarrow (3)$$

Typically, $0.004 < C_0 < .02$

$C_1 << C_0 \ (s^2/m^2)$

$C_0 mg$ = maximum rolling resistance at Standstill.

$$Sin(v_{xT}) = \begin{cases} 1 & \text{if } v_{xT} \geq 0 \\ -1 & \text{if } v_{xT} < 0. \end{cases}$$

→ The aerodynamic drag force is the viscous resistance of the air working against the motion vehicle.

$$F_{AD} = Sin(v_{xT})\,\{0.5\,\rho\, C_D A_F (v_{xT} + v_0)^2\} \quad \rightarrow (4)$$

where,

ρ = air density in kg/m^3

C_D = Aerodynamic drag Coefficient $(0.2 < C_D < 0.4)$

A_F = Equivalent frontal area of vehicle.

v_0 = head-wind velocity

2 b)

Maximum Gradability:—

→ The maximum Grade that a vehicle will be able to overcome with the max. force available from the propulsion unit is nothing but "max. Gradability

→ The vehicle is Expected to move forward very slowly when climbing a steep slope, and hence we can make the following assumptions for max. gradability:

1. Vehicle moves very slowly, $\Rightarrow v \cong 0$
2. F_{AD}, F_{roll} are negligible.
3. The vehicle is not accelerating, $\frac{dv}{dt} = 0$.
4. F_{TR} is max. tractive force delivered by motor at near to zero speed.

WKT,

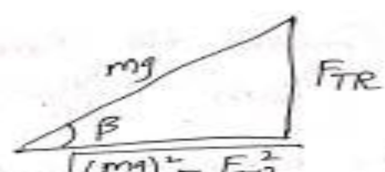
$$F_{TR} - F_{AD} - F_{roll} - F_{gxT} = 0$$

At near stall Condition, under the above assumptions,

$$\sum F = 0 \Rightarrow F_{TR} - F_{gxT} = 0$$

$$\Rightarrow F_{TR} = mg \sin\beta$$

$$\therefore \sin\beta = F_{TR}/mg$$



The maximum, % grade $= 100 \tan\beta$

$$\text{max } \% \text{ grade} = 100\left(\frac{\sin\beta}{\cos\beta}\right)$$

$$= 100 \cdot \frac{F_{TR}/mg}{\frac{\sqrt{(mg)^2 - F_{TR}^2}}{mg}}$$

$$\text{max. \% grade} = \frac{100\, F_{TR}}{\sqrt{mg^2 - F_{TR}^2}}$$

3 b)

Quarter - Car Model :-

→ The study of laws of motion starts from Consideration of quarter-Car, i.e., one quarter of vehicle.

→ Quarter Car model allows analyzing Seperately the behaviour of driving wheels and trailing wheels.

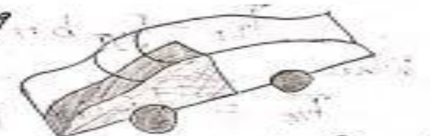


fig:- Quarter Car model

→ As chassis & wheel dynamics are treated Seperately, this model is Especially used to study Suspension dynamics if Vertical Components of forces are taken into account.

→ The X- Component of newton's law of motion applied to Chassis is.

$$F_{EW} - F_{AD} + F_{gxT} - F_{Rr} = m \frac{dv}{dt} \longrightarrow (1)$$

where,

F_{RW} = Reaction force Exchanged b/w wheel & Chassis.

m = mass of quarter of Chassis.

F_{Rr} = Reaction force b/w front & rear portion of Car

→ The Components on y-axis are omitted as Suspension dynamics will not be analyzed here.

The wheel dynamics Can be achieved from the balance of forces and torques as

$$-F_{ew} - F_{roll} = m_w \frac{dv}{dt} \longrightarrow (2)$$

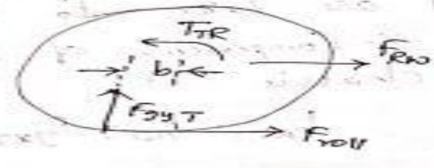
$$T_{TR} - F_{gyT}\, b + F_{roll} \cdot r_{wh} = J_w \frac{d\omega_{wh}}{dt} \longrightarrow (3)$$

where,

T_{TR} = driving / tractive torque

m_w = mass of wheel.

r_{wh} = wheel radius.



$T_{TR} = 0$ for trailing wheel.

→ The tractive force is force Supplied by Em in EV & by Combination of Em & IC Engine is a Hybrid vehicle to overcome Road load.

Sub. Eq (2) & (3) in Eq (1)

$$\frac{T_{TR}}{r_{wh}} - F_{gyr}\frac{b}{r_{wh}} - F_{AD} - F_{gxr} - F_{Rr} = (m+m_w)\frac{dv}{dt} + \frac{J_w}{r_{wh}}\frac{d\omega_{wh}}{dt} \rightarrow (4)$$

Here, T_{TR}/r_{wh} plays role of traction force.

→ To accelerate vehicle, traction force has to overcome rolling resistance, Aerodynamic load, weight component + Equivalent force of rear potion of Car.

Eq (4) Can be written as

$$(m+m_w)\frac{dv}{dt} + \frac{J_w}{r_{wh}}\frac{d\omega_{wh}}{dt} = K_m m\frac{dv}{dt} \rightarrow (5)$$

Here, $K_m m(dv/dt)$ = Equivalent force of inertia accounting for translating mass of quarter Car plus inertia of rotating masses.

The dynamic Eq. of motion is tangential direction assumes,

$$K_m m\frac{dv}{dt} = F_{TR} - F_{RL} - F_{Rr} \rightarrow (6)$$

K_m = rotational inertia Coefficient.

3 a)

1. Constant F_{TR}, Level Road:—

→ Here, we assume a level Road Condition, where the propulsion unit for an EV. Exerts a Constant tractive force.

→ The level road Condition is Considered to be

$\beta(s) = 0$

Where, $\beta(s)$ = Roadway input = slope of a roadway.

→ We assume, EV is initially at rest, i.e., $v(0) = 0$.

[forces acting on vehicle on level road]

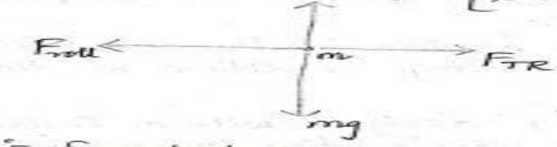


fig :- free body diagram at $t = 0$.

fig :- forces on vehicle at $t > 0$.

→ The initial tractive force is capable of overcoming the initial rolling resistance.

i.e., $F_{TR}(0) = F_{TR} > C_0 mg$

$C_0 mg$ = max rolling resistance at standstill.

$$\therefore \sum F(0) = m a(0) = m \frac{dv(0)}{dt}.$$

$$\Rightarrow F_{TR} - C_0 mg = m \frac{dv(0)}{dt}.$$

→ Since, $F_{TR}(0) > C_0 mg$, $\frac{dv(0)}{dt} > 0$ and velocity 'v' is increasing.

→ Once the vehicle starts to move, the forces acting on it changes. At $t > 0$,

$$\sum F = m \frac{dv}{dt}$$

$$F_{TR} - F_{AD} - F_{roll} = m \frac{dv}{dt}$$

$$F_{TR} - \sin[v(t)] \frac{\rho}{2} C_D A_F v^2(t) - \sin[v(t)] mg [C_0 + C_1 v^2(t)] = m \frac{dv}{dt}.$$

Assuming, $v(t) > 0$ ~~and~~ for $t > 0$ and solving for dv/dt,

$$\frac{dv}{dt} = \frac{F_{TR}}{m} - \frac{\rho}{2m} C_D A_F v^2 - \cancel{\sin} g [C_0 + C_1 v^2]$$

$$= \frac{F_{TR}}{m} - \frac{\rho}{2m} C_D A_F v^2 - g C_0 - g C_1 v^2$$

$$\frac{dv}{dt} = \left[\frac{F_{TR}}{m} - g C_0\right] - v^2 \left[\frac{\rho}{2m} C_D A_F + g C_1\right]$$

where, assuming ~~Cons~~ K_1 & K_2 are Constants for a Constant F_{TR} acceleration,

$$\boxed{\frac{dv}{dt} = K_1 - K_2 v^2}$$

$$\therefore K_1 = \frac{F_{TR}}{m} - g C_0$$

$$K_2 = \frac{\rho}{2m} C_D A_F + g C_1$$

Distance traversed:—

→ The distance traversed by the vehicle can be obtained from Equation.

$$v(t) = \sqrt{\frac{K_1}{K_2}} \tanh\left(\sqrt{K_1 K_2}\, t\right) \longrightarrow (1)$$

and also, WKT, $\sqrt{\frac{K_1}{K_2}} = V_T$.

$$\cancel{V_T} \quad \sqrt{K_1} = V_T \sqrt{K_2} \longrightarrow (2)$$

Sub. Eq (2) in Eq (1)

$$v(t) = V_T \tanh\left(V_T \sqrt{K_2}\sqrt{K_2}\, t\right)$$

$$v(t) = V_T \tanh\left(V_T K_2 t\right)$$

and also, $\frac{ds(t)}{dt} = v(t)$.

$$\therefore \frac{ds(t)}{dt} = V_T \tanh(K_2 V_T t). \longrightarrow (3)$$

on integrating Eq (3), we can obtain distance as a function of time. $\int \tanh(at)\,dt = \frac{1}{a}\ln[\cosh(at)]$

$$\int \frac{ds(t)}{dt}\,dt = \int V_T \tanh(K_2 V_T t)\,dt.$$

$$S(t) = \frac{V_T}{a} \cdot \ln\cosh(K_2 V_T t). \quad [\text{Consider}, K_2 V_T = a]$$

$$S(t) = \frac{1 V_T}{K_2 V_T} \ln[\cosh(K_2 V_T t)]. \qquad S(t) = \frac{1}{K_2} \ln[\cosh(K_2 V_T t)]$$

$S(t) \Rightarrow$

The starting acceleration is specified as 0 to V_f m in t_f sec,

where, v_f = desired velocity at end of specified time t_f sec

→ The time to reach the desired velocity + distance traversed during the time is

$$t_f = \frac{1}{K_2 V_T} \cosh^{-1}\left[e^{(K_2 S_f)}\right]$$

and $$S_f = \frac{1}{K_2} \ln\left[\cosh(K_2 V_T t_f)\right]$$

---

and t_f can also be Expressed as,

$$t_f = \frac{1}{\sqrt{K_1 K_2}} \tanh^{-1}\left[\sqrt{\frac{K_2}{K_1}}\, v_f\right]$$

t_f = desired time

4 a)

Slip:-

→ The friction forces that are fundamental to vehicle traction depend on difference between the tire rolling speed and its linear speed of travel.

→ The tire rolling speed is related to the wheel angular speed and is given by

$$v_{tire} = \omega_{wh} r_{wh}.$$

where, v_{tire} = Spin velocity of wheel.
ω_{wh} = wheel speed.
r_{wh} = driven wheel radius

→ The wheel speed of vehicle is Equivalent to vehicle translatory speed v.

→ vehicle linear velocity 'v' and tire speed v_{tire} differ in magnitude & direction.

→ The angle between tire velocity & vehicle velocity is known as "slip angle 'α'."

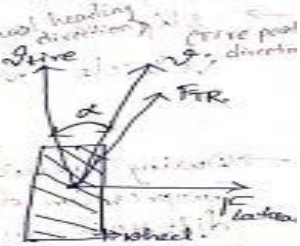


Fig: Speed & forces in tire road contact path area.

→ The vehicle traction force F_{TR} works in longitudinal direction.

→ The tire treads gradually Enter the Contact path of a moving vehicle when traction torque is applied to driven wheel.

→ The tire treads in front end of the Contact patch undergoes high levels of Compression Compared to the rear End due to traction torque applied at the axle.

→ Mathematically, slip of vehicle is

$$S = \left(1 - \frac{v}{\omega_{wh} r_{wh}}\right)$$

→ Since vehicle velocity 'v' is Smaller than linear velocity $\omega_{wh} r_{wh}$, the slip is positive number b/w 0 and 1.0 for propulsion.

→ slip is alway Expressed in percentage, the vehicle velocity is given by

$$v = \omega_{wh} r_{wh} (1-s).$$

→ During vehicle braking, the tire is Subjected to Similar Compression due to the applied braking torque, and a difference in vehicle Speed and wheel angular Speed occurs Enabling the vehicle stop.

→ The slip during braking is given as

$$S = \left(1 - \frac{\omega_{wh} r_{wh}}{v}\right).$$

4 b)

Force Transmission at Tire - Road Interface :—

→ The tire road interface can be thought of a gear mechanism responsible for the generation of traction force.

→ The conversion of rotary motion of the wheels to linear motion of vehicle can be compared to ball-screw arrangement shown in fig, which is a gear mechanism used for force transmission along with conversion of rotary motion to linear motion.

fig:- Ball-Screw gear arrangement (for converting rotary to linear motion).

→ Let us consider that, power is being transmitted from the rotary gear to linear gear with the latter held stationary.

→ The rotary gear is driven along its spin axis by a rotary motor, while enables the rotary system of motor and gear to move along the linear gear train.

→ When torque is applied by the rotary motor, friction force is developed in the gear train that helps the rotary system move forward or backward.

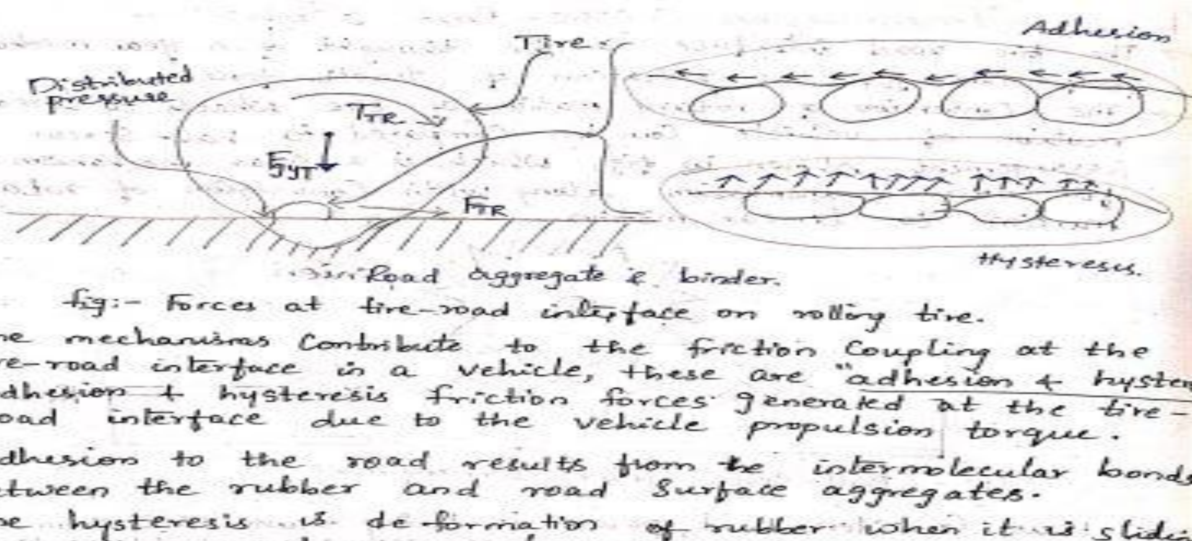


fig:- Forces at tire-road interface on rolling tire.

→ The mechanisms contribute to the friction coupling at the tire-road interface in a vehicle, these are "adhesion & hysteresis"

→ Adhesion + hysteresis friction forces generated at the tire-road interface due to the vehicle propulsion torque.

→ Adhesion to the road results from the intermolecular bonds between the rubber and road surface aggregates.

→ The hysteresis is deformation of rubber when it is sliding over the road aggregates, the friction component comes from energy loss due to hysteresis.

→ The friction forces of adhesion and hysteris depend heavily on the deformation of tire, at tire-road interface due to tire properties and interaction of forces.

→ Some slip is essential for vehicle traction, but too much slippage reduces the traction force limit.

5 a)

Sol a) force balance Eq.

$$F_{TR} - F_{AD} - F_{roll} = m\frac{dv}{dt}$$

$$F_{TR}(t) = m\frac{dv}{dt} + F_{AD} + F_{roll}$$

$$= m\frac{dv}{dt} + \frac{\rho}{2} C_D A_f V^2 + mg(C_0 + C_1 V^2)$$

$$F_{TR}(t) = 800 \times \frac{d}{dt}(0.29055t^2) + \frac{1.18}{2} \times 0.2 \times 2.2 \times (0.29055t^2)^2 + 800 \times 9.81\left(0.008 + (1 \times 0.6 \times 10^{-6})(0.29 t^2)\right)$$

$$\cancel{= 464.88t}$$

$$= 800 \times (2 \times 0.29055)t + \frac{1.18}{2} \times 0.2 \times 2.2 \times (0.29055)^2 t^4 + 800 \times 9.81\left[0.008 + (0.6 \times 10^{-6})(0.29055)^2 t^4\right]$$

$$= 464.88t + 0.02192t^4 + 62.784 + 3.9751 \times 10^{-4} t^4$$

$$F_{TR}(t) = (0.02232\, t^4 + 464.88\, t + 62.784)\ N$$

b) $$P_{TR}(t) = F_{TR}(t) \times V(t)$$

$$= (0.02232t^4 + 464.88t + 62.784) \times 0.29055t^2$$

$$P_{TR}(t) = (0.00648\, t^6 + 135.07\, t^3 + 18.24\, t^2)\ W$$

c) Energy loss due to nonconservative forces

c) $$E_{loss} = \int_0^{10} v(F_{AD} + F_{roll})\,dt$$

$$= \int_0^{10} 0.29055t^2(0.02192t^4 + 62.784 + 3.9751 \times 10^{-4} t^4)\,dt$$

$$= \int_0^{10} (0.00636\, t^6 + 18.24t^2 + 0.0001154\, t^6)\,dt$$

$$= 0.00648\left(\frac{t^7}{7}\right)_0^{10} + 18.24\left(\frac{t^3}{3}\right)_0^{10} \cancel{+ 0.0001154\frac{t^7}{}}$$

$$= 9250 + 6080$$

$$E_{loss} = 15330\ J$$

(d) The KE of vehicle is,

$$\Delta KE = \frac{1}{2} m \left[v(10)^2 - v(0)^2 \right]$$

$$= \frac{1}{2} \times 800 \left[(0.29055(10)^2 - 0.29055(0)^2 \right]$$

$$= \frac{1}{2} \times 800 \left[(0.29055\, t^2)^2 \right]_0^{10}$$

$$= \frac{1}{2} \times 800 \left[0.29055\, t^4 \right]_0^{10}$$

$$= \frac{1}{2} \times 800 \left[0.29055\, (10)^4 \right]$$

$$\Delta KE = 337677.21 \text{ J.}$$

∴ Change in tractive Energy is

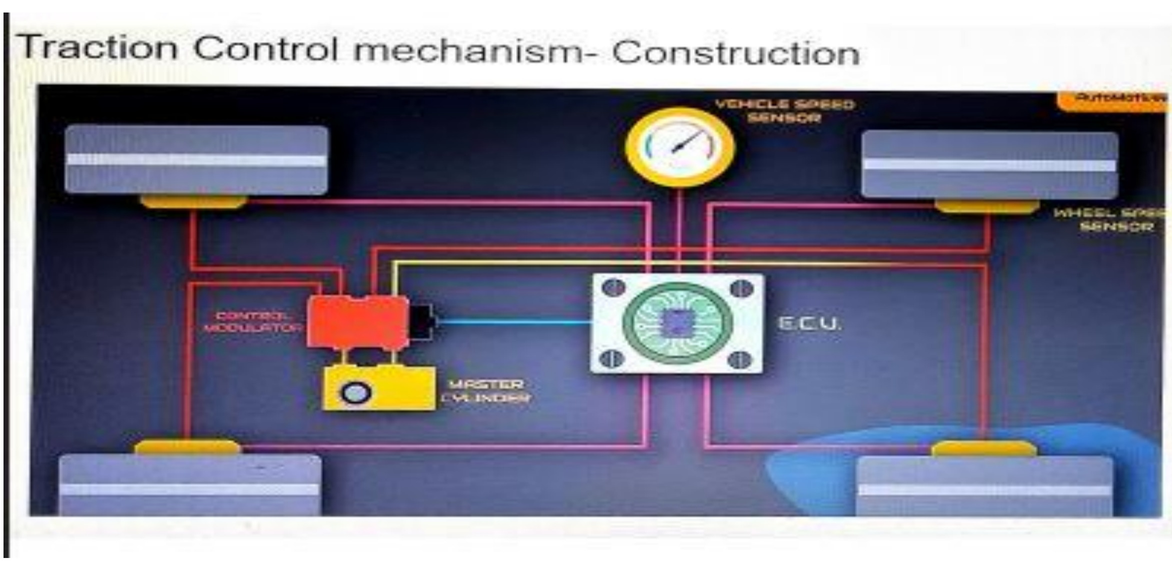
$$\therefore \Delta E_{TR} = \Delta KE + E_{Loss}$$

$$= 337677.21 + \overset{15330}{\cancel{4249.81}}$$

$$\cancel{\Delta E_{TR} = 341927.02 \text{ J}}$$

$$\Delta E_{TR} = 3,53,007.21 \text{ J}$$

5 b) block diagram with proper explanation.



6 a)

Propulsion System Design

- The steady-state maximum velocity, maximum gradeability, and the velocity equations can be used in the design stage to specify the power requirement of a particular vehicle.
- Let us consider the tractive power requirement for initial acceleration, which plays a significant role in determining the rated power of the propulsion unit.
- The design problem is to solve for FTR starting with a set of variables including vehicle mass, rolling resistance, aerodynamic drag coefficient, percent grade, wheel radius, etc., some of which are known, while others have to be assumed.
- The acceleration of the vehicle in terms of these variables is given by

$$a = \frac{dv}{dt} = \frac{F_{TR} - F_{RL}}{m}$$

The tractive force output of the electric motor for an electric vehicle or the combination of electric motor and internal combustion engine for a hybrid electric vehicle will be a function of the vehicle velocity. Furthermore, the road load characteristics are also function of velocity, resulting in a transcendental equation to be solved to determine the desired tractive power from the propulsion unit.

6 b)

Non-Constant F_{TR}, General Acceleration :-

→ Generally, with non-constant F_{TR} & an arbitrary velocity profile, force can be calculated as

→ $$\sum F = m\frac{dv}{dt}$$

$$F_{TR} - F_{AD} - F_{roll} - F_{gxT} = m\frac{dv}{dt}$$

$$F_{TR} = m\frac{dv}{dt} + mg\sin\beta + F_{AD} + F_{roll}.$$

$$F_{TR} = m\frac{dv}{dt} + mg\sin\beta + \left[mgC_0 + \frac{\rho}{2}A_F C_D\right] + mgC_0 \rightarrow (1)$$

> Instantaneous tractive power, $P_{TR}(t)$ is

$$P_{TR}(t) = F_{TR}(t)\,v(t)$$

$$\hookrightarrow = mv\frac{dv}{dt} + v\left[F_{gxT} + F_{AD} + F_{roll}\right] \rightarrow (2)$$

> The change in tractive Energy, Δe_{TR} is

$$\Delta e_{TR} = \int_{t_i}^{t_f} P_{TR}(t)\,dt$$

$$\Delta e_{TR} = m\int_{v(t_i)}^{v(t_f)} v\,dv + \int_{t_i}^{t_f} v\,F_{gxT}\,dt + \int_{t_i}^{t_f} v(F_{AD}+F_{roll})\,dt \rightarrow (3)$$

The Energy Supplied by the propulsion unit is Converted into various forms of Energy, Some of which are Stored in Vehicle System, while others are lost due to the non-Constructive forces.

Let us Consider the first term on right Side of Eq (3)

$$m\int_{v(t_i)}^{v(t_f)} v\,dv = m\left[\frac{v^2}{2}\right]_{v(t_i)}^{v(t_f)}$$

$$= \frac{m}{2}\left[v^2(t_f) - v^2(t_i)\right]$$

$$= \Delta \text{ (Kinetic Energy)}.$$

Also,

$$\int_{t_i}^{t_f} v F_{gxT}\,dt = \int_{t_i}^{t_f} v\, mg \sin\beta\, dt$$

$$= mg\int_{t_i}^{t_f} v \sin\beta\, dt.$$

$$= mg\int_{s(t_i)}^{s(t_f)} \sin\beta\, ds \qquad \left[\because v = \frac{ds}{dt},\ ds = v\,dt\right]$$

$$= mg\int_{f(t_i)}^{f(t_f)} df \qquad \left[\because \sin\beta\, ds = df\right]$$

$$= mg\left[f(t_f) - f(t_i)\right]$$

$$\left[\cancel{\int_{t_i}^{t_f} v F_{gxT}\, dt}\right] = \Delta \text{ (Potential Energy)}.$$

The above term represents change in vertical displacement multiplying by mg, which is change in potential Energy.

from Eq (3), F_{AD} + F_{roll} represents the Energy required to overcome the non-Constructive forces that include the rolling resistance + the aerodynamic drag force. The Energy represented in these terms is Essentially the loss term.

$$\therefore \int_{t_i}^{t_f} v\,(F_{AD} + F_{roll})\,dt = E_{loss}$$

Let, $k_3 = mgC_0$, $k_4 = mgC_1 + (\rho/2)\,C_D A_F$, for $v(t) > 0$

$t_i \le t \le t_f$.

$$E_{loss} = k_3\int_{t_i}^{t_f} \frac{d\cancel{x}}{dt}\,dt + k_4\int_{t_i}^{t_f} v\,dt$$

$$E_{loss} = k_3 \Delta S + k_4 \int_{t_i}^{t_f} v \, dt$$

In summary, we can write

$$\Delta e_{TR} = \frac{1}{2} m \left[v^2(t_f) - v^2(t_i)\right] + mg\left[f(t_f) - f(t_i)\right] + \int_{t_i}^{t_f} v \left(F_{AD} + F_{roll}\right) dt$$

(or)

$$\Delta e_{TR} = \Delta(\text{Kinetic Energy}) + \Delta(\text{Potential Energy}) + E_{loss} \quad //$$

7)

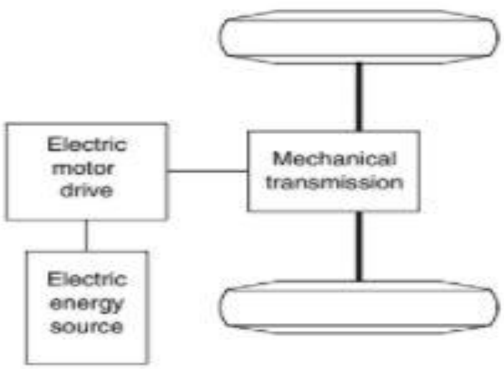


FIGURE 4.1
Primary electric vehicle power train

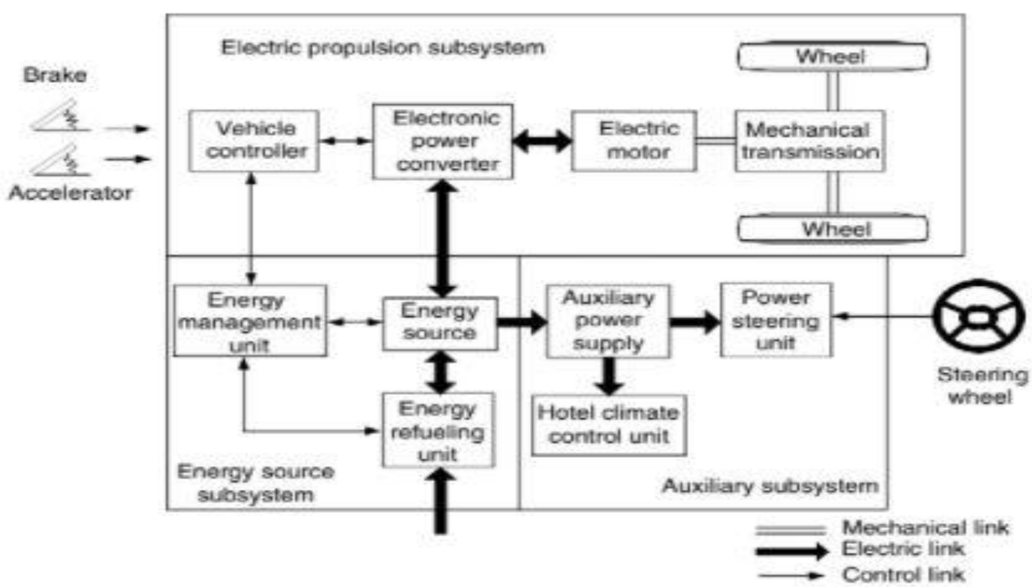


FIGURE 4.2
Conceptual illustration of general EV configuration

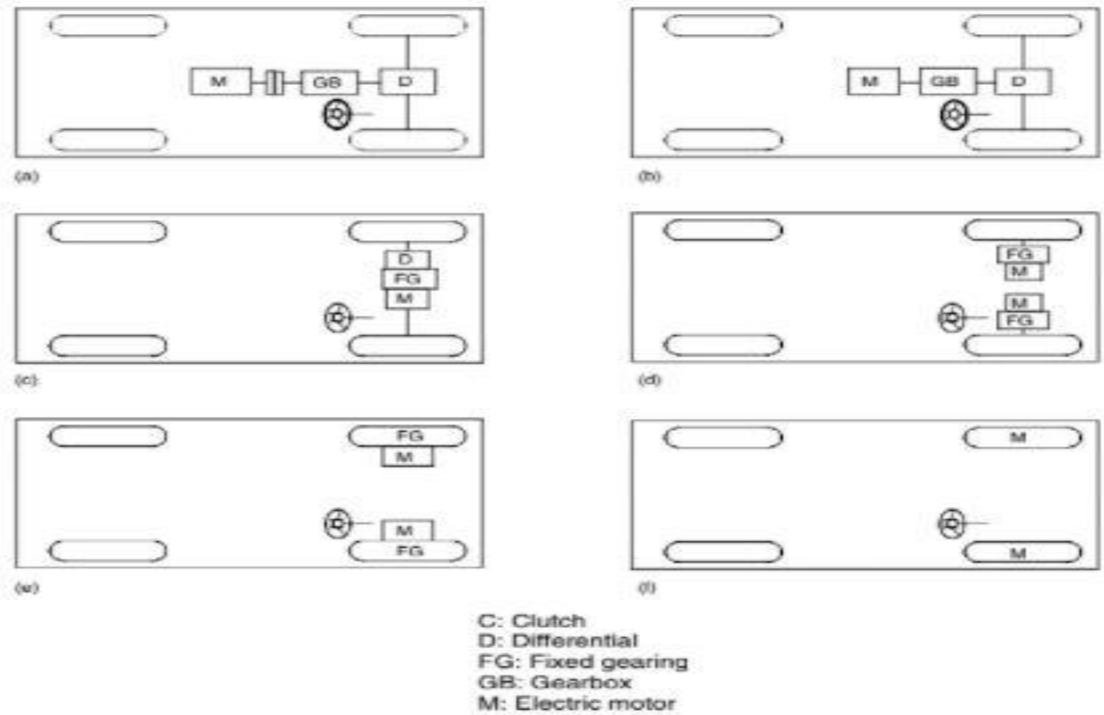


FIGURE 4.3
Possible EV configurations

4.4 Energy Consumption

In transportation, the unit of energy is usually kilowatt-hour (kWh) rather than joule or kilojoule (J or kJ). The energy consumption per unit distance in kWh/km is generally used to evaluate the vehicle energy consumption. However, for ICE vehicles the commonly used unit is a physical unit of fuel volume per unit distance, such as liters per 100 km (l/100 km). In the U.S., the distance per unit volume of fuel is usually used; this is expressed as miles per gallon (mpg). On the other hand, for battery-powered EVs, the original energy consumption unit in kWh, measured at the battery terminals, is more suitable. The battery energy capacity is usually measured in kWh and the driving range per battery charge can be easily calculated. Similar to ICE vehicles, l/100 km (for liquid fuels) or kg/100 km (for gas fuels, such as hydrogen) or mpg, or miles per kilogram is a more suitable unit of measurement for vehicles that use gaseous fuels.

Energy consumption is an integration of the power output at the battery terminals. For propelling, the battery power output is equal to resistance power and any power losses in the transmission and the motor drive, including power losses in electronics. The power losses in transmission and motor drive are represented by their efficiencies η_t and η_m, respectively. Thus, the battery power output can be expressed as

$$P_{b\text{-}out} = \frac{V}{\eta_t \eta_m}\left(M_v g (f_r + i) + \frac{1}{2}\rho_a C_D A_f V^2 + M\delta \frac{dV}{dt}\right). \tag{4.16}$$

Here, the nontraction load (auxiliary load) is not included. In some cases, the auxiliary loads may be too significant to be ignored and should be added to the traction load. When regenerative braking is effective on an EV, a part of that braking energy — wasted in conventional vehicles — can be recovered by operating the motor drive as a generator and restoring it into the batteries. The regenerative braking power at the battery terminals can also be expressed as

$$P_{b\text{-}in} = \frac{\alpha V}{\eta_t \eta_m}\left(M_v g (f_r + i) + \frac{1}{2}\rho_a C_D A_f V^2 + M\delta \frac{dV}{dt}\right), \tag{4.17}$$

where road grade i or acceleration dV/dt or both of them are negative, and α $(0 < \alpha < 1)$ is the percentage of the total braking energy that can be applied by the electric motor, called the regenerative braking factor. The regenerative braking factor α is a function of the applied braking strength and the design of the power train, which will be discussed in detail in the later chapters. The net energy consumption from the batteries is

$$E_{out} = \int_{traction} P_{b\text{-}out}\,dt + \int_{braking} P_{b\text{-}in}\,dt. \tag{4.18}$$

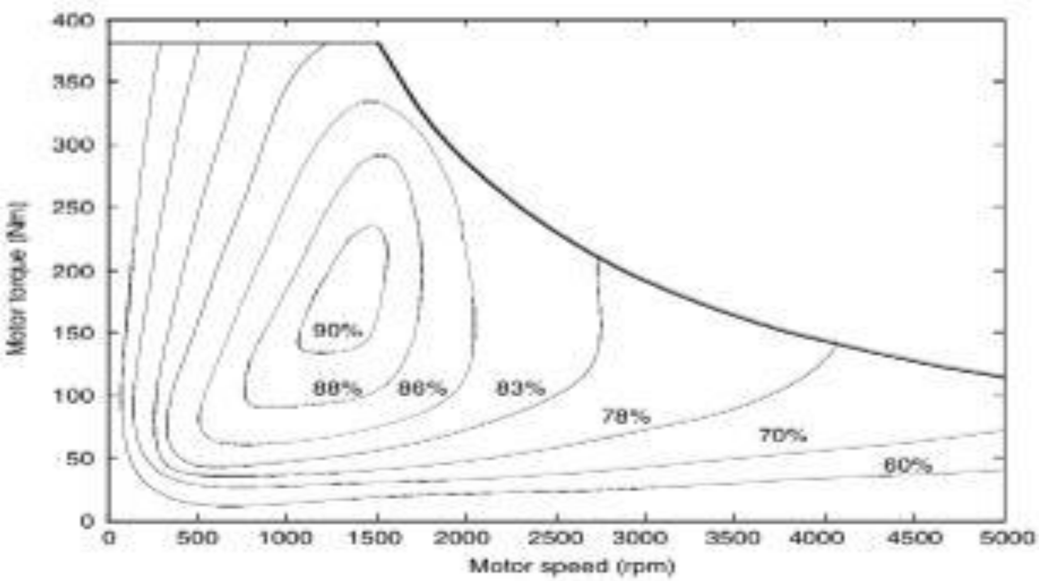


FIGURE 4.14
Typical electric motor efficiency characteristics