
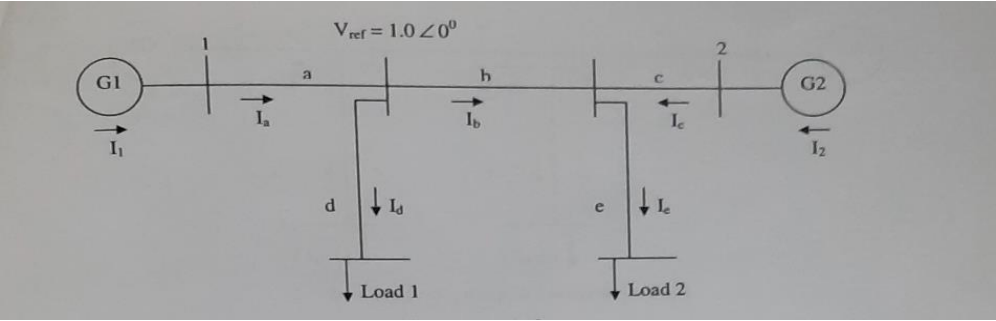


Internal Assessment Test - III

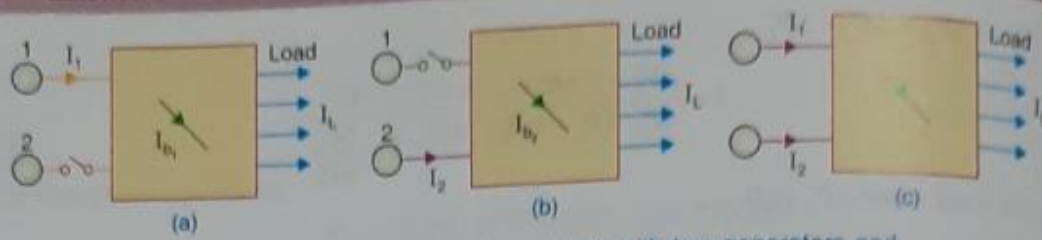
Sub:	Power System Analysis II	Code:	18EE71/17EE71
Date:	24/01/2022	Duration:	90 mins
		Max Marks:	50
		Sem:	7
		Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE																
			CO	RBT															
1	With a usual notation derive generalized transmission loss formula and B – coefficients.	[10]	CO3	L3															
2	Explain the method of equal incremental cost for the economic operation of generators with transmission loss considered.	[10]	CO3	L2															
3	A two bus system is shown in fig. If a load of 120 MW is transmitted from plant 1 to the load, a loss of 16 MW is incurred. Determine the generation schedule and the load demand if the cost of received power is Rs 25 /MWhr. Solve the problem using coordination equations and the penalty factor method approach. The incremental production costs of the plants are $dF_1/dP_1=0.03 P_1+20$ $dF_2/dP_2=0.04 P_2+15$ <div style="text-align: center; margin-top: 10px;">  </div>	[10]	CO4	L2															
4	Derive the algorithm for the formation of bus impedance matrix $Z_{bus}$ for a single phase system when a link element is added to the partial network.	[10]	CO5	L3															
5	For the positive sequence network data shown in table below, obtain Z bus by building procedure.	[10]	CO5	L4															
	<table border="1" style="width: 100%; border-collapse: collapse; margin-top: 10px;"> <thead> <tr> <th>#</th> <th>P-Q(nodes)</th> <th>Pos.Seq. reactance in pu</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0-1</td> <td>0.25</td> </tr> <tr> <td>2</td> <td>0-3</td> <td>0.20</td> </tr> <tr> <td>3</td> <td>1-2</td> <td>0.08</td> </tr> <tr> <td>4</td> <td>2-3</td> <td>0.06</td> </tr> </tbody> </table>	#	P-Q(nodes)	Pos.Seq. reactance in pu	1	0-1	0.25	2	0-3	0.20	3	1-2	0.08	4	2-3	0.06			
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3	1-2	0.08																	
4	2-3	0.06																	
6	Calculate the loss coefficient in pu and $MW^{-1}$ on a base of 50 MVA for the network as shown in figure. Corresponding data is given below.	[10]	CO4	L3															
	<div style="text-align: center; margin-top: 10px;">  </div>																		

$I_a=1.2 - j0.4$ pu	$Z_a= 0.02+j 0.08$ pu
$I_b=0.4 - j0.2$ pu	$Z_b= 0.08+j 0.32$ pu
$I_c=0.8 - j0.1$ pu	$Z_c= 0.02+j 0.08$ pu
$I_d=0.8 - j0.2$ pu	$Z_d= 0.03+j 0.12$ pu
$I_e=1.2 - j0.3$ pu	$Z_e= 0.03+j 0.12$ pu

CO5	L3
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**Fig. 19.6** Schematic diagram of a network with two generators and large number of loads. One branch of the network is shown.

Consider one three phase line  $b$ . The total load current  $I_L$  is supplied by source 1 and the current in line  $b$  is  $I_{b1}$

Let 
$$\alpha_{b1} = \frac{I_{b1}}{I_L} \quad (19.13)$$

Similarly when source 2 alone supplies load current  $I_L$  and if  $I_{b2}$  is the current through the same line  $b$ , then

$$\alpha_{b2} = \frac{I_{b2}}{I_L} \quad (19.14)$$

When both the plants are connected to supply the load current, the current through the branch  $b$ , using the principle of superposition, will be

$$I_b = \alpha_{b1} I_1 + \alpha_{b2} I_2 \quad (19.15)$$

where  $I_1$  and  $I_2$  are the currents from plant 1 and 2 respectively and  $\alpha_{b1}$  and  $\alpha_{b2}$  are the current distribution factors.

Here we make two simplifying assumptions for deriving the loss coefficients.

- (i) The ratio  $X/R$  for all the transmission lines is the same.
- (ii) The phase angle of all the load currents is the same.

The net effect of these two assumptions is that the load currents and branch currents are in phase and hence the current distribution factors are real.

Let  $I_1 = |I_1| \cos \theta_1 + j |I_1| \sin \theta_1$   
and  $I_2 = |I_2| \cos \theta_2 + j |I_2| \sin \theta_2$  (19.16)

where  $\theta_1$  and  $\theta_2$  are the phase angles of currents  $I_1$  and  $I_2$  with reference to a common phasor.

$\therefore I_b = \alpha_{b_1} I_1 + \alpha_{b_2} I_2$   
 $= \alpha_{b_1} |I_1| \cos \theta_1 + \alpha_{b_2} |I_2| \cos \theta_2 + j[\alpha_{b_1} |I_1| \sin \theta_1 + \alpha_{b_2} |I_2| \sin \theta_2]$   
or  $I_b^2 = \alpha_{b_1}^2 |I_1|^2 + \alpha_{b_2}^2 |I_2|^2 + 2\alpha_{b_1} \alpha_{b_2} |I_1| |I_2| \cos (\theta_1 - \theta_2)$  (19.17)

Since  $I_1 = \frac{P_1}{\sqrt{3} |V_1| \cos \phi_1}$  and  $I_2 = \frac{P_2}{\sqrt{3} |V_2| \cos \phi_2}$  and if  $R_b$  is the resistance of branch  $b$ , the total loss will be

$$P_L = \sum_{b=1}^b 3I_b^2 R_b$$

where  $\Sigma$  is the summation of losses in all the branches and  $\phi_1$  and  $\phi_2$  are the p.f. angles of plants 1 and 2 respectively.

$$P'_L = \sum \alpha_{b_1}^2 \frac{P_1^2 R_b}{|V_1|^2 \cos^2 \phi_1} + \sum \alpha_{b_2}^2 \frac{P_2^2 R_b}{|V_2|^2 \cos^2 \phi_2} + \sum \frac{2P_1 P_2 R_b \alpha_{b_1} \alpha_{b_2} \cos (\theta_1 - \theta_2)}{|V_1| |V_2| \cos \phi_1 \cos \phi_2}$$

$$= B_{11} P_1^2 + B_{22} P_2^2 + 2B_{12} P_1 P_2$$
 (19.18)

where

$$B_{11} = \sum \frac{\alpha_{b_1}^2 R_b}{|V_1|^2 \cos^2 \phi_1} = \frac{1}{|V_1|^2 \cos^2 \phi_1} \sum \alpha_{b_1}^2 R_b$$

$$B_{22} = \frac{1}{|V_2|^2 \cos^2 \phi_2} \sum \alpha_{b_2}^2 R_b$$

and

$$B_{12} = \frac{1}{|V_1| |V_2| \cos \phi_1 \cos \phi_2} \sum \alpha_{b_1} \alpha_{b_2} R_b$$

Therefore, if there are  $n$  number of sources the general loss coefficients will be

$$B_{nn} = \frac{1}{|V_n|^2 \cos^2 \phi_n} \sum \alpha_{b_n}^2 R_b$$
 (19.19)

and

$$B_{mn} = \frac{1}{|V_m| |V_n| \cos \phi_m \cos \phi_n} \sum \alpha_{b_m} \alpha_{b_n} R_b$$
 (19.20)

Whenever there are wide variations in the operating conditions, the various assumptions for evaluation of  $B$ -coefficients may cause errors in loss calculation, it is desirable to obtain one or two additional sets of loss coefficients for such conditions.

Many power companies, however, obtain reasonably accurate results assuming one set of coefficients corresponding to a typical operating condition.

### 19.3 OPTIMUM LOAD DISPATCH INCLUDING TRANSMISSION LOSSES

Before an optimum strategy for load scheduling is derived, the need for inclusion of losses is further stressed by the following example.

Consider Fig. 19.3 which consists of two identical generators *i.e.*, generators with identical incremental production cost. If generator 2 has a local load, according to equal incremental production criterion, the total load must be shared equally by both the generators, *i.e.*, each generator should supply half of the total load. The common sense tells us that it is more economical to let generator 2 supply most of the local load because generator 1 has to supply in addition to the load, the transmission losses also. Therefore, the criterion of sharing load by equal incremental production cost does not hold good under such situation and a strategy must be evolved which takes into account the transmission losses also.

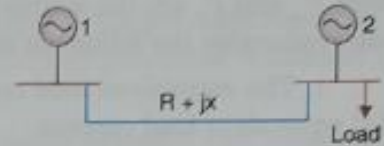


Fig. 19.3 Two identical generators connected through a transmission link.

The optimal load dispatch problem including transmission losses is defined as

$$\text{Min } F_T = \sum_{n=1}^n F_n \quad (19.4)$$

$$\text{Subject to } P_D + P_L - \sum_{n=1}^n P_n = 0 \quad (19.5)$$

where  $P_L$  is the total system loss which is assumed to be a function of generation and the other term have their usual significance.

Making use of the Lagrangian multiplier  $\lambda$ , the auxiliary function is given by

$$F = F_T + \lambda(P_D + P_L - \sum P_n)$$

The partial differential of this expression when equated to zero gives the condition for optimal load dispatch, *i.e.*,

$$\frac{\partial F}{\partial P_n} = \frac{\partial F_T}{\partial P_n} + \lambda \left( \frac{\partial P_L}{\partial P_n} - 1 \right) = 0$$

or

$$\frac{dF_n}{dP_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda \quad (19.6)$$

Here the term  $\frac{\partial P_L}{\partial P_n}$  is known as the incremental transmission loss at plant  $n$  and  $\lambda$  is

known as the incremental cost of received power in ₹ per MWhr.

The equation (19.6) is a set of  $n$  equations with  $(n + 1)$  unknowns. Here  $n$  generations are unknown and  $\lambda$  is also unknown. These equations are known as coordination equations because they coordinate the incremental transmission losses with the incremental cost of production.

To solve these equations the loss formula equation (19.7) is expressed in terms of generations and is approximately expressed as

$$P_L = \sum_m \sum_n P_m B_{mn} P_n \quad (19.7)$$

where  $P_m$  and  $P_n$  are the source loadings,  $B_{mn}$  the transmission loss coefficients. The formula is derived under the following assumptions:

1. The equivalent load current at any bus remains a constant complex fraction of the total equivalent load current.
2. The generator bus voltage magnitudes and angles are constant.
3. The power factor of each source is constant.

The solution of coordination equation (19.6) requires the calculation of  $\partial P_L / \partial P_n$  which is obtained from equation (19.7) as

$$\frac{\partial P_L}{\partial P_n} = 2 \sum_m B_{mn} P_m \quad (19.8)$$

Also

$$\frac{dF_n}{dP_n} = F_{nn} P_n + f_n$$

∴ The coordination equations can be rewritten as

$$F_{nn} P_n + f_n + \lambda \sum_m 2B_{mn} P_m = \lambda \quad (19.9)$$

Collecting all coefficients of  $P_n$ , we obtain

$$P_n (F_{nn} + 2\lambda B_{nn}) = -\lambda \left( \sum_{m \neq n} 2B_{mn} P_m \right) - f_n + \lambda$$

Solving for  $P_n$  we obtain

$$P_n = \frac{1 - \frac{f_n}{\lambda} \sum_{m \neq n} 2B_{mn} P_m}{\frac{F_{nn}}{\lambda} + 2B_{nn}} \quad (19.10)$$

To arrive at an optimal load dispatching solution, the simultaneous solution of the coordination equations along with the equality constraint (19.5) should suffice and any standard matrix inversion subroutine could be used. But, because of the fact that plants might go beyond their loading conditions, it becomes necessary to solve a new set of equations and thus by the

The following steps are required for the iterative procedure:

1. Assume a suitable value of  $\lambda^0$ . This value should be more than the largest intercept of the incremental production cost of the various generators.
2. Calculate the generations based on equal incremental production cost.
3. Calculate the generation at all the buses using the equation

$$P_n = \frac{1 - \frac{f_n}{\lambda} - \sum_{m \neq n} 2B_{mn} P_m}{\frac{F_{nn}}{\lambda} + 2B_{nn}}$$

It is to be noted that the powers to be substituted on the right hand side during zeroth iteration correspond to the values as calculated in step 2. For subsequent iterations the values of powers to be substituted correspond to the powers as calculated in the previous iteration. In case any of the generations violates the limit the generation of that generator is fixed at the limit violated.

4. Check if the difference in power at all generator buses between two consecutive iterations is less than a prespecified value. If not, go back to step 3.
5. Calculate losses using the relation

$$P_L = \sum_m \sum_n P_n B_{mn} P_m$$

and calculate

$$\Delta P = | \Sigma P_G - P_L - P_D |$$

6. If  $\Delta P$  is less than  $\epsilon$ , stop calculation and calculate cost of generation with these values of powers.
7. Update value of  $\lambda$  and go back to step 3. The flow chart is given in Fig. 19.4.

Co-ordination eq<sup>n</sup> for plant 1,

$$\frac{dF_1}{dP_1} + \lambda \frac{\partial P_L}{\partial P_1} = \lambda$$

$$0.03P_1 + 20 + 25(0.002P_1) = 25$$

$$0.03P_1 + 20 + 0.05P_1 = 25$$

$$0.08P_1 = 5$$

$$P_1 = \underline{\underline{62.5 \text{ MW}}}$$

Co-ordination eq<sup>n</sup> for plant 2,

$$\frac{dF_2}{dP_2} + \lambda \frac{\partial P_L}{\partial P_2} = \lambda$$

$$\frac{\partial P_L}{\partial P_2} = 0$$

$$0.04P_2 + 15 = 25$$

$$P_2 = \underline{\underline{250 \text{ MW}}}$$

Transmission loss

$$\textcircled{1} \Rightarrow P_L = 0.001 P_1^2$$

$$= 0.001 (62.5)^2$$

$$P_L = 3.906 \text{ MW}$$

$$\text{load } P_D = 62.5 + 250 - 3.906$$

$$P_D = \underline{\underline{308.594 \text{ MW}}}$$

~~Solution using Penalty factor,~~

$$\frac{dF_1}{dP_1} \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \lambda$$



$$\frac{0.03P_1 + 20}{1 - 0.002P_1} = 25.$$

$$0.03P_1 + 20 = 25 - 0.05P_1$$

$$0.08P_1 = 5$$

$$P_1 = \underline{\underline{62.5 \text{ MW}}}$$

Since  $\frac{\partial P_1}{\partial P_2} = 0 \therefore L_2$  is unity

incremental cost of received power equals incremental cost of production.

$$0.04P_2 + 15 = 25$$

$$P_2 = \underline{\underline{250 \text{ MW}}}$$

4

#### Addition of a link

If the added element  $p-q$  is a link, the procedure for recalculating the elements of the bus impedance matrix is to connect in series with the added element a voltage source  $e_l$  as shown in Fig. 4.4. This creates a

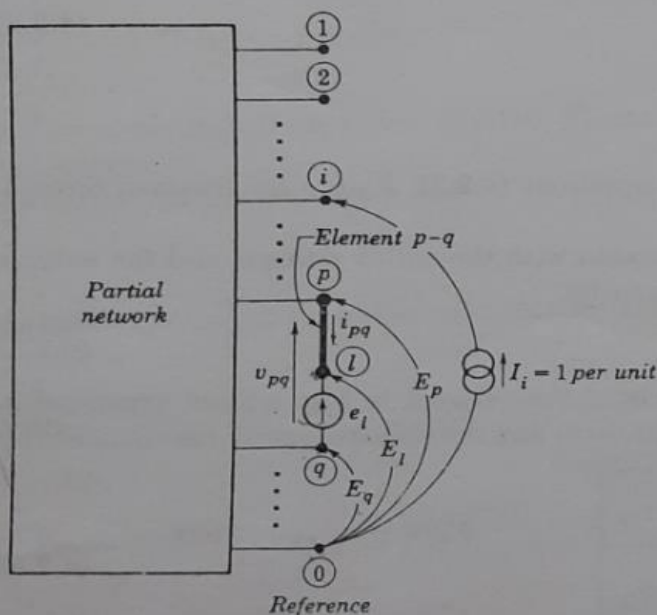


Fig. 4.4 Injected current voltage source in series with added link and bus voltages for calculation of  $Z_{ii}$ .

The performance equation for the partial network with the added element  $p-l$  and the series voltage source  $e_l$  is

		1	$p$				$m$	$l$		
$E_1$	1	$Z_{11}$	$Z_{12}$	$\dots$	$Z_{1p}$	$\dots$	$Z_{1m}$	$Z_{1l}$	$I_1$	
$E_2$		$Z_{21}$	$Z_{22}$	$\dots$	$Z_{2p}$	$\dots$	$Z_{2m}$	$Z_{2l}$	$I_2$	
$\dots$		$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	
$E_p$	$= p$	$Z_{p1}$	$Z_{p2}$	$\dots$	$Z_{pp}$	$\dots$	$Z_{pm}$	$Z_{pl}$	$I_p$	
$\dots$		$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	
$E_m$	$m$	$Z_{m1}$	$Z_{m2}$	$\dots$	$Z_{mp}$	$\dots$	$Z_{mm}$	$Z_{ml}$	$I_m$	
$e_l$	$l$	$Z_{l1}$	$Z_{l2}$	$\dots$	$Z_{lp}$	$\dots$	$Z_{lm}$	$Z_{ll}$	$I_l$	

(4.2.13)

$$\tilde{e}_l = E_l - E_q$$

the element  $Z_{li}$  can be determined by injecting a current at the  $i$ th bus and calculating the voltage at the  $l$ th node with respect to bus  $q$ . Since all other bus currents equal zero, it follows from equation (4.2.13) that

$$\begin{aligned} E_k &= Z_{ki} I_i & k &= 1, 2, \dots, m \\ e_l &= Z_{li} I_i \end{aligned} \quad (4.2.14)$$

Letting  $I_i = 1$  per unit in equations (4.2.14),  $Z_{li}$  can be obtained directly by calculating  $e_l$ .

The series voltage source is

$$e_l = E_p - E_q - v_{pl} \quad (4.2.15)$$

Since the current through the added link is

$$i_{pq} = 0$$

the element  $p-l$  can be treated as a branch. The current in this element in terms of primitive admittances and the voltages across the elements is

$$i_{pl} = y_{pl,p} v_{pl} + \bar{y}_{pl,\rho\sigma} \bar{v}_{\rho\sigma}$$

where

$$i_{pl} = i_{pq} = 0$$

Therefore

$$v_{pl} = - \frac{\bar{y}_{pl,\rho\sigma} \bar{v}_{\rho\sigma}}{y_{pl,pl}}$$

Since

$$\bar{y}_{pl,\rho\sigma} = \bar{y}_{pq,\rho\sigma} \quad \text{and} \quad y_{pl,pl} = y_{pq,pq}$$

then

$$v_{pl} = - \frac{\bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma}}{y_{pq,pq}} \quad (4.2.16)$$

Substituting in order from equations (4.2.16), (4.2.6), and (4.2.14) with  $I_i = 1$  into equation (4.2.15) yields

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,\rho\sigma} (\bar{Z}_{\rho i} - \bar{Z}_{\sigma i})}{y_{pq,pq}} \quad \begin{array}{l} i = 1, 2, \dots, m \\ i \neq l \end{array} \quad (4.2.17)$$

The element  $Z_{li}$  can be calculated by injecting a current at the  $l$ th bus with bus  $q$  as reference and calculating the voltage at the  $l$ th bus with respect to bus  $q$ . Since all other bus currents equal zero, it follows from equation (4.2.13) that

$$\begin{aligned} E_k &= Z_{kl} I_l \quad k = 1, 2, \dots, m \\ e_l &= Z_{ll} I_l \end{aligned} \quad (4.2.18)$$

Letting  $I_l = 1$  per unit in equation (4.2.18),  $Z_{li}$  can be obtained directly by calculating  $e_l$ .

The current in the element  $p-l$  is

$$i_{pl} = -I_l = -1$$

This current in terms of primitive admittances and the voltages across the elements is

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,\rho\sigma} \bar{v}_{\rho\sigma} = -1$$

Again, since

$$\bar{y}_{pl,\rho\sigma} = \bar{y}_{pq,\rho\sigma} \quad \text{and} \quad y_{pl,pl} = y_{pq,pq}$$

then

$$v_{pl} = - \frac{1 + \bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma}}{y_{pq,pq}} \quad (4.2.19)$$

Substituting in order from equations (4.2.19), (4.2.6), and (4.2.18) with  $I_l = 1$  into (4.2.15) yields

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq, pq}(Z_{pl} - Z_{ql})}{\bar{y}_{pq, pq}} \quad (4.2.20)$$

If there is no mutual coupling between the added element and other elements of the partial network, the elements of  $\bar{y}_{pq, pq}$  are zero and

$$z_{pq, pq} = \frac{1}{\bar{y}_{pq, pq}}$$

It follows from equation (4.2.17) that

$$Z_{li} = Z_{pi} - Z_{qi} \quad \begin{array}{l} i = 1, 2, \dots, m \\ i \neq l \end{array}$$

and from equation (4.2.20),

$$Z_{ll} = Z_{pl} - Z_{ql} + z_{pq, pq}$$

Furthermore, if there is no mutual coupling and  $p$  is the reference node,

$$Z_{pi} = 0 \quad \begin{array}{l} i = 1, 2, \dots, m \\ i \neq l \end{array}$$

and

$$Z_{li} = -Z_{qi} \quad \begin{array}{l} i = 1, 2, \dots, m \\ i \neq l \end{array}$$

Also

$$Z_{pl} = 0$$

and therefore,

$$Z_{ll} = -Z_{ql} + z_{pq, pq}$$

The elements in the  $l$ th row and column of the bus impedance matrix for the augmented partial network are found from equations (4.2.17) and (4.2.20). It remains to calculate the required bus impedance matrix to include the effect of the added link. This can be accomplished by modifying the elements  $Z_{ij}$ , where  $i, j = 1, 2, \dots, m$ , and eliminating the  $l$ th row and column corresponding to the fictitious node.

The fictitious node  $l$  is eliminated by short circuiting the series voltage source  $e_l$ . From equation (4.2.13),

$$\bar{E}_{BUS} = Z_{BUS} \bar{I}_{BUS} + Z_{ul} I_l \quad (4.2.21)$$

and

$$e_l = Z_{lj} \bar{I}_{BUS} + Z_{ul} I_l = 0 \quad (4.2.22)$$

where  $i, j = 1, 2, \dots, m$ . Solving for  $I_i$  from equation (4.2.22) and substituting into (4.2.21),

$$\bar{E}_{BUS} = \left( Z_{BUS} - \frac{\bar{Z}_i \bar{Z}_{ij}}{Z_u} \right) \bar{I}_{BUS}$$

which is the performance equation of the partial network including the link  $p-q$ . It follows that the required bus impedance matrix is

$$Z_{BUS(\text{modified})} = Z_{BUS(\text{before elimination})} - \frac{\bar{Z}_i \bar{Z}_{ij}}{Z_u}$$

where any element of  $Z_{BUS(\text{modified})}$  is

$$Z_{ij(\text{modified})} = Z_{ij(\text{before elimination})} - \frac{Z_i Z_{ij}}{Z_u}$$

5

① Add branch ⑥ to ①  $Z_{bus} = \boxed{0.25}$

② Add branch ① to ②  $Z_{bus} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0.25 & 0.25 \\ 2 & 0.25 & 0.33 \end{array}$

③ Add branch ⑤ to ③  $Z_{bus} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.25 & 0.25 & 0 \\ 2 & 0.25 & 0.33 & 0 \\ 3 & 0 & 0 & 0.2 \end{array}$

$Z_{g1} = Z_{p1} = 0$   
 $Z_{g2} =$   
 $Z_{g3} =$

~~$Z_{g2}$~~

$$Z_{13} = 0 - \frac{0.25 \times 0.2}{0.59} = 0.084$$

$$Z_{22} = 0.33 - \frac{0.33 \times 0.33}{0.59} = 0.145$$

$$Z_{23} = 0 - \frac{0.33 \times 0.2}{0.59} = 0.112$$

$$Z_{33} = -0.2 - \frac{-0.2 \times 0.2}{0.59} = \cancel{0.268} \cdot 132$$

$Z_{bus}$

0.114	0.11	0.084
0.11	0.145	0.112
0.08	0.112	<del>0.268</del> -132

6

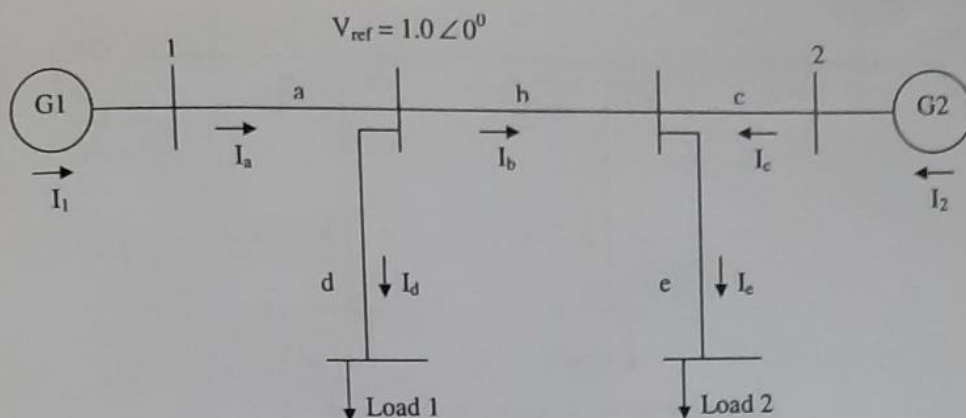


Fig : Example 8

**Solution:**

Total load current

$$I_L = I_d + I_e = 2.0 - j 0.5 = 2.061 \angle -14.03^\circ \text{ A}$$

$$I_{L1} = I_d = 0.8 - j 0.2 = 0.8246 \angle -14.03^\circ \text{ A}$$

$$\frac{I_{L1}}{I_L} = 0.4; \quad \frac{I_{L2}}{I_L} = 1.0 - 0.4 = 0.6$$

If generator 1, supplies the load then  $I_1 = I_L$ . The current distribution is shown in Fig a.

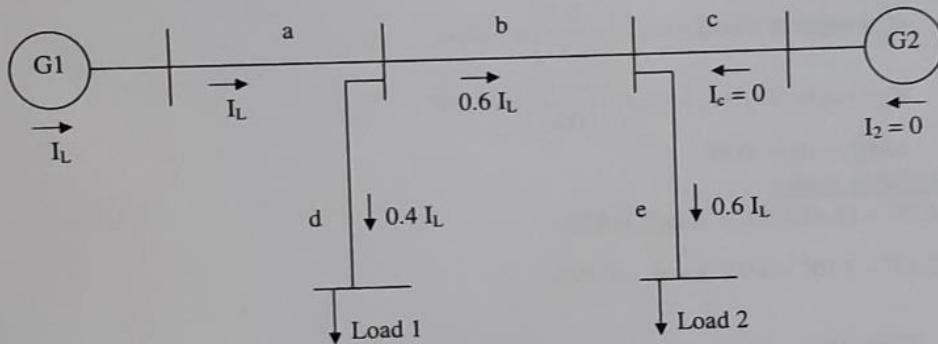


Fig a : Generator 1 supplying the total load

$$N_{a1} = \frac{I_a}{I_L} = 1.0; N_{b1} = \frac{I_b}{I_L} = 0.6; N_{c1} = 0; N_{d1} = 0.4; N_{e1} = 0.6.$$

Similarly the current distribution when only generator 2 supplies the load is shown in Fig b.

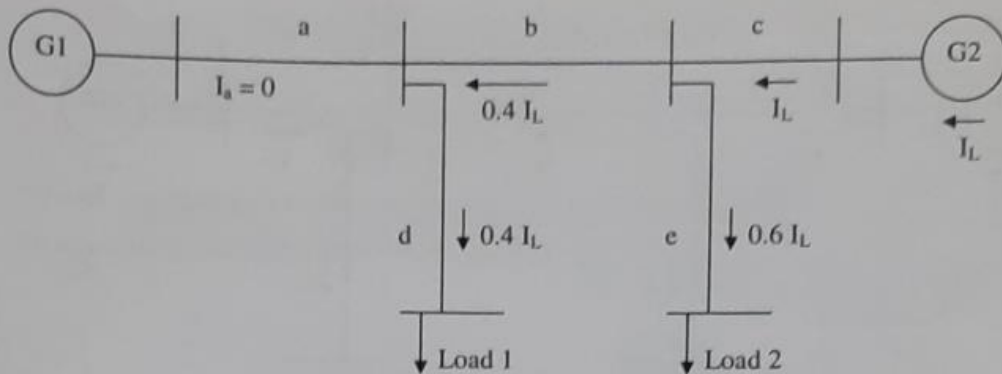


Fig b: Generator 2 supplying the total load

$$N_{a2} = 0; N_{b2} = -0.4; N_{c2} = 1.0; N_{d2} = 0.4; N_{e2} = 0.6$$

$$\text{From Fig 8.10, } V_1 = V_{ref} + Z_a I_a$$

$$= 1 \angle 0^\circ + (1.2 - j 0.4) (0.02 + j 0.08)$$

$$= 1.06 \angle 4.78^\circ = 1.056 + j 0.088 \text{ pu.}$$

$$V_2 = V_{ref} - I_b Z_b + I_c Z_c$$

$$= 1.0 \angle 0^\circ - (0.4 - j 0.2) (0.08 + j 0.32) + (0.8 - j 0.1) (0.02 + j 0.08)$$

$$= 0.928 - j 0.05 = 0.93 \angle -3.10^\circ \text{ pu.}$$

### Current Phase angles

$$\sigma_1 = \text{angle of } I_1 (=I_a) = \tan^{-1} \left( \frac{-0.4}{1.2} \right) = -18.43^\circ$$

$$\sigma_2 = \text{angle of } I_2 (=I_c) = \tan^{-1} \left( \frac{-0.1}{0.8} \right) = -7.13^\circ$$

$$\cos(\sigma_1 - \sigma_2) = 0.98$$

### Power factor angles

$$\phi_1 = 4.78^\circ + 18.43^\circ = 23.21^\circ; \cos \phi_1 = 0.92$$

$$\phi_2 = 7.13^\circ - 3.10^\circ = 4.03^\circ; \cos \phi_2 = 0.998$$

$$B_{11} = \frac{\sum_k N_{k1}^2 R_k}{|V_1|^2 (\cos \phi_1)^2} = \frac{1.0^2 \times 0.02 + 0.6^2 \times 0.08 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(1.06)^2 (0.920)^2}$$

$$= 0.0677 \text{ pu}$$

$$= 0.0677 \times \frac{1}{50} = 0.1354 \times 10^{-2} \text{ MW}^{-1}$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|(\cos \phi_1)(\cos \phi_2)} \sum_k N_{k1} N_{k2} R_k$$

$$= \frac{0.98}{(1.06)(0.93)(0.998)(0.92)} [-0.4 \times 0.6 \times 0.08 + 0.4 \times 0.4 \times 0.03 + 0.6 \times 0.6 \times 0.03]$$

$$= -0.00389 \text{ pu}$$

$$= -0.0078 \times 10^{-2} \text{ MW}^{-1}$$

$$B_{22} = \frac{\sum_k N_{k2}^2 R_k}{|V_2|^2 (\cos \phi_2)^2}$$

$$= \frac{(-0.4)^2 \times 0.08 + 1.0^2 \times 0.02 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(0.93)^2 (0.998)^2}$$

$$= 0.056 \text{ pu} = 0.112 \times 10^{-2} \text{ MW}^{-1}$$