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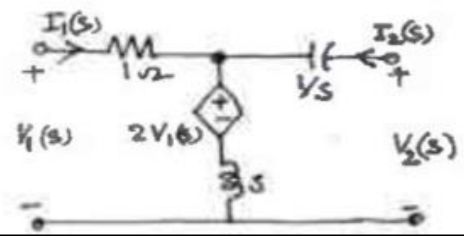
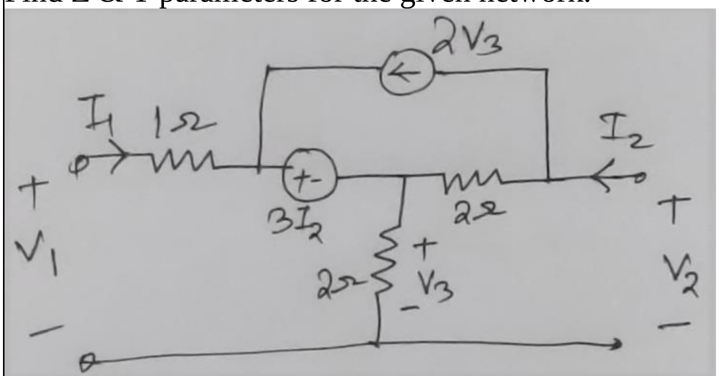
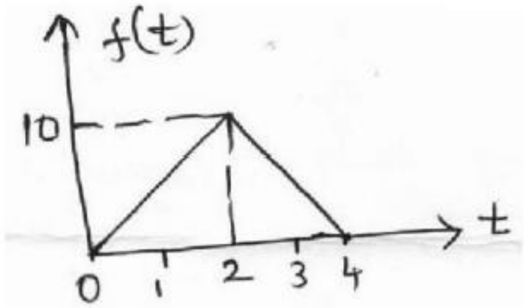
Internal Assesment Test - III

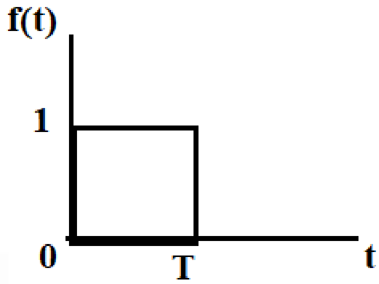
Sub:	**Electric Circuit Analysis**							Code:	18EE32
Date:	07/03/2022	Duration:	90 mins	Max Marks:	50	Sem:	3rd	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1	Find Transmission Line Parameters for the given network. 	10	CO6	L3
2	Find Z & T parameters for the given network. 	10	CO6	L3
3	a) Determine the line currents for a delta connected load of Z_{ab}=10 60° Ω, Z_{bc} = 20 90° Ω and Z_{ca}=25 30° Ω. Assume 3-phase 400V, ABC sequence.	4	CO6	L3
	b) A three phase, 400V, 4 wire system has a star connected load with Z_A= (10+ j0)Ω, Z_B = (15+j10)Ω, Z_C= (0+j5)Ω. Find the line currents & current through neutral wire.	6	CO6	L3
4	Determine the line currents, neutral current, Real & Reactive Power in an unbalanced star connected load supplied from a symmetrical 3 phase, 440V system. The branch impedances are Z_R = 4 30° Ω Z_Y=10 45° Ω and Z_B=10 60° Ω. The phase sequence is RYB	10	CO6	L3
5	State & prove Initial & Final Value Theorem for any function x(t). Accordingly find the initial & final values of the following functions: (a) $\dfrac{s^3+7s^2+5}{s(s^3+3s^2+4s+2)}$ (b) $\dfrac{s^2+5}{s(s^2+4s+4)}$	10	CO5	L3

<table>
<tr><td>6</td><td>Find the Laplace transform of the function given by the following waveforms.

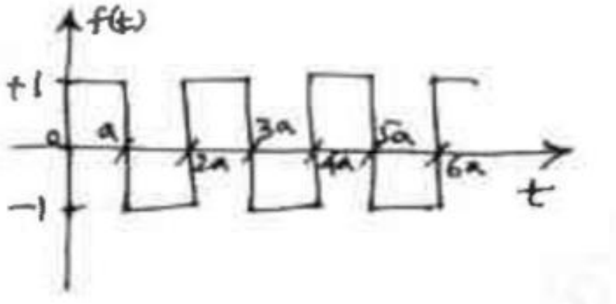




</td><td>10
(7+3)</td><td>CO5</td><td>L3</td></tr>
<tr><td>7</td><td>a) Find Inverse Laplace of following functions:

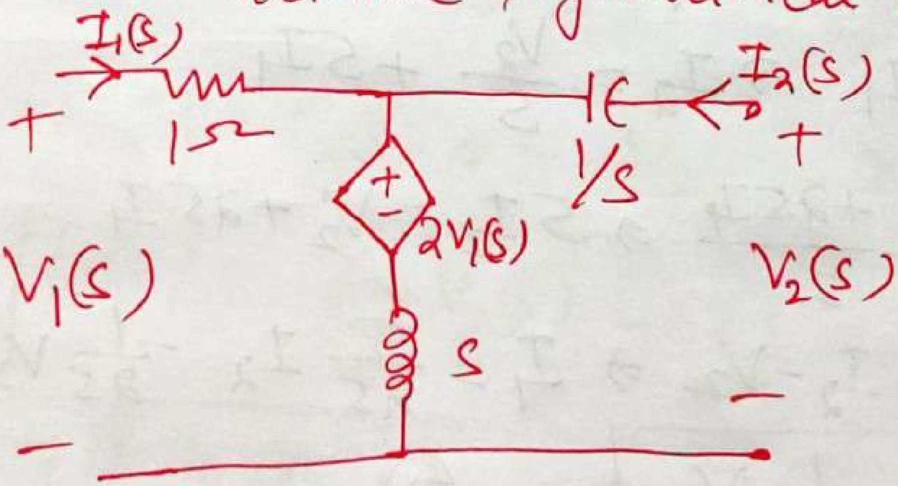
(a) $F(s) = \dfrac{s+2}{s(s+3)(s+4)}$

(b) $F(s) = \dfrac{2s+6}{s^2+6s+25}$
</td><td>7</td><td>CO5</td><td>L3</td></tr>
<tr><td></td><td>b) Find Laplace of the following functions:
(i) $\partial(t)$ (ii) u(t) (iii) sin ωt</td><td>3</td><td>CO5</td><td>L2</td></tr>
<tr><td>8</td><td>(a) Find Laplace Transform of the given Periodic function:

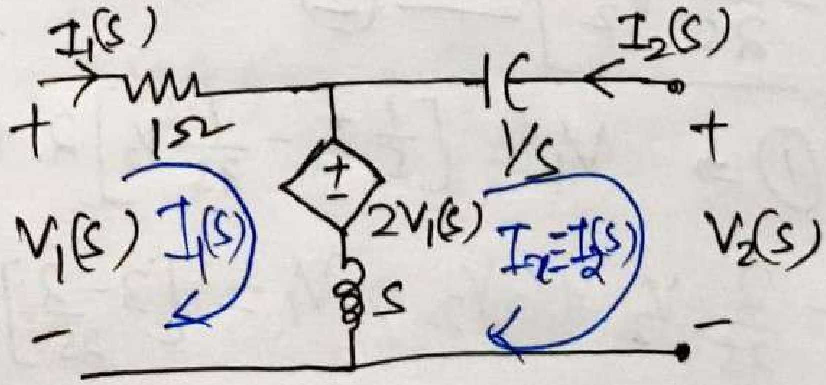


</td><td>8</td><td>CO5</td><td>L3</td></tr>
<tr><td></td><td>(b) Draw the following functions:
(i) tu(t-T)
(ii) (t-T)u(t-T)
(iii) u(-t)
(iv) tu(t+T)</td><td>2</td><td>CO5</td><td>L3</td></tr>
</table>

Q: For the given network find Y-Parameters. Is this network symmetrical?



Ans:



$I_x = -I_2(s)$

Mesh I_1 eqn $\Rightarrow V_1(s) - 1\, I_1(s) - 2V_1(s) - s(I_1(s) - I_x(s)) = 0$

$\Rightarrow V_1(s) - I_1(s) - 2V_1(s) - sI_1(s) + sI_x(s) = 0 \quad [I_x = -I_2]$

$\Rightarrow V_1(s) - I_1(s) - 2V_1(s) - sI_1(s) - sI_2(s) = 0$

$\Rightarrow -V_1(s) - I_1(s)[s+1] - sI_2(s) = 0$

$\Rightarrow \boxed{I_1(s) = -V_1(s) - sI_2(s)}$ — (1)

Mesh I_2 eqn: $\quad [I_x = -I_2]$

$-V_2(s) - s(I_x(s) - I_1(s)) + 2V_1(s) - I_x(s)\frac{1}{s} = 0$

$\Rightarrow -V_2(s) - sI_x(s) + sI_1(s) + 2V_1(s) - \frac{I_x(s)}{s} = 0$

$\Rightarrow -V_2(s) + sI_2(s) + sI_1(s) + 2V_1(s) + \frac{I_2(s)}{s} = 0$

$\Rightarrow \left(s + \frac{1}{s}\right) I_2(s) + sI_1(s) + 2V_1(s) - V_2(s) = 0$

$\Rightarrow \left(\frac{s^2+1}{s}\right) I_2(s) = V_2(s) - 2V_1(s) - sI_1(s)$

$\Rightarrow \boxed{I_2(s) = \frac{s}{s^2+1} V_2(s) - \frac{2s}{s^2+1} V_1(s) - \frac{s^2}{s^2+1} I_1(s)}$ — (2)

Putting (2) in (1) we get:

$I_1(s) = -V_1(s) - s\left[\frac{s}{s^2+1} V_2(s) - \frac{2s}{s^2+1} V_1(s) - \frac{s^2}{s^2+1} I_1(s)\right]$

$\Rightarrow I_1(s) = -V_1(s) - \frac{s^2}{s^2+1} V_2(s) + \frac{2s^2}{s^2+1} V_1(s) + \frac{s^3}{s^2+1} I_1(s)$

$$\Rightarrow I_1(s) = V_1(s)\left[\frac{2s^2}{s^2+1} - 1\right] - \frac{s^2}{s^2+1}V_2(s) + \frac{s^3}{s^2+1}I_1(s)$$

$$\Rightarrow I_1(s) - I_1(s)\left(\frac{s^3}{s^2+1}\right) = V_1(s)\left[\frac{2s^2-s^2-1}{s^2+1}\right] - \frac{s^2}{s^2+1}V_2(s)$$

$$\Rightarrow I_1(s)\left[1 - \frac{s^3}{s^2+1}\right] = V_1(s)\left[\frac{s^2-1}{s^2+1}\right] - \left(\frac{s^2}{s^2+1}\right)V_2(s)$$

$$\Rightarrow I_1(s)\left[\frac{s^2+1-s^3}{s^2+1}\right] = V_1(s)\left[\frac{s^2-1}{s^2+1}\right] - \left(\frac{s^2}{s^2+1}\right)V_2(s)$$

$$\Rightarrow I_1(s) = \frac{V_1(s)\,[s^2-1]}{\cancel{(s^2+1)}} \times \frac{\cancel{(s^2+1)}}{-s^3+s^2+1} - \left(\frac{s^2}{\cancel{s^2+1}}\right) \times \frac{\cancel{s^2+1}}{-s^3+s^2+1}V_2(s)$$

$$\Rightarrow \boxed{I_1(s) = \left[\frac{s^2-1}{-s^3+s^2+1}\right]V_1(s) - \left[\frac{s^2}{-s^3+s^2+1}\right]V_2(s)} \quad —③$$

Y-Parameter $\Rightarrow$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$\boxed{Y_{11} = \frac{s^2-1}{-s^3+s^2+1}}, \quad \boxed{Y_{12} = \frac{-s^2}{-s^3+s^2+1} = \frac{s^2}{s^3-s^2-1}}$$

Putting ③ in ② $\Rightarrow$

$$I_2(s) = \frac{s}{s^2+1}V_2(s) - \frac{2s}{s^2+1}V_1(s) - \frac{s^2}{s^2+1}I_1(s)$$

$$\Rightarrow \cancel{I_2(s) - \left(\frac{s}{s^2+1}\right)V_2(s) - \frac{2s}{}V_1(s) - \frac{s^2}{}}$$

$$I_2(s) = \frac{s}{s^2+1}V_2(s) - \frac{2s}{s^2+1}V_1(s) - \frac{s^2}{s^2+1}\left[\frac{s^2-1}{-s^3+s^2+1}V_1(s) - \frac{s^2}{-s^3+s^2+1}V_2(s)\right]$$

$$\Rightarrow I_2(s) = \frac{s}{s^2+1}V_2(s) - \frac{2s}{s^2+1}V_1(s) - \frac{s^2(s^2-1)\;V_1(s)}{(s^2+1)(-s^3+s^2+1)} + \frac{s^4\;V_2(s)}{(s^2+1)(-s^3+s^2+1)}$$

$$\Rightarrow I_2(s) = V_2(s)\left[\frac{s}{s^2+1} + \frac{s^4}{(s^2+1)(-s^3+s^2+1)}\right] - V_1(s)\left[\frac{2s}{s^2+1} + \frac{s^2(s^2-1)}{(s^2+1)(-s^3+s^2+1)}\right]$$

$$\Rightarrow I_2(s) = V_2(s)\left[\frac{s(-s^3+s^2+1)+s^4}{(s^2+1)(-s^3+s^2+1)}\right] - V_1(s)\left[\frac{2s(-s^3+s^2+1)+s^4-s^2}{(s^2+1)(-s^3+s^2+1)}\right]$$

$$\Rightarrow I_2(s) = V_2(s)\left[\frac{\cancel{-s^4}+s^3+s+\cancel{s^4}}{(s^2+1)(-s^3+s^2+1)}\right] - \left[\frac{-2s^4+2s^3+2s+s^4-s^2}{(s^2+1)(-s^3+s^2+1)}\right]V_1(s)$$

$$\Rightarrow I_2(s) = V_2(s)\left[\frac{s^3+s}{(s^2+1)(-s^3+s^2+1)}\right] - \left[\frac{-s^4+2s^3-s^2+2s}{(s^2+1)(-s^3+s^2+1)}\right]V_1(s)$$

$$\Rightarrow I_2(s) = V_1(s)\left[\frac{s^4-2s^3+s^2-2s}{(s^2+1)(-s^3+s^2+1)}\right] + V_2(s)\left[\frac{s\cancel{(s^2+1)}}{\cancel{(s^2+1)}(-s^3+s^2+1)}\right]$$

$$\Rightarrow I_2(s) = V_1(s)\left[\frac{s^3(s-2)+s(s-2)}{(s^2+1)(-s^3+s^2+1)}\right] + \frac{s}{-s^3+s^2+1}V_2(s)$$

$$\Rightarrow I_2(s) = V_1(s)\left[\frac{(s^3+s)(s-2)}{(s^2+1)(-s^3+s^2+1)}\right] + \left[\frac{s}{-s^3+s^2+1}\right]V_2(s)$$

$$= V_1(s)\left[\frac{s\cancel{(s^2+1)}(s-2)}{\cancel{(s^2+1)}(-s^3+s^2+1)}\right] + \left(\frac{s}{-s^3+s^2+1}\right)V_2(s)$$

$$\Rightarrow I_2(s) = \left[\frac{s(s-2)}{-s^3+s^2+1}\right]V_1(s) + \left(\frac{s}{-s^3+s^2+1}\right)V_2(s)$$

Comparing with $I_2 = Y_{21} V_1 + Y_{22} V_2$

$$Y_{21} = \frac{s(s-2)}{-s^3+s^2+1} \quad , \quad Y_{22} = \frac{s}{-s^3+s^2+1}$$

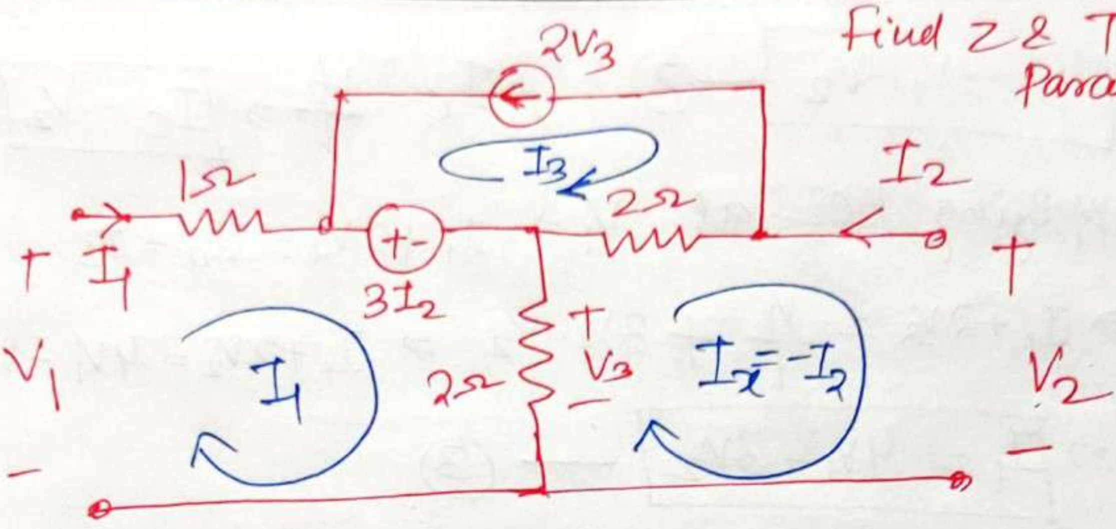
$$\Rightarrow \quad Y_{11} = \frac{s^2-1}{-s^3+s^2+1} \quad , \quad Y_{12} = \frac{s^2}{s^3-s^2-1}$$

$$Y = \begin{bmatrix} \frac{s^2-1}{-s^3+s^2+1} & \frac{s^2}{s^3-s^2-1} \\ \frac{s(s-2)}{-s^3+s^2+1} & \frac{s}{-s^3+s^2+1} \end{bmatrix}$$

Here $Y_{11} \neq Y_{22}$ & $Y_{21} \neq Y_{12}$ So not symmetric.

Q.

Q:- Find Z & T Parameters.



Ans:- Applying Mesh Analysis in I_1 Mesh

$\Rightarrow V_1 - I_1 - 3I_2 - 2(I_1 - I_x) = 0$

$\Rightarrow V_1 - I_1 - 3I_2 - 2I_1 + 2I_x = 0$

$\Rightarrow V_1 - I_1 - 3I_2 - 2I_1 - 2I_2 = 0$

$\Rightarrow \boxed{V_1 = 3I_1 + 5I_2}$ — ① $\quad [I_x = -I_2]$

From I_3 Mesh: $\quad I_3 = -2V_3 \;\; \& \;\; V_3 = 2(I_1 - I_x)$

$\Rightarrow I_3 = -2 \times 2(I_1 - I_x) = -4(I_1 + I_2)$

$\Rightarrow \boxed{I_3 = -4I_1 - 4I_2}$ ← ②

From I_x Mesh: $\quad -V_2 - 2(I_x - I_1) - 2(I_x - I_3) = 0$

$\Rightarrow -V_2 - 2I_x + 2I_1 - 2I_x + 2I_3 = 0$

$\Rightarrow V_2 = -2(-I_2) + 2I_1 - 2(-I_2) + 2I_3$

Putting I_3 value from ② ⇒

$V_2 = 2I_2 + 2I_1 + 2I_2 + 2(-4I_1 - 4I_2)$

$$V_2 = 4I_2 + 2I_1 - 8I_1 - 8I_2 \Rightarrow \boxed{V_2 = -6I_1 - 4I_2} \quad \text{--- (3)}$$

From (1) & (3)

$$V_1 = 3I_1 + 5I_2$$
$$V_2 = -6I_1 - 4I_2$$

Comparing with Z-Parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

So

$$Z_{11} = 3,\ Z_{12} = 5,\ Z_{21} = -6,\ Z_{22} = -4$$

$$\boxed{[Z] = \begin{bmatrix} 3 & 5 \\ -6 & -4 \end{bmatrix}} \Rightarrow \Delta Z = \begin{vmatrix} 3 & 5 \\ -6 & -4 \end{vmatrix} = -12 - (-30)$$

$$\Rightarrow \Delta Z = 18$$

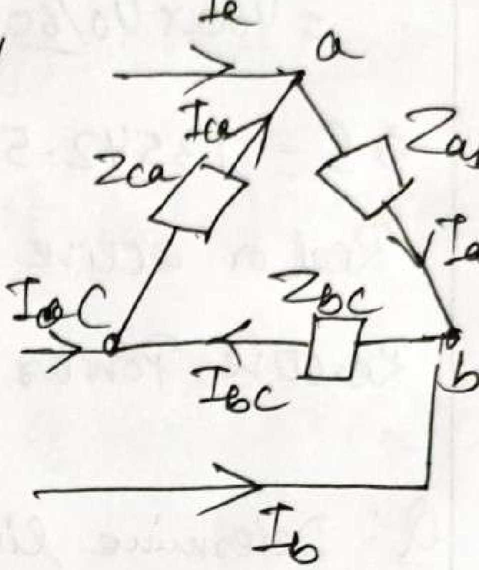
$$A = \frac{Z_{11}}{Z_{21}} = \frac{3}{-6} \Rightarrow \boxed{A = -\frac{1}{2}},\quad B = \frac{\Delta Z}{Z_{21}} = \frac{18}{-6} \cancel{\Rightarrow B = 3}$$

$$\boxed{B = -3},\quad \boxed{C = \frac{1}{Z_{21}} = \frac{1}{5}},\quad D = \frac{Z_{22}}{Z_{21}} = \frac{-4}{-6} \Rightarrow \boxed{D = \frac{2}{3}}$$

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \Rightarrow \boxed{[T] = \begin{bmatrix} -\frac{1}{2} & -3 \\ \frac{1}{5} & \frac{2}{3} \end{bmatrix}}$$

Q: Determine the line currents & total Power supplied to a delta connected load of $Z_{ab} = 10\angle 60^\circ\ \Omega$, $Z_{bc} = 20\angle 90^\circ\ \Omega$, $Z_{ca} = 25\angle 30\ \Omega$. Assume 3-Phase 400V, ABC sequence.

Delta Load



Ans: Sequence: ABC, $V_\ell = 400V$

$V_{AB} = V_\ell \angle 0 = 400\angle 0$

$V_{BC} = V_\ell \angle -120 = 400\angle -120$

$V_{CA} = V_\ell \angle 120 = 400\angle 120$

Phase Currents:

$Z_{ab} = 10\angle 60^\circ$

$Z_{bc} = 20\angle 90^\circ$

$Z_{ca} = 25\angle 30^\circ$

$$I_{ab} = \frac{V_{AB}}{Z_{ab}} = \frac{400\angle 0}{10\angle 60} = 40\angle -60^\circ$$

$$I_{bc} = \frac{V_{BC}}{Z_{BC}} = \frac{400\angle -120}{20\angle 90} = 20\angle \cancel{-210} = 20\angle -210$$

$$I_{ca} = \frac{Z_{CA}}{Z_{ca}} = \frac{400\angle 120}{25\angle 30} = 16\angle 90$$

Line Currents: KCL at a $\Rightarrow I_a + I_{ca} = I_{ab}$

$\Rightarrow I_a = I_{ab} - I_{ca} = 40\angle -60 - 16\angle 90 \Rightarrow \boxed{I_a = 54.44\angle -68.44}$

KCL at b $\Rightarrow I_{ab} + I_b = I_{bc} \Rightarrow I_b = I_{bc} - I_{ab} = 20\angle -210 - 40\angle -60$

$\Rightarrow \boxed{I_b = 58.18\angle 129.89}$

KCL at c $\Rightarrow I_c + I_{bc} = I_{ca} \Rightarrow I_c = I_{ca} - I_{bc} = 16\angle 90 - 20\angle -210$

$\Rightarrow \boxed{I_c = 18.33\angle 19.1}$

Power:- $\cancel{S = V_{AB} I_{ab}^{*} + V_{BC} I_{bc}}$

$$S = V_{AB} I_{ab}^{*} + V_{BC} I_{bC}^{*} + V_{CA} I_{Ca}^{*}$$

$$= 400\angle 0 \times [40\angle -60]^{*} + 400\angle -120 \times [20\angle -210]^{*} + 400\angle 120 \times [16\angle 90]^{*}$$

$$= 400 \times 40\angle 60 + 400\angle -120 \times 20\angle 210 + 400\angle 120 \times 16\angle -90$$

$$\Rightarrow S = 13542.56 + 25056.4j \quad VA$$

Real or active Power $= P = 13542.56$ W $= Re[S] \Leftarrow$ Real Part of S

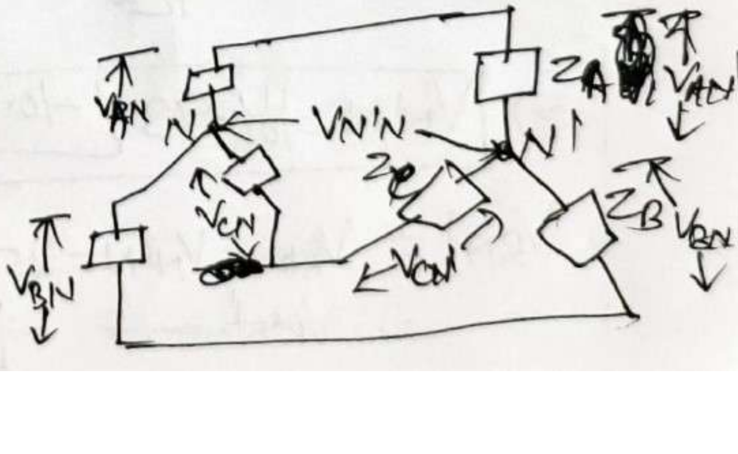
Reactive Power $= Q = Im[S] = 25056.4$ VAR $\Leftarrow$ Imaginary part of S.

Q:- A 3-Phase 400V 4 wire system has a star Connected load with $Z_A = (10+j0)\Omega$, $Z_B = (15+j10)\Omega$, $Z_C = (0+j5)\Omega$. Find line currents & current through neutral wire.

Ans:- <u>3-Phase</u> <u>Star</u>:

At balanced source side

$V_p = \frac{V_l}{\sqrt{3}}$, $V_l = 400V$



Line Voltages are: Considering Phase sequence as ABC.

$V_{AB} = V_L\angle 0 = 400\angle 0$, $V_{BC} = 400\angle -120$, $V_{CA} = 400\angle 120$

Phase Voltages: $V_P = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94$

* angle of phase voltages lags 30 to line voltage.

$V_{AN} = 230.94$ [angle lags by 30 to V_{AB}] $= 230.94\angle 0-30$

$\Rightarrow V_{AN} = 230.94\angle -30$

$V_{BN} = 230.94$ [angle 30 lags by V_{BC}] $= 230.94\angle -120-30$

$\Rightarrow V_{BN} = 230.94\angle -150$

$V_{CN} = 230.94$ [angle 30 lags by V_{CA}] $= 230.94\angle 120-30$

$\Rightarrow V_{CN} = 230.94\angle 90$

Neutral Shift Voltage:

$$V_{N'N} = \frac{V_{AN}Y_A + V_{BN}Y_B + V_{CN}Y_C}{Y_A + Y_B + Y_C} = \frac{\frac{V_{AN}}{Z_A} + \frac{V_{BN}}{Z_B} + \frac{V_{CN}}{Z_C}}{\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C}}$$

$$\Rightarrow V_{N'N} = \frac{\frac{230.94\angle -30}{10+j0} + \frac{230.94\angle -150}{15+j10} + \frac{230.94\angle 90}{0+j5}}{\frac{1}{10+j0} + \frac{1}{15+j10} + \frac{1}{0+j5}}$$

$$\Rightarrow \boxed{V_{N'N} = 199.4\angle 46.29}$$

Actual Phase Voltages

$V_{AN'} = V_{AN} - V_{N'N} = 230.94\angle -30 - 199.4\angle 46.29$ ~~$\Rightarrow V_{AN'} =$~~

$$\boxed{V_{AN'} = 266.95 \angle -76.52}$$

$$V_{BN'} = V_{BN} - V_{N'N} = 230.94 \angle -150 - 199.4 \angle 46.29$$

$$\Rightarrow \boxed{V_{BN'} = 426.02 \angle -142.45}$$

$$V_{CN'} = V_{CN} - V_{N'N} = 230.94 \angle 90 - 199.4 \angle 46.29$$

$$\Rightarrow \boxed{V_{CN'} = 162.85 \angle 147.78}$$

Line Currents = Phase Currents.

$$I_a = \frac{V_{AN'}}{Z_a} = \frac{266.95 \angle -76.52}{10 + j0} \Rightarrow \boxed{I_a = 26.695 \angle -76.52}$$

$$I_b = \frac{V_{BN'}}{Z_b} = \frac{426.02 \angle -142.45}{15 + j10} \Rightarrow \boxed{I_b = 23.63 \angle -176.14}$$

$$I_C = \frac{V_{CN'}}{Z_C} = \frac{162.85 \angle 147.78}{0 + j5} \Rightarrow \boxed{I_C = 32.57 \angle 57.78}$$

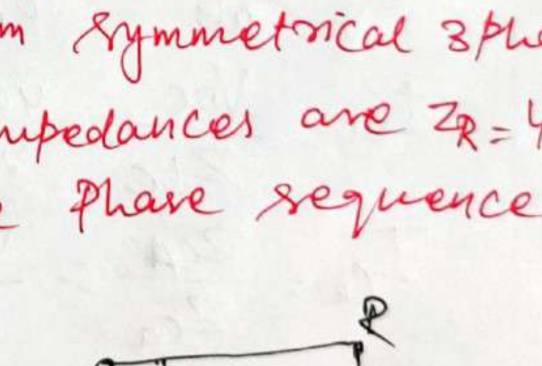
Current through Neutral Wire:

$$I_N = -(I_A + I_B + I_C) = -1 \times [26.695 \angle -76.52 + 23.63 \angle -176.14 + 32.57 \angle 57.78]$$

$$\Rightarrow \boxed{I_N = 0.012 \angle -160}$$

Q:- Determine line currents in an unbalanced star connected load supplied from symmetrical 3 phase, 440V system. The branch impedances are $Z_R = 40\angle 30^\circ$, $Z_Y = 10\angle 45^\circ$ & $Z_B = 10\angle 60^\circ$. The phase sequence is RYB.

Ans: 3-Phase Star



At source side phase voltage $V_P = \frac{V_L}{\sqrt{3}}$

& lags the line voltage by 30°

$V_L = 440V$.

Sequence in R-Y-B $\Rightarrow$ $V_{RY} = V_\ell \angle 0 = 440\angle 0$ Line Voltages

$V_{YB} = 440\angle -120$, $V_{BR} = 440\angle 120$

Phase Voltage $= V_p = \frac{V_\ell}{\sqrt{3}} = 254.03\,V$

$\boxed{V_{RN} = 254.03 \text{ [angle is 30° lagging of } V_{RY}] = 254.03\angle 0-30}$

$V_{YN} = 254.03$ [angle is 30° lagging of V_{YB}] $= 254.03\angle -120-30$

$\Rightarrow \boxed{V_{YN} = 254.03\angle -150}$

$V_{BN} = 254.03$ [angle is 30° lagging of V_{BR}] $= 254.03\angle 120-30$

$\Rightarrow \boxed{V_{BN} = 254.03\angle 90}$

$Z_R = 4\angle 30$, $Z_Y = 10\angle 45$, $Z_B = 10\angle 60$

Neutral Shift Voltage:

$$V_{N'N} = \frac{V_{RN}Y_R + V_{YN}Y_Y + V_{BN}Y_B}{Y_R + Y_Y + Y_B} = \frac{\frac{V_{RN}}{Z_R} + \frac{V_{YN}}{Z_Y} + \frac{V_{BN}}{Z_B}}{\frac{1}{Z_R} + \frac{1}{Z_Y} + \frac{1}{Z_B}}$$

$$= \frac{\frac{254.03\angle -30}{4\angle 30} + \frac{254.03\angle -150}{10\angle 45} + \frac{254.03\angle 90}{10\angle 60}}{\frac{1}{4\angle 30} + \frac{1}{10\angle 45} + \frac{1}{10\angle 60}}$$

$\Rightarrow \boxed{V_{N'N} = 104.93\angle -10.78}$

Actual Phase Voltages

$V_{RN'} = V_{RN} - V_{N'N} = 254.03\angle -30 - 104.93\angle -10.78$

$\Rightarrow$ ~~$V_{RN'} = 15$~~ $\Rightarrow \boxed{V_{RN'} = 158.75\angle -42.56}$

$$V_{YN'} = V_{YN} - V_{N'N} = 254.03\angle{-150} - 104.93\angle{-10.78}$$

$$\Rightarrow \boxed{V_{YN'} = 340.45\angle{-161.61}}$$

$$V_{BN'} = V_{BN} - V_{N'N} = 254.03\angle{90} - 104.93\angle{-10.78}$$

$$\Rightarrow \boxed{V_{BN'} = 292.42\angle{110.63}}$$

Line Currents = Phase Currents

$$\cancel{I_a = \frac{V_{aN'}}{Z_a}} \qquad I_R = \frac{V_{RN'}}{Z_R} = \frac{158.75\angle{-42.56}}{4\angle{30}}$$

$$\Rightarrow \boxed{I_R = 39.68\angle{-72.56}}$$

$$\boxed{I_Y = \frac{V_{YN'}}{Z_Y} = \frac{340.45\angle{-161.61}}{10\angle{45}} = 34.04\angle{-206.61}}$$

$$I_B = \frac{V_{BN'}}{Z_B} = \frac{292.42\angle{110.63}}{10\angle{60}} \Rightarrow \boxed{I_B = 29.24\angle{50.63}}$$

(viii) Initial Value Theorem:-

- It is used to find initial value of function
- If $x(t)$ is a function, this can be used to find $x(0)$ if $X(s)$ is given.

If $L\{x(t)\} = X(s)$, then $x(0) = \lim_{s\to\infty} sX(s)$

Proof:- We know:- $L\left\{\frac{dx(t)}{dt}\right\} = sX(s) - x(0)$

$$L\left\{\frac{dx(t)}{dt}\right\} = \int_0^\infty \frac{dx(t)}{dt} e^{-st}\, dt = sX(s) - x(0)$$

Applying $\lim_{s\to\infty}$ on both sides we get:-

$$\Rightarrow \lim_{s\to\infty} \int_0^\infty \frac{dx(t)}{dt} e^{-st}\, dt = \lim_{s\to\infty} [sX(s) - x(0)]$$

$$\Rightarrow 0 = \lim_{s\to\infty} [sX(s)] - x(0) \quad \left[e^{-st}\Big|_{s=\infty} = 0 \right]$$

$$\Rightarrow \boxed{x(0) = \lim_{s\to\infty} sX(s)}$$

(viii) Final Value Theorem:-

- It is used to find the final value of function.
- If $x(t)$ is a function, this can be used to find $x(\infty)$ if $X(s)$ is given.

If $L\{x(t)\} = X(s)$, then $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$

Proof:-

$$L\left\{\frac{dx(t)}{dt}\right\} = \int_0^\infty \frac{dx(t)}{dt} e^{-st}\, dt = sX(s) - x(0)$$

Taking $\lim_{s\to 0}$ both the sides we get:-

$$\lim_{s\to 0} \int_0^\infty \frac{dx(t)}{dt} e^{-st}\, dt = \lim_{s\to 0} [sX(s) - x(0)] \quad \text{—(1)}$$

$$\Rightarrow \int_0^\infty \frac{dx(t)}{dt} \left[\lim_{s\to 0} e^{-st}\right] dt = \int_0^\infty \frac{dx(t)}{dt} \cdot 1\, dt = [x(t)]_0^\infty$$

$$= x(\infty) - x(0) \quad \text{—(2)}$$

Applying this in (1) we get:-

$$x(\infty) - \cancel{x(0)} = \lim_{s\to 0} [sX(s) - x(0)] = \lim_{s\to 0} sX(s) - x(0)$$

$$\Rightarrow \boxed{x(\infty) = \lim_{s\to 0} sX(s)}$$

Q: Using Initial & final value theorem find $f(0)$ & $f(\infty)$ for following functions.

(i) $\dfrac{s^3+7s+5}{s(s^3+3s^2+4s+2)}$ (ii) $\dfrac{s(s+4)(s+8)}{(s+1)(s+6)}$

Ans: (i) $F(s) = \dfrac{s^3+7s+5}{s(s^3+3s^2+4s+2)}$

Initial Value theorem ⇒

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \cancel{s}\,\frac{s^3+7s^2+5}{\cancel{s}(s^3+3s^2+4s+2)}$$

$$= \lim_{s\to\infty} \frac{\cancel{s^3}\left[1+\frac{7}{s}+\frac{5}{s^3}\right]}{\cancel{s^3}\left[1+\frac{3}{s}+\frac{4}{s^2}+\frac{2}{s^3}\right]} = \frac{1+\frac{7}{\infty}+\frac{5}{\infty^3}}{1+\frac{3}{\infty}+\frac{4}{\infty^2}+\frac{2}{\infty^3}}$$

$$\Rightarrow \boxed{f(0) = 1}$$

Final value theorem:

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \cancel{s}\cdot\frac{s^3+7s^2+5}{\cancel{s}(s^3+3s^2+4s+2)}$$

$$\Rightarrow f(\infty) = \frac{0^3+7\times0^2+5}{0^3+3\times0^2+4\times0+2} \Rightarrow \boxed{f(\infty) = \frac{5}{2}}$$

(ii) $F(s) = \dfrac{s(s+4)(s+8)}{(s+1)(s+6)}$

Initial Value Theorem:

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} s\cdot\frac{s(s+4)(s+8)}{(s+1)(s+6)}$$

$$f(0) = \lim_{s\to\infty} \frac{s^2\cdot s\left(1+\frac{4}{s}\right) s\left(1+\frac{8}{s}\right)}{s\left(1+\frac{1}{s}\right) s\left(1+\frac{6}{s}\right)} = \lim_{s\to\infty} \frac{s^{2}\left(1+\frac{4}{s}\right)\left(1+\frac{8}{s}\right)}{\left(1+\frac{1}{s}\right)\left(1+\frac{6}{s}\right)}$$

$$\Rightarrow f(0) = \frac{\infty^2\left[1+\frac{4}{\infty}\right]\left[1+\frac{8}{\infty}\right]}{\left(1+\frac{1}{\infty}\right)\left(1+\frac{6}{\infty}\right)} \Rightarrow \boxed{f(0) = \infty}$$

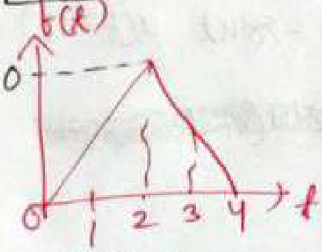
Final Value Theorem:

$$f(\infty) = \lim_{s\to 0} s\cdot F(s) = \lim_{s\to 0} s\cdot\frac{s(s+4)(s+8)}{(s+1)(s+6)}$$

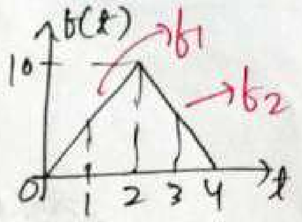
$$= \lim_{s\to 0} \frac{s^2(s+4)(s+8)}{(s+1)(s+6)} = \frac{0^2(0+4)(0+8)}{(0+1)(0+6)}$$

$$\rightarrow \boxed{f(\infty) = 0}$$

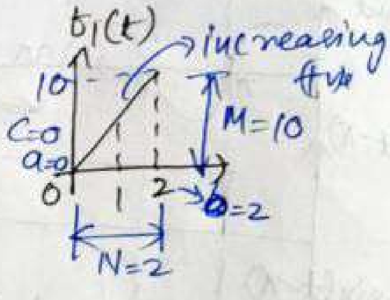
Q:- Find Laplace of f(t).



Ans:-



f1

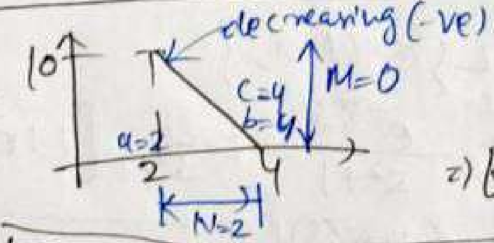


$f_1 = +\frac{M}{N}(t-c)[u(t-a) - u(t-b)]$

$\Rightarrow f_1 = \frac{10}{2}(t-0)[u(t-0) - u(t-2)]$

$\Rightarrow \boxed{f_1 = 5t\,u(t) - 5t\,u(t-2)}$

f2



$f_2 = -\frac{M}{N}(t-c)[u(t-a) - u(t-b)]$

$\Rightarrow f_2 = -\frac{10}{2}(t-4)[u(t-2) - u(t-4)]$

$\Rightarrow \boxed{f_2 = -5(t-4)u(t-2) - 5(t-4)u(t-4)}$

$f(t) = f_1(t) + f_2(t) = 5t\,u(t) - 5t\,u(t-2) - 5(t-4)u(t-2) - 5(t-4)\,u(t-4)$

$= 5r(t) - 5u(t-2)[t + t - 4] - 5r(t-4)$

$= 5r(t) - 5u(t-2)(2t-4) - 5r(t-4)$

$= 5r(t) - 10u(t-2)(t-2) - 5r(t-4)$

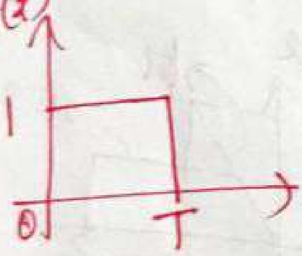
$\Rightarrow f(t) = 5r(t) - 10r(t-2) - 5r(t-4)$

$\Rightarrow F(s) = L\{5r(t)\} - L\{10r(t-2)\} - L\{5r(t-4)\}$

$\Rightarrow \boxed{F(s) = \frac{5}{s^2} - \frac{10e^{-2s}}{s^2} - \frac{5e^{-4s}}{s^2}}$

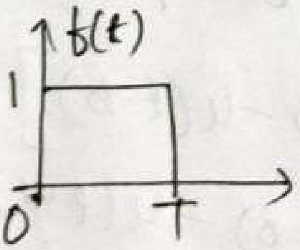
$\Rightarrow \boxed{F(s) = \frac{5}{s^2}\left[1 - 2e^{-2s} - e^{-4s}\right]}$

Q:



Find Laplace of following wave form.

Ans:



$$f(t) = 1\left[u(t) - u(t-T)\right]$$

Taking Laplace

$$f(t) = u(t) - u(t-T)$$

$$F(s) = L\{f(t)\} = L\{u(t)\} - L\{u(t-1)\}$$

$$\Rightarrow F(s) = \frac{1}{s} - \frac{e^{-s}}{s} \Rightarrow \boxed{F(s) = \frac{1}{s}\left(1 - e^{-s}\right)}$$

Q:: Find inverse Laplace of following:

(i) $\frac{s^2+5}{s(s^2+4s+4)}$ (ii) $\frac{2s+6}{s^2+6s+25}$

Ans:

$$\frac{s^2+5}{s(s^2+4s+4)} = \frac{s^2+5}{s(s^2+2\cdot s\cdot 2+2^2)}$$

$$F(s) = \frac{s^2+5}{s(s+2)^2} \Rightarrow \text{using partial fraction}$$

$$\frac{s^2+5}{s(s+2)^2} = \frac{A_1}{s} + \frac{A_2}{(s+2)^2} + \frac{A_3}{(s+2)}$$

$$\Rightarrow A_1 = \cancel{s}\cdot\frac{s^2+5}{\cancel{s}(s+2)^2}\Big|_{s=0} \Rightarrow A_1 = \frac{5}{4}$$

$$A_2 = \cancel{(s+2)^2}\cdot\frac{s^2+5}{s\cancel{(s+2)^2}}\Big|_{s+2=0 \text{ or } s=-2} \Rightarrow A_2 = \frac{(-2)^2+5}{-2}$$

$$\Rightarrow \boxed{A_2 = -\frac{9}{2}}$$

$$A_3 = \frac{d}{ds}\left[\cancel{(s+2)^2}\cdot\frac{s^2+5}{s\cancel{(s+2)^2}}\right]\Big|_{s+2=0 \text{ or } s=-2} = \frac{d}{ds}\left[\frac{s^2+5}{s}\right]\Big|_{s=-2}$$

$$\Rightarrow A_3 = \frac{s\left[\frac{d}{ds}(s^2+5)\right]-(s^2+5)\frac{d}{ds}s}{s^2}\Big|_{s=-2}$$

$$\left[\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right]$$

$$= \frac{2s^2-s^2-5}{s^2}\Big|_{s=-2} = \frac{s^2-5}{s^2}\Big|_{-2}$$

$$A_3 = \frac{(-2)^2-5}{(-2)^2} \Rightarrow A_3 = -\frac{1}{4}$$

$$\text{So } F(s) = \frac{s^2+5}{s(s+2)^2} = \frac{A_1}{s} + \frac{A_2}{(s+2)^2} + \frac{A_3}{s+2}$$

$$\Rightarrow F(s) = \frac{5/4}{s} + \frac{-9/2}{(s+2)^2} + \frac{-1/4}{s+2}$$

Inverse Laplace

$$\Rightarrow L^{-1}[F(s)] = L^{-1}\left[\frac{5/4}{s}\right] - L^{-1}\left[\frac{9/2}{(s+2)^2}\right] - L^{-1}\left[\frac{1/4}{s+2}\right]$$

$$\Rightarrow \cancel{f(t) = \frac{5}{4}u(t)}$$

$$\boxed{f(t) = \frac{5}{4}u(t) - \frac{9}{2}te^{-2t} - \frac{1}{4}e^{-2t}u(t)}$$

$$\left[\begin{aligned} &L^{-1}\left[\frac{1}{s}\right] = u(t) \\ &L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}u(t) \\ &L^{-1}\left[\frac{1}{(s+a)^2}\right] = e^{-at}t\,u(t)\end{aligned}\right.$$

ii) $F(s) = \frac{2s+6}{s^2+6s+25}$

$$= \frac{2s+6}{s^2+2\times3\times s+[9+16]} = \frac{2(s+3)}{s^2+2\times3\times s+9+16}$$

$$F(s) = \frac{2(s+3)}{(s+3)^2+4^2}$$

$$\left[L^{-1}\left[\frac{s+a}{(s+a)^2+\omega^2}\right] = \cos\omega t\, e^{-at}\right.$$

Inverse Laplace

$$L^{-1}[F(s)] = L^{-1}\left[\frac{2(s+3)}{(s+3)^2+4^2}\right] \Rightarrow \cancel{f(t) = 2e^{-3t}\sin 4t}$$

$$\boxed{f(t) = 2e^{-3t}\cos 4t}$$