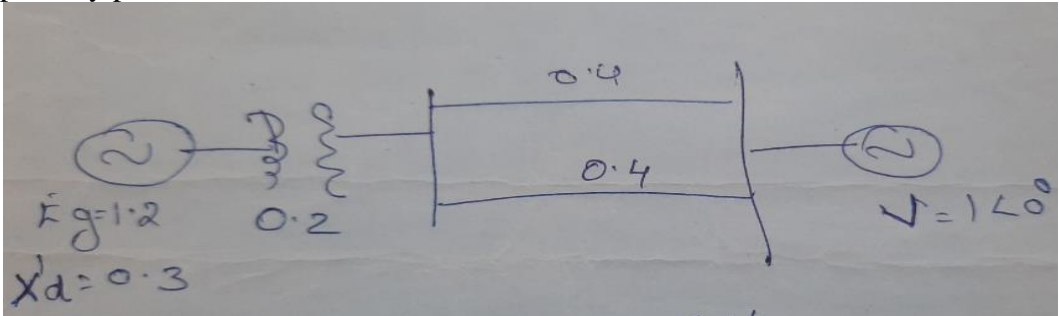


Internal Assessment Test - II

Sub:	Power System Analysis II	Code:	18EE71/17EE71
Date:	16/12/2021	Duration:	90 mins
		Max Marks:	50
		Sem:	7
		Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE																																													
			CO	RBT																																												
1	Deduce the fast decoupled load flow model clearly stating all the assumptions made	[10]	CO3	L3																																												
2	<p>A 50 Hz synchronous generator having an inertia constant $H=5.2$ MJ/MVA and $x_d' = 0.3$ pu is connected to an infinite bus through a double circuit line as shown in fig.1 .The reactance of the connecting HT transformer is 0.2 pu and reactance of each line is 0.4 pu. $E_g = 1.2$ pu and $V = 1.0$ pu and $P_e = 0.8$ pu. Plot the swing curve if a 3 phase fsult occurs at the middle of one of the transmission lines by point by point method.</p>  <p style="text-align: center;">Fig.1</p>	[10]	CO6	L4																																												
3	Derive the expression for all elements of Jacobian matrix in polar form	[10]	CO3	L3																																												
4	<p>Calculate the voltages at all buses for the 3 bus system as shown in fig at the end of first iteration by NR method.</p> <table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th>From bus</th> <th>To bus</th> <th>R(pu)</th> <th>X(pu)</th> <th>B_c/2</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2</td> <td>0</td> <td>0.1</td> <td>0</td> </tr> <tr> <td>1</td> <td>3</td> <td>0</td> <td>0.2</td> <td>0</td> </tr> <tr> <td>2</td> <td>3</td> <td>0</td> <td>0.2</td> <td>0</td> </tr> </tbody> </table> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Bus no</th> <th>P_G</th> <th>Q_G</th> <th>P_L</th> <th>Q_L</th> <th>V_{sp}</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-</td> <td>-</td> <td>-</td> <td>-</td> <td>1.0</td> </tr> <tr> <td>2</td> <td>5.3217</td> <td>-</td> <td>0</td> <td>0</td> <td>1.1</td> </tr> <tr> <td>3</td> <td>-</td> <td>-</td> <td>3.6392</td> <td>0.5339</td> <td>1.0</td> </tr> </tbody> </table>	From bus	To bus	R(pu)	X(pu)	B _c /2	1	2	0	0.1	0	1	3	0	0.2	0	2	3	0	0.2	0	Bus no	P _G	Q _G	P _L	Q _L	V _{sp}	1	-	-	-	-	1.0	2	5.3217	-	0	0	1.1	3	-	-	3.6392	0.5339	1.0	[10]	CO3	L4
From bus	To bus	R(pu)	X(pu)	B _c /2																																												
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1	3	0	0.2	0																																												
2	3	0	0.2	0																																												
Bus no	P _G	Q _G	P _L	Q _L	V _{sp}																																											
1	-	-	-	-	1.0																																											
2	5.3217	-	0	0	1.1																																											
3	-	-	3.6392	0.5339	1.0																																											
5	With the help of flow chart , explain the load flow study procedure with expressions as per Gauss Seidal method for power system having all types of buses	[10]	CO2	L3																																												

6	<p>Determine the voltages at the end of first iteration by Gauss seidal method. Take $\alpha=1.6$. Following is the system data for the load flow solution.</p> <p>The line admittances:</p> <table border="1"> <thead> <tr> <th>Bus code</th> <th>Admittance</th> </tr> </thead> <tbody> <tr> <td>1-2</td> <td></td> </tr> <tr> <td>1-3</td> <td>$2-j8.0$</td> </tr> <tr> <td>2-3</td> <td>$1-j4.0$</td> </tr> <tr> <td>2-4</td> <td>$0.666-j2.664$</td> </tr> <tr> <td>3-4</td> <td>$1-j4.0$</td> </tr> <tr> <td></td> <td>$2-j8.0$</td> </tr> </tbody> </table> <p>The schedule of active and reactive powers:</p> <table border="1"> <thead> <tr> <th>Bus code</th> <th>P</th> <th>Q</th> <th>V</th> <th>Remarks</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-</td> <td>-</td> <td></td> <td></td> </tr> <tr> <td>2</td> <td>0.5</td> <td>0.2</td> <td>1.06</td> <td>Slack</td> </tr> <tr> <td>3</td> <td>0.4</td> <td>0.3</td> <td>$1+j0.0$</td> <td>PQ</td> </tr> <tr> <td>4</td> <td>0.3</td> <td>0.1</td> <td>$1+j0.0$</td> <td>PQ</td> </tr> </tbody> </table> <p>Determine the voltages at the end of first iteration by Gauss seidal method.</p>	Bus code	Admittance	1-2		1-3	$2-j8.0$	2-3	$1-j4.0$	2-4	$0.666-j2.664$	3-4	$1-j4.0$		$2-j8.0$	Bus code	P	Q	V	Remarks	1	-	-			2	0.5	0.2	1.06	Slack	3	0.4	0.3	$1+j0.0$	PQ	4	0.3	0.1	$1+j0.0$	PQ	[10]	CO2	L4
Bus code	Admittance																																										
1-2																																											
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4	0.3	0.1	$1+j0.0$	PQ																																							

Solutions

1

7.8 DECOUPLED
 In the NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected power systems, alternative solution methods are possible, taking into account certain observations made of practical systems. These are:

- Change in voltage magnitude $|V_j|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that $\frac{\partial Q_i}{\partial |V_j|}$ is much larger than $\frac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.
- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\frac{\partial P_i}{\partial \delta_j}$ is much larger than $\frac{\partial Q_i}{\partial \delta_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

These observations reduce (7.60) to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix} \quad \dots(7.87)$$

From (7.87) it is obvious that the voltage angle corrections $\Delta \delta$ are obtained using real power residues ΔP and the voltage magnitude corrections $\frac{\Delta |V|}{|V|}$ are obtained from reactive power residues ΔQ . (7.87) can be solved in two ways.

7.9 FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of Fast Decoupled Load Flow (FDLF) method by B. Stott in 1974. Certain assumptions are made based on observations of practical power systems. They are:

- $B_{ij} \gg G_{ij}$ (Since, the X/R ratio of transmission lines is high in well designed systems)
- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i \ll B_{ii} |V_i|^2$

With these assumptions the elements of the Jacobian become

$$\begin{aligned} H_{ik} &= L_{ik} = -|V_i| |V_k| B_{ik} \quad (i \neq k) \\ H_{ii} &= L_{ii} = -B_{ii} |V_i|^2 \end{aligned}$$

The matrix (7.87) reduces to

$$[\Delta P] = [|V_i| |V_j| B'_{ij}] [\Delta \delta] \quad \dots(7.88a)$$

$$[\Delta Q] = [|V_i| |V_j| B'_{ij}] \left[\frac{\Delta |V|}{|V|} \right] \quad \dots(7.88b)$$

where, B'_{ij} and B''_{ij} are negative of the susceptances of respective elements of the bus admittance matrix. In (7.88) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get

$$\left[\frac{\Delta P}{|V|} \right] = [B'_{ij}] [\Delta \delta] \quad \dots(7.89a)$$

$$\left[\frac{\Delta Q}{|V|} \right] = [B''_{ij}] \left[\frac{\Delta |V|}{|V|} \right] \quad \dots(7.89b)$$

Equations (7.89a) and (7.89b) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers.
- Setting off-nominal turns ratio of transformers to 1.0.
- In forming B'_j , omitting the effect of shunt reactors and capacitors which mainly affect reactive power.
- Ignoring series resistance of lines in forming the Y_{bus} .

With these assumptions we obtain a loss less network. If further, all voltage magnitudes are assumed to be 1.0 pu, we obtain a DC power flow model. This model is acceptable where only approximate solutions are required like in planning expansions and in contingency studies. In the *FDLF* method, the matrices $[B']$ and $[B'']$ are constants and need to be inverted only once at the beginning of the iterations. Separate convergence tests can be applied for real power and reactive power, as $\max [\Delta P] \leq \epsilon_p$ and $\max [\Delta Q] \leq \epsilon_q$. Generally, the tolerances for power mismatch are 0.001 pu.

2

Solution:

Before fault transfer reactance between generator and infinite bus

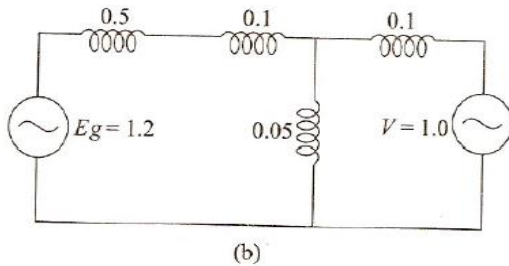
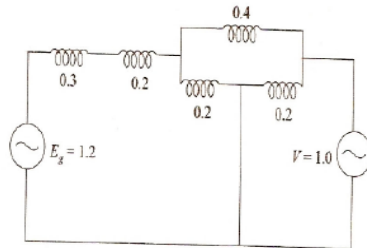
$$X_1 = 0.3 + 0.2 + \frac{0.4}{2} = 0.7 \text{ pu}$$

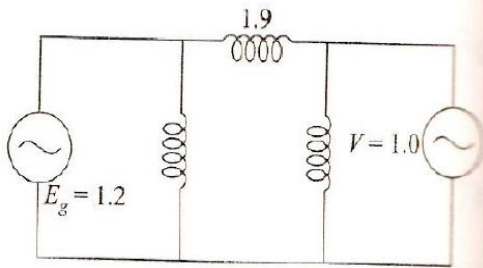
$$P_{\max 1} = \frac{1.2 \times 1.0}{0.7} = 1.714 \text{ pu.}$$

$$\text{Initial } P_e = 0.8 \text{ pu} = P_m$$

$$\text{Initial operating angle } \delta_0 = \sin^{-1} \frac{0.8}{1.714} = 27.82^\circ = 0.485 \text{ rad.}$$

When fault occurs at middle of one of the transmission lines, the network and its reduction is as shown in Fig a to Fig c.





The transfer reactance is 1.9 pu.

$$P_{\max \text{ II}} = \frac{1.2 \times 1.0}{1.9} = 0.63 \text{ pu}$$

After fault

$x_3 = 0.9$

$$P_3 = \frac{1.2 \times 1.0}{0.9} = 1.333 \text{ pu}$$

At $t = 0$, transition from 'no fault' to 'fault'

$(t=0^-)$ $(t=0^+)$

So $P_a = \frac{P_a^{(0^-)} + P_a^{(0^+)}}{2}$

$$P_a^{0^-} = 0.8 - 1.714 \sin 27.82 = 0$$

$$P_a^{0^+} = 0.8 - 0.63 \sin 27.82 = 0.506$$

$$P_a = \frac{0 + 0.506}{2} = \underline{0.253}$$

$$\Delta \omega = \frac{P_a \times \Delta t}{m} = \frac{0.253 \times 0.05}{0.00054} = 23.426$$

$$\omega_1 = \omega_0 + \Delta \omega = 0 + 23.426 = \underline{23.426}$$

$$\Delta \delta_1 = \omega_1 \times \Delta t = 23.426 \times 0.05 = \underline{1.1713}$$

$$\delta_1 = \delta_0 + \Delta \delta_1 = 27.82 + 1.1713 = \underline{28.9913}$$

} 1st interval

At $t = 0.05$

$$P_a = 0.8 - 0.63 \sin 28.9913$$
$$= \underline{0.4947}$$

$$\Delta \omega_2 = \frac{P_a \times \Delta t}{m} = \frac{0.4947 \times 0.05}{0.00054} = \underline{45.806}$$

$$\omega_2 = \omega_1 + \Delta \omega_2 = 23.426 + 45.806$$
$$= 69.2316$$

$$\Delta \delta_2 = \omega_2 \times \Delta t = 3.4615$$

$$\delta_2 = \delta_1 + \Delta \delta_2 = \underline{32.4528}$$

At $t = 0.1 \text{ sec}$

$$P_a = 0.8 - 0.63 \sin 32.45$$
$$= 0.4619$$

$$\Delta \omega_3 = \frac{0.4619 \times 0.05}{0.00054} = \underline{42.80}$$

$$\omega_3 = 69.23 + 42.80 = 112.03$$

$$\Delta \delta_3 = 112.03 \times 0.05 = 5.601$$

$$\delta_3 = 32.45 + 5.601 = 38.05$$

At $t = 0.152\text{sec}$.

$$P_a^{0.15-} = 0.8 - 0.63 \sin 38.04 = 0.4117$$

$$P_a^{0.15+} = 0.8 - 1.33 \sin 38.04 = -0.0195$$

$$P_a = \frac{P_a^{0.15-} + P_a^{0.15+}}{2} = 0.1961$$

$$\Delta\omega = \frac{0.1961 \times 0.05}{0.00054} = 18.157$$

$$\omega = \frac{112}{69.2316} + 18.157 = \frac{130.15}{87.38}$$

$$\Delta\delta = \frac{130.15}{87.38} \times 0.05 = 4.369^\circ \quad 6.5$$

$$\delta = 38.05 + 4.369^\circ = \frac{42.419^\circ}{44.55^\circ}$$

3

has been designed:

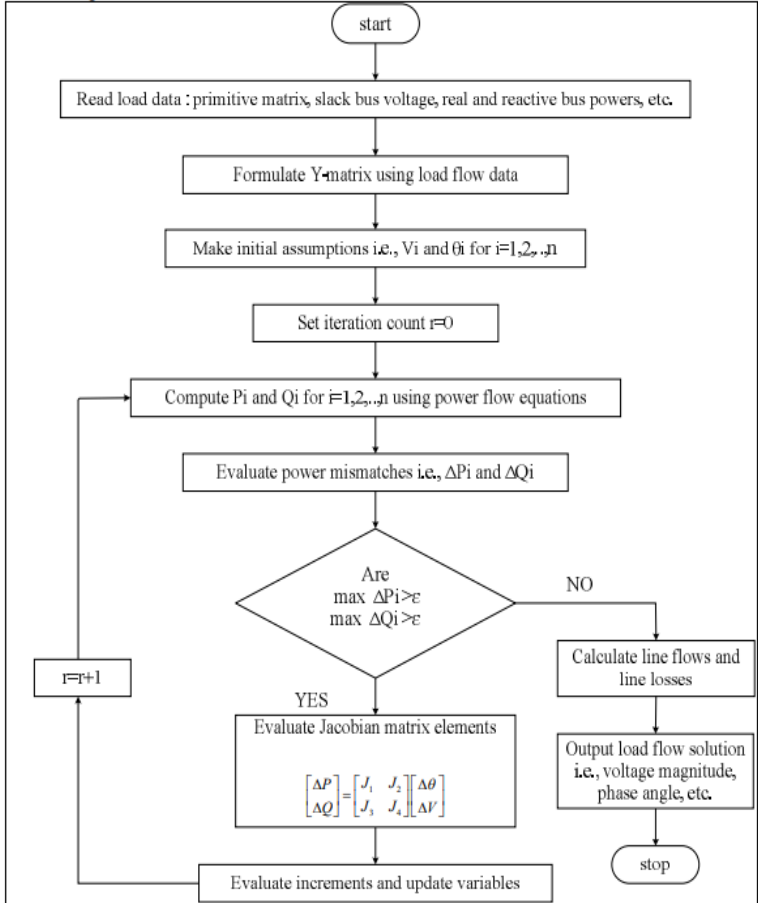


Figure 4.1 Detailed flow chart of Newton-Raphson method

7.7.2 Algorithm for NR Method in Polar Coordinates

1. Formulate the Y_{bus} .
2. Assume initial voltages as follows:

$$V_i = |V_{i,sp}| \angle 0^\circ \text{ (at all PV buses)}$$

$$V_i = 1 \angle 0^\circ \text{ (at all PQ buses)}$$
3. At $(r+1)^{th}$ iteration, calculate $P_i^{(r+1)}$ at all the PV and PQ buses and $Q_i^{(r+1)}$ at all the PQ buses, using voltages from previous iteration, $V_i^{(r)}$. The formulae to be used are:

$$P_{i,cal} = P_i = G_{ii} |V_i|^2 + \sum_{\substack{k=1 \\ k \neq i}}^n |V_i| |V_k| (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik})$$

$$Q_{i,cal} = Q_i = -B_{ii} |V_i|^2 + \sum_{\substack{k=1 \\ k \neq i}}^n |V_i| |V_k| (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik})$$

4. Calculate the power mismatches (power residues).

$$\Delta P_i^{(r)} = P_{i,sp} - P_{i,cal}^{(r+1)} \text{ (at PV and PQ buses)}$$

$$\Delta Q_i^{(r)} = Q_{i,sp} - Q_{i,cal}^{(r+1)} \text{ (at PQ buses)}$$
5. Calculate the Jacobian $[J^{(r)}]$ using $V_i^{(r)}$.

6. Compute

$$\begin{bmatrix} \Delta \delta^{(r)} \\ \frac{\Delta |V^{(r)}|}{|V|} \end{bmatrix} = [J^{(r)}]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$$

7. Update the variables as follows:

$$\delta_i^{(r+1)} = \delta_i^{(r)} + \Delta \delta_i^{(r)} \text{ (at all buses)}$$

$$|V_i|^{(r+1)} = |V_i|^{(r)} + \Delta |V_i|^{(r)}$$
8. Go to step 3 and iterate till the power mismatches are within acceptable tolerance.

Formulate Y_{bus} :

$$Y_{bus} = \begin{bmatrix} -j15.0 & j10.0 & j5.0 \\ j10.0 & -j15.0 & j5.0 \\ j5.0 & j5.0 & -j10.0 \end{bmatrix}$$

Assumes:

$$V_1 = 1.0 + j0.0 = 1.0 \angle 0^\circ$$

$$V_2 = 1.1 + j0.0 = 1.1 \angle 0^\circ$$

$$V_3 = 1.0 + j0.0 = 1.0 \angle 0^\circ$$

$$P_{2,sp} = 5.3217; P_{3,sp} = -3.6392 \text{ (since it is a load } P_{sp} \text{ is negative)}$$

$$P_{2,cat} = P_2 = G_{22}|V_2|^2 + |V_2||V_1|(G_{21} \cos \delta_{21} + B_{21} \sin \delta_{21}) \\ + |V_2||V_3|(G_{23} \cos \delta_{23} + B_{23} \sin \delta_{23})$$

$$\delta_{21} = \delta_2 - \delta_1 = 0^\circ; \delta_{23} = \delta_2 - \delta_3 = 0^\circ; G_{22} = 0.0$$

$$P_{2,cat} = 0.0$$

$$\Delta P_2 = 5.3217 - 0.0 = 5.3217.$$

$$P_{3,cat} = P_3 = G_{33}|V_3|^2 + |V_3||V_1|(G_{31} \cos \delta_{31} + B_{31} \sin \delta_{31}) \\ + |V_3||V_2|(G_{32} \cos \delta_{32} + B_{32} \sin \delta_{32})$$

$$= 0.0$$

$$\Delta P_3 = -3.6392 - 0.0 = -3.6392.$$

$$Q_{3,sp} = -0.5339.$$

$$Q_{3,cat} = Q_3 = -B_{33}|V_3|^2 + |V_3||V_1|(G_{31} \sin \delta_{31} - B_{31} \cos \delta_{31}) \\ + |V_3||V_2|(G_{32} \sin \delta_{32} - B_{32} \cos \delta_{32})$$

$$= 10.0(1.0)^2 + (1.0 \times 1.0 \times -5.0)(1.0 \times 1.1 \times -5.0)$$

$$= 10.0 - 5.0 - 5.5 = -0.5 \text{ pu.}$$

$$\Delta Q_3 = -0.5339 - (-0.5) = -0.0339 \text{ pu.}$$

The matrix for solution is given by

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} H_{22} & H_{23} & N_{23} \\ H_{32} & H_{33} & N_{33} \\ M_{32} & M_{33} & L_{33} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \frac{\Delta |V_3|}{|V_3|} \end{bmatrix}$$

* The suffixes in the Jacobian element indicate the bus numbers of the variables (for example H_{22} indicates $\frac{\partial P_2}{\partial \delta_2}$) and not their positions in the matrix.

$$\begin{aligned} H_{22} &= -Q_2 - B_{22} |V_2|^2 \\ Q_2 &= -B_{22} |V_2|^2 + |V_2| |V_1| (G_{21} \sin \delta_{21} - B_{21} \cos \delta_{21}) \\ &\quad + |V_2| |V_3| (G_{23} \sin \delta_{23} - B_{23} \cos \delta_{23}) \\ &= 15 (1.1)^2 + (1.1 \times 1.0 - 10.0) + (1.1 \times 1.0 \times -5.0) \\ &= 18.15 - 11.0 - 5.5 = 1.65 \\ H_{22} &= -1.65 + 15 \times (1.1)^2 = 16.5 \\ H_{23} &= |V_2| |V_3| (G_{23} \sin \delta_{23} - B_{23} \cos \delta_{23}) \\ &= 1.1 \times 1.0 \times -5.0 = -5.5 \\ H_{32} &= |V_3| |V_2| (G_{32} \sin \delta_{32} - B_{32} \cos \delta_{32}) \\ &= 1.1 \times 1.0 \times -5.0 = -5.5 \\ H_{33} &= -Q_3 - B_{33} |V_3|^2 = 0.5 + 10 = 10.5 \\ N_{23} &= |V_2| |V_3| (G_{23} \cos \delta_{23} + B_{23} \sin \delta_{23}) = 0.0 \\ N_{33} &= P_3 + G_{33} |V_3|^2 = 0.0 \\ M_{32} &= -N_{23} = 0.0 \\ M_{33} &= P_3 - G_{33} |V_3|^2 = 0.0 \\ L_{33} &= Q_3 - B_{33} |V_3|^2 = -0.5 + 10.0 = 9.5 \end{aligned}$$

We therefore get the matrix,

$$\begin{bmatrix} 5.3217 \\ -3.6392 \\ -0.0339 \end{bmatrix} = \begin{bmatrix} 16.5 & -5.5 & 0.0 \\ -5.5 & 10.5 & 0.0 \\ 0.0 & 0.0 & 9.5 \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \frac{\Delta |V_3|}{|V_3|} \end{bmatrix}$$

$$[J]^{-1} = \begin{bmatrix} 0.0734 & 0.0385 & 0.0 \\ 0.0385 & 0.1154 & 0.0 \\ 0.0 & 0.0 & 0.1053 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \frac{\Delta|V_3|}{|V_3|} \end{bmatrix} = \begin{bmatrix} 0.0734 & 0.0385 & 0.0 \\ 0.0385 & 0.1154 & 0.0 \\ 0.0 & 0.0 & 0.1053 \end{bmatrix} \begin{bmatrix} 5.3217 \\ -3.6392 \\ -0.0339 \end{bmatrix} = \begin{bmatrix} 0.2508 \\ -0.2152 \\ -0.0036 \end{bmatrix}$$

$$\Delta\delta_2 = 0.2508 \text{ rad} = 14.37^\circ$$

$$\delta_2 = 0.0 + 14.37^\circ = 14.37^\circ$$

$$\Delta\delta_3 = -0.2152 \text{ rad} = -12.33^\circ$$

$$\delta_3 = 0.0 - 12.33^\circ = -12.33^\circ$$

$$\frac{\Delta|V_3|}{|V_3|} = -0.0036; \Delta|V_3| = -0.0036 \times 1.0 = -0.0036.$$

$$|V_3| = 1.0 - 0.0036 = 0.9964.$$

rst iteration

$$V_1 = 1.0 \angle 0^\circ = 1.0 + j0.0$$

$$V_2 = 1.1 \angle 14.37^\circ = 1.065584 + j0.273$$

$$V_3 = 0.9964 \angle -12.33^\circ = 0.97342 - j0.212773$$

Development of load flow equations.

Nodal current equations

$$I_p = \sum_{q=1}^n Y_{pq} V_q \quad p=1, 2, \dots, n$$

$$I_p = Y_{pp} V_p + \sum_{\substack{q=1 \\ q \neq p}}^n Y_{pq} V_q$$

$$V_p \left(\frac{I_p}{Y_{pp}} - \sum_{\substack{q=1 \\ q \neq p}}^n Y_{pq} V_q \right) = Y_{pp} V_p$$

$$V_p = \frac{I_p}{Y_{pp}} - \frac{1}{Y_{pp}} \sum_{\substack{q=1 \\ q \neq p}}^n Y_{pq} V_q$$

$$V_p I_p^* = P_p + jQ_p$$

$$V_p^* I_p = P_p - jQ_p$$

$$I_p = \frac{P_p - jQ_p}{V_p^*}$$

Substituting.

$$V_p = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{V_p^*} - \sum_{\substack{q=1 \\ q \neq p}}^n Y_{pq} V_q \right] \quad p=1, 2, \dots, n$$

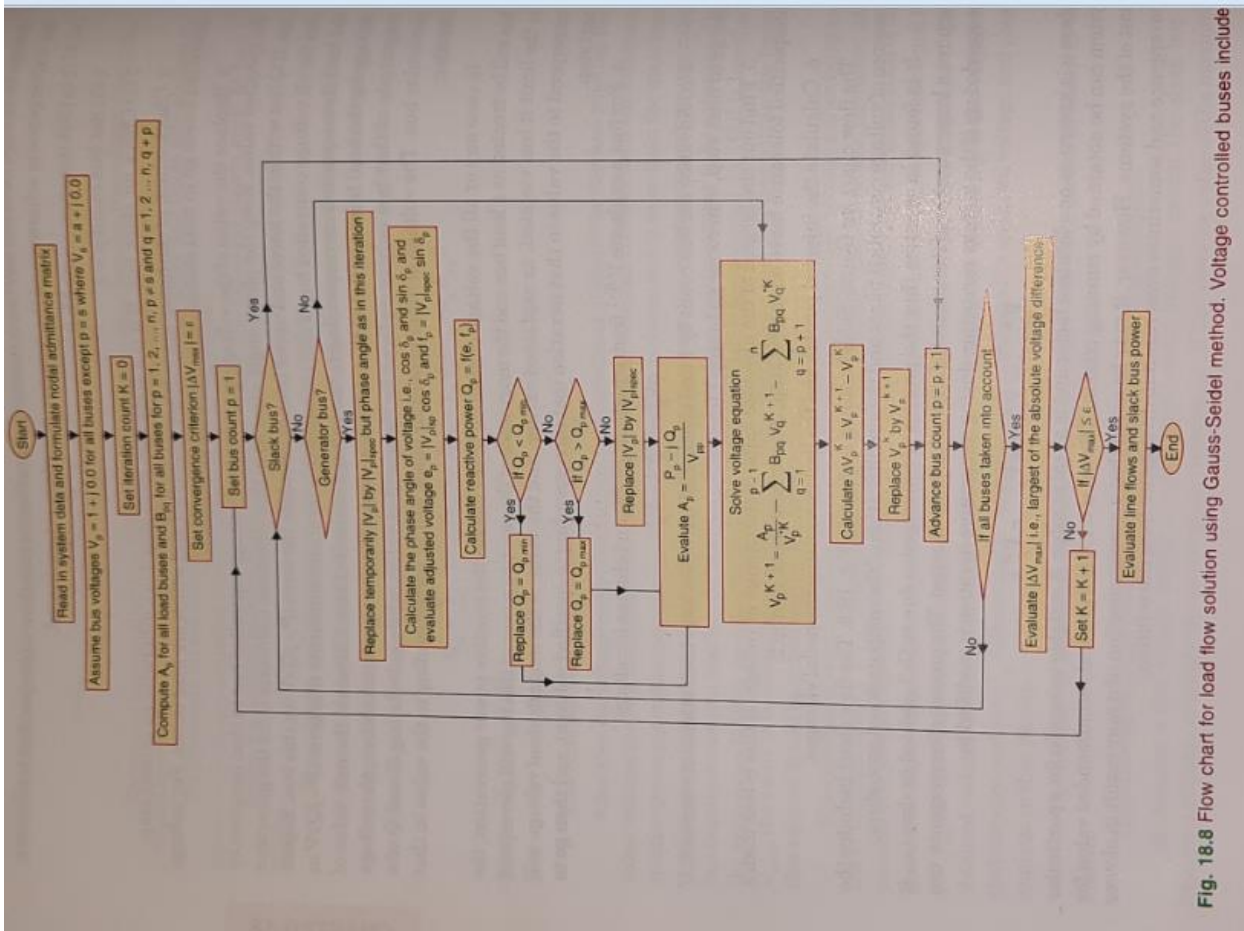


Fig. 18.8 Flow chart for load flow solution using Gauss-Seidel method. Voltage controlled buses include

Example 18.1: The following is the system data for a load flow solution:
The line admittances:

Bus code	Admittance
1-2	
1-3	$2-j8.0$
2-3	$1-j4.0$
2-4	$0.666-j2.664$
3-4	$1-j4.0$
	$2-j8.0$

The schedule of active and reactive powers:

Bus code	P	Q	V	Remarks
1	-	-		
2	0.5	0.2	1.06	Slack
3	0.4	0.3	$1+j0.0$	PQ
4	0.3	0.1	$1+j0.0$	PQ
			$1+j0.0$	PQ

Determine the voltages at the end of first iteration using Gauss-Seidel method. Take $\alpha = 1.6$.

Solution: The admittance matrix will be as given below:

$$Y_{pq} = \begin{bmatrix} 3-j12.0 & -2+j8.0 & -1+j4.0 & 0.0 \\ -2+j8.0 & 3.666-j14.664 & -0.666+j2.664 & -1+j4.0 \\ -1+j4.0 & -0.666+j2.664 & 3.666-j14.664 & -2+j8.0 \\ 0.0 & -1+j4.0 & -2+j8.0 & 3-j12.0 \end{bmatrix}$$

The powers for load buses are to be taken as negative and that for generator buses as positive.

For the system given

$$\begin{aligned} V_2^1 &= \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1^0 - Y_{23}V_3^0 - Y_{24}V_4^0 \right] \\ &= \frac{1}{(3.666 - j14.664)} \left[\frac{-0.5 + j0.2}{1 - j0.0} - 1.06(-2 + j8) - 1.0 \right. \\ &\quad \left. (-0.666 + j2.664) - (-1 + j4.0)1.0 \right] \\ &= (1.01187 - j0.02888) \\ V_{2\text{acc}}^1 &= (1.0 + j0.0) + 1.6(1.01187 - j0.02888 - 1.0 - j0.0) \\ &= 1.01899 - j0.046208 \quad \text{Ans.} \end{aligned}$$

$$V_3^1 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{V_3^{0*}} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0 \right]$$

$$= \frac{1}{(3.666 - j14.664)} \left[\frac{-0.4 + j0.3}{1 - j0.0} - (-1 + j4.0)1.06 \right. \\ \left. - (-0.666 + j2.664)(1.01899 - j0.046208) - (-2 + j8)(1 + j0.0) \right]$$

$$= 0.994119 - j0.029248$$

$$V_{3\text{acc}}^1 = (1 + j0.0) + 1.6[0.994119 - j0.029248 - 1 - j0.0]$$

$$= 0.99059 - j0.0467968 \quad \text{Ans.}$$

$$V_4^1 = \frac{1}{Y_{44}} \left[\frac{P_4 - jQ_4}{V_4^{0*}} - Y_{42}V_2^1 - Y_{43}V_3^1 \right]$$

$$= \frac{1}{(3 - j12)} \left[\frac{-0.3 + j0.1}{1 - j0.0} - (-1 + j4.0)(1.01899 - j0.046208) \right. \\ \left. - (-2 + j8)(0.99059 - j0.0467968) \right]$$

$$= 0.9716032 - j0.064684$$

$$V_{4\text{acc}}^1 = 1.0 + j0.0 + 1.6[0.9716032 - j0.064684 - 1 - j0.0]$$

$$= 0.954565 - j0.1034944 \quad \text{Ans.}$$