CMR INSTITUTE OF TECHNOLOGY	USN											

Internal Assesment Test III – Jan 2022

Sub:	Signals and Systems							Code:	18EE54
Date:	27/01/2022	Duration:	90 mins	Max Marks:	50	Sem:	V	Section	EEE (A &B)

Note: Answer any **five FULL** Questions

Sketch neat figures wherever necessary. Answer to the point. **Good luck!**

		Marks	OBE CO	OBE RBT
1	Sketch the Direct form-I and Direct form-II implementations for the differential/difference equations $(a)\ \frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4\,y(t) = \frac{dx(t)}{dt}$ $(b)\ y[n] + \frac{1}{2}\,y[n-1] - y[n-3] = 3\,x[n-1] + 2x[n-2]$	[10 M]	CO3	L3
2 (a)	Show that: If $x(t) \xleftrightarrow{FT} X(j\omega), then\ \frac{d}{dt}[x(t)] \xleftrightarrow{FT} j\omega\, X(j\omega)$	[5 M]	CO4	L3
2 (b)	Find the Fourier Transform of unit step function	[5 M]	CO4	L2
3 (a)	Find the Fourier transform of $x(t) = e^{-a\|t\|}$ and Draw its spectrum	[4 M]	CO4	L3
3(b)	The impulse response of a continuous time LTI system is given by $h(t) = \frac{1}{RC}e^{-\frac{t}{RC}}\,u(t).$ Find the Frequency response and plot the magnitude and Phase response.	[6 M]	CO4	L3
4(a)	Find the Inverse Fourier transform of $X(j\omega) = \frac{-j\omega}{(j\omega)^2+3j\omega+2}$	[4 M]	CO4	L3
4(b)	Find the frequency response and the impulse response of the system described by the differential equation $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6\,y(t) = \frac{-dx(t)}{dt}$	[6 M]	CO4	L3
5(a)	State and Prove (i) Time shifting and (ii) Parseval's theorem in discrete time domain.	[6M]	CO4	L2
5(b)	Find the DTFT of the signal $x[n] = \alpha^n u[n]\ ;\ \|\alpha\| < 1.$ Draw Magnitude spectrum	[4 M]	CO4	L3
6	Obtain the frequency response and impulse response of the system having the output $y[n] = \frac{1}{4}\left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[n]$ for the input $x[n] = \left(\frac{1}{2}\right)^n u[n]$	[10M]	CO4	L3
7(a)	Find the frequency and the impulse response of the system described by the difference equation $y[n] + \frac{1}{2}y[n-1] = x[n] - 2x[n-1]$	[5M]	CO4	L3
7(b)	Evaluate the Fourier transform for the following sequence $x[n] = \cos\left(\frac{\pi}{4}n\right)\left(\frac{1}{2}\right)^n u[n-2]$	[5M]	CO4	L3
8(a)	Find the DTFT of the following using appropriate properties $(i)x[n] = \left(\frac{1}{4}\right)^n u[n-4]$ $(ii)x[n] = \left(\frac{1}{3}\right)^n u[n] * \left(\frac{1}{4}\right)^n u[n]$	[5M]	CO4	L3
8 (b)	Determine the impulse response for the given frequency response $H(e^{j\Omega}) = 1 + \frac{e^{-j\Omega}}{(1-\frac{1}{2}e^{-j\Omega})(1+\frac{1}{4}e^{-j\Omega})}$	[5M]	CO4	L3

CI CCI HOD

① Sketch Direct form-I and Direct from-II.

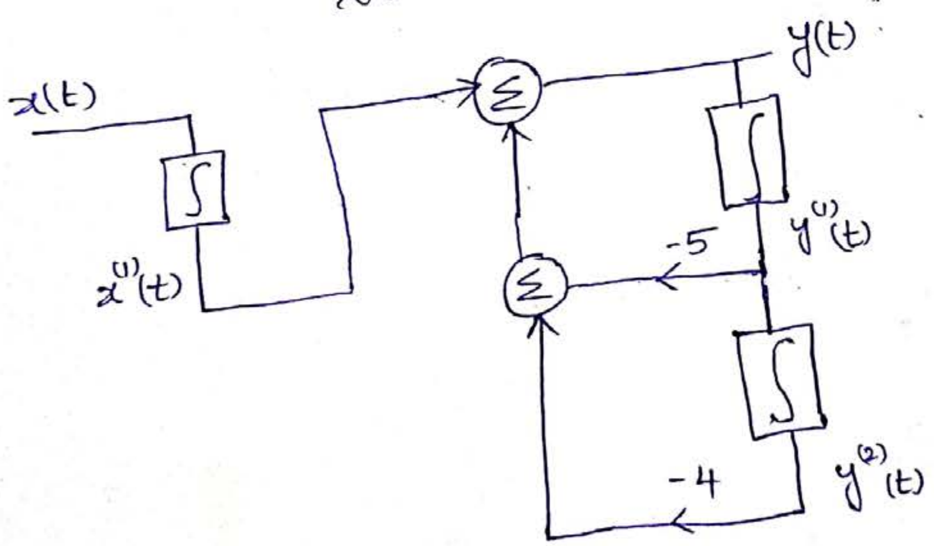
(a) $\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4\, y(t) = \frac{dx(t)}{dt}$

Integeating the above differential equation '2' times,

$$y(t) + 5\, y^{(1)}(t) + 4\, y^{(2)}(t) = x^{(1)}(t)$$

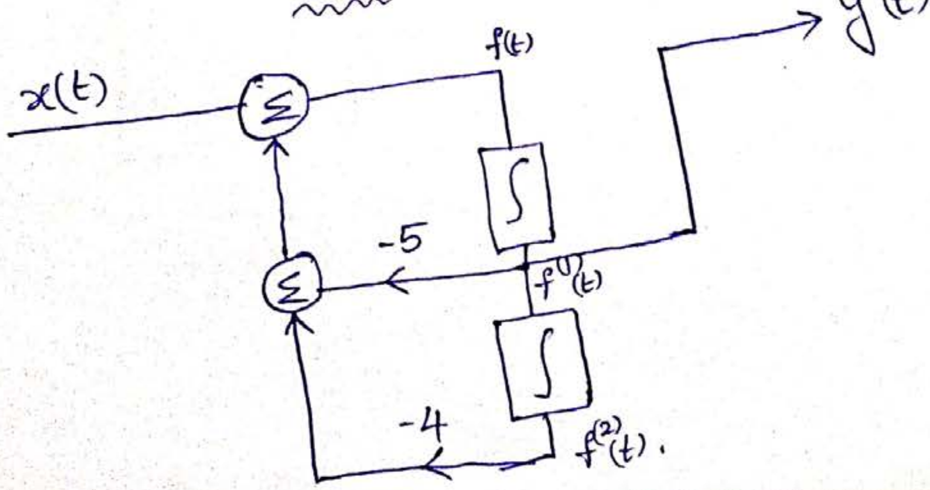
$$y(t) = x^{(1)}(t) - 5\, y^{(1)}(t) - 4\, y^{(2)}(t).$$

Direct form-I



— 2 ½ M

Direct from-II



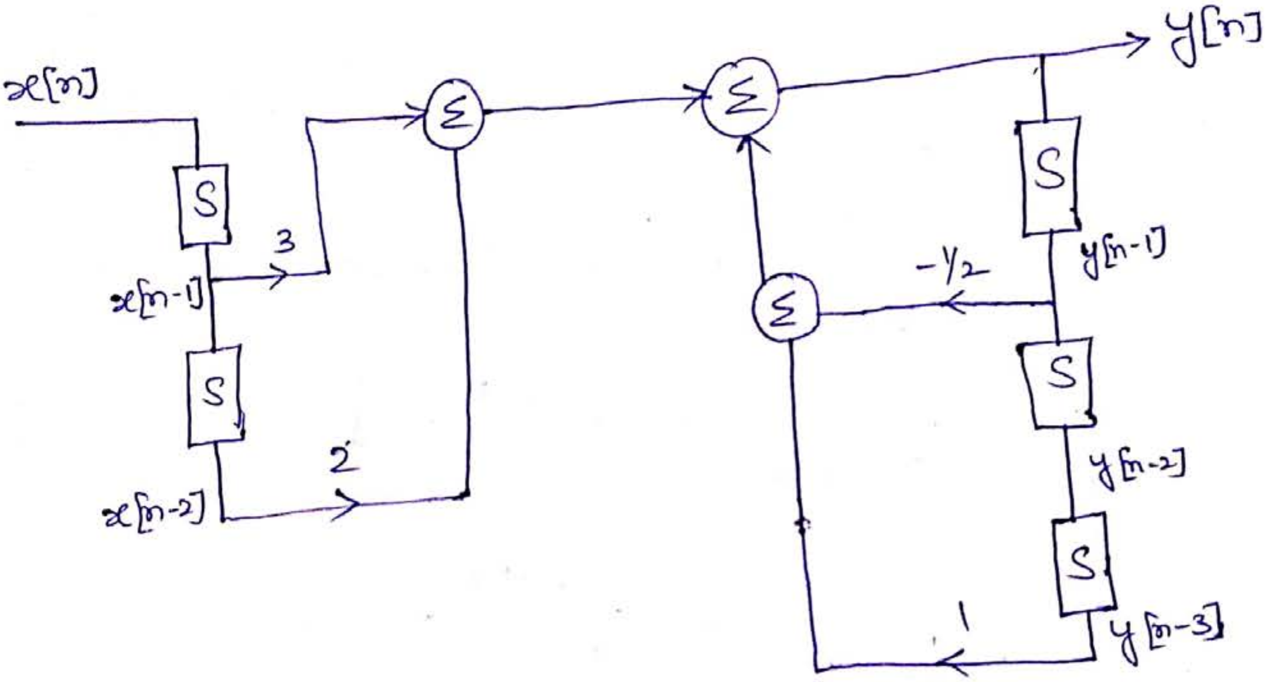
— 2 ½ M

5M

(b) $y[n] + \frac{1}{2} y[n-1] - y[n-3] = 3x[n-1] + 2x[n-2]$ (2)

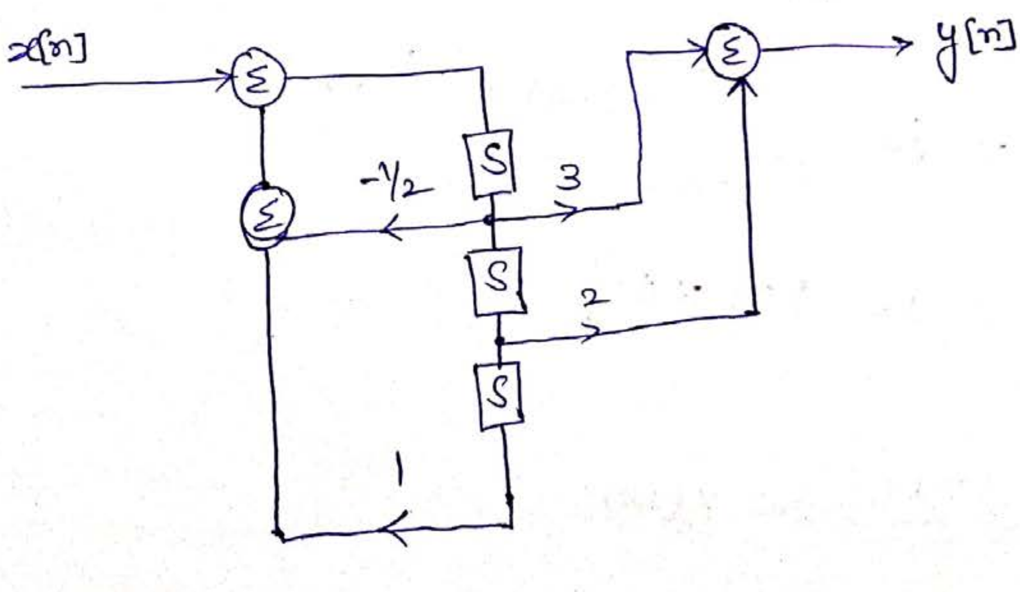
$$y[n] = 3x[n-1] + 2x[n-2] - \frac{1}{2} y[n-1] + y[n-3]$$

Direct form-I.



— 2 ½ M

Direct form-II



— 2 ½ M

5M.

② (a) Time differention property

$$x(t) \xleftrightarrow{CTFT} X(j\omega)$$

$$\frac{dx(t)}{dt} \xleftrightarrow{CTFT} j\omega\, X(j\omega).$$ — 1M

Proof:-

$$x(t) \xleftrightarrow{CTFT} X(j\omega).$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$ — 1M

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}\left[\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega\right]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, \frac{\partial(e^{j\omega t})}{\partial t}\, d\omega$$ — 1M

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,(j\omega)\, e^{j\omega t}\, d\omega$$ — 1M

$$\frac{dx(t)}{dt} = j\omega\, \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega.$$ — 1M

$$= j\omega\, FT[x(t)]$$

$$\frac{dx(t)}{dt} \xleftrightarrow{CTFT} j\omega\, X(j\omega).$$

5M

2 ⓑ fourier transform of unit step function

$$x(t) = u(t)$$

$$x(t) = \frac{1}{2} + \frac{1}{2}\,Sgn(t). \quad -①$$

Apply Fourier transform,

$$1 \xleftrightarrow{CTFT} 2\pi\delta(\omega)$$

$$\frac{1}{2} \xleftrightarrow{CTFT} \pi\delta(\omega). \quad -②$$

— 2M

$$e^{-at}u(t) - e^{at}u(-t) \xleftrightarrow{FT} \frac{1}{a+j\omega} - \frac{1}{a-j\omega}$$

let a=0

$$u(t) - u(-t) \xleftrightarrow{FT} \frac{1}{j\omega} - \left(\frac{1}{-j\omega}\right)$$

$$u(t) - u(-t) \xleftrightarrow{FT} \frac{2}{j\omega}$$

$$Sgn(t) \xleftrightarrow{FT} \frac{2}{j\omega}$$

$$\frac{1}{2}\,Sgn(t) \xleftrightarrow{FT} \frac{1}{j\omega} \quad -③.$$

— 2M

From ①, ② & ③,

$$u(t) = \frac{1}{2} + \frac{1}{2}Sgn(t) \xleftrightarrow{FT} \pi\delta(\omega) + \frac{1}{j\omega}.$$

$$\therefore\ u(t) \xleftrightarrow{CTFT} \pi\delta(\omega) + \frac{1}{j\omega}.$$

— 1M

5M

$$Sgn(t) = u(t) - u(-t)$$

$$= u(t) - (1 - u(t))$$

$$= 2u(t) - 1$$

$$1 + Sgn(t) = 2u(t)$$

$$\frac{1}{2} + \frac{1}{2}Sgn(t) = u(t)$$

$$\delta(t) \xleftrightarrow{FT} 1.$$

using duality property.

$$1 \xleftrightarrow{FT} 2\pi\delta(-\omega)$$

$$1 \xleftrightarrow{FT} 2\pi\delta(\omega)$$

$$\frac{1}{2} \xleftrightarrow{FT} \frac{1}{\cancel{2}}\cancel{2}\pi\delta(\omega)$$

$$\cancel{e^{-at}u(t)}$$

3 (a) Fourier transform of $x(t) = e^{-a|t|}$

$$x(t) = e^{-a|t|} \qquad |t| = \begin{cases} t & t>0 \\ -t & t<0. \end{cases}$$

$$x(t) = e^{-at}u(t) + e^{-a(-t)}u(-t)$$

$$x(t) = e^{-at}u(t) + e^{at}u(-t).$$

$$e^{-at}u(t) \xleftrightarrow{FT} \frac{1}{a+j\omega}$$

$$e^{at}u(-t) \xleftrightarrow{FT} \frac{1}{a-j\omega}. \quad \text{— 1M}$$

~~x(t) =~~ Apply Fourier transform,

$$X(j\omega) = \frac{1}{a+j\omega} + \frac{1}{a-j\omega}$$

$$X(j\omega) = \frac{a-j\omega+a+j\omega}{(a+j\omega)(a-j\omega)}$$

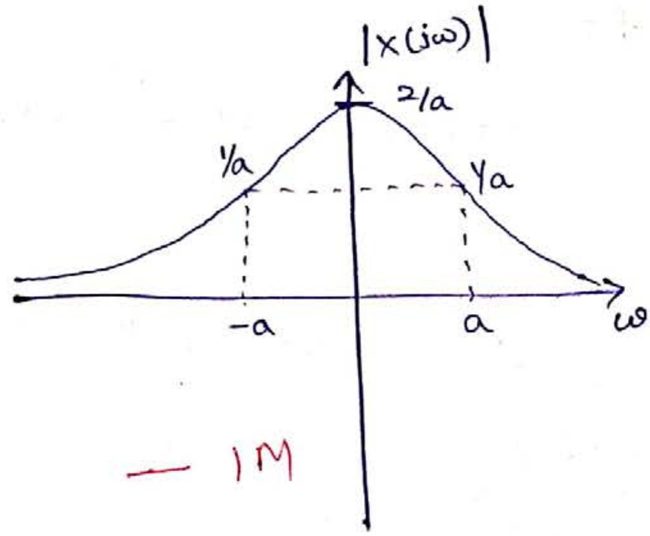
$$X(j\omega) = \frac{2a}{a^2+\omega^2}. \quad \text{— 2M}$$

$$|X(j\omega)| = \frac{2a}{a^2+\omega^2}, \quad \angle X(j\omega) = 0.$$

$$\omega = 0 \Rightarrow |X(j\omega)| = \frac{2a}{a^2} = \frac{2}{a}$$

$$\omega = a \Rightarrow |X(j\omega)| = \frac{2a}{2a^2} = \frac{1}{a}$$

$$\omega = -a \Rightarrow |X(j\omega)| = \frac{2a}{2a^2} = \frac{1}{a}.$$



— 1M

4M

3b) find frequency response for the given impulse response. (6)

$$h(t) = \frac{1}{RC} e^{-t/RC} u(t).$$

Sol:- frequency response $= H(j\omega)$

$= FT[h(t)]$.

$$h(t) \xleftrightarrow{FT} H(j\omega).$$

Standard fourier transform,

$$e^{-at} u(t) \xleftrightarrow{FT} \frac{1}{a+j\omega}$$

$$e^{-t/RC} u(t) \xleftrightarrow{FT} \frac{1}{\frac{1}{RC}+j\omega}$$

$$\frac{1}{RC} e^{-t/RC} u(t) \xleftrightarrow{FT} \frac{1/RC}{1/RC + j\omega}$$

$$h(t) \xleftrightarrow{FT} H(j\omega).$$

$$\therefore H(j\omega) = \frac{1/RC}{1/RC+j\omega} = \frac{(1/RC)}{\left(\frac{1+j\omega RC}{RC}\right)}$$

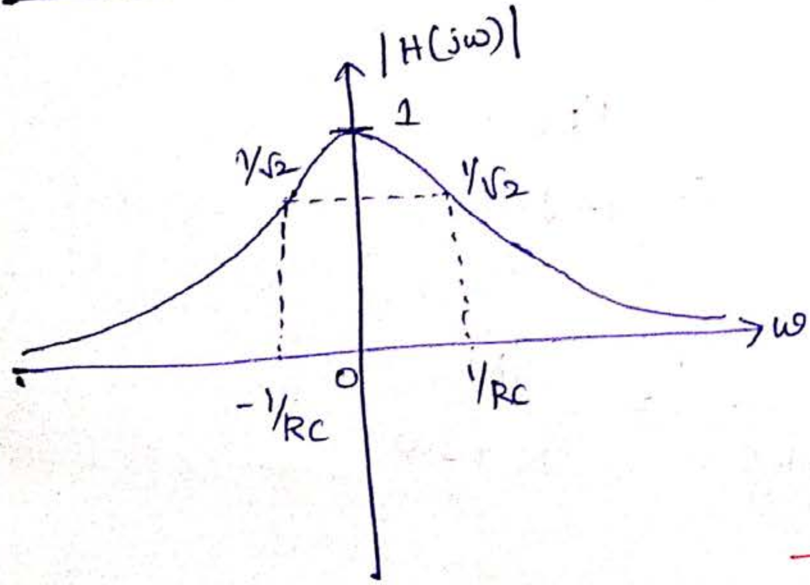
$$H(j\omega) = \frac{1}{1+j\omega RC} = \frac{1/RC}{1/RC + j\omega}$$ — 3M

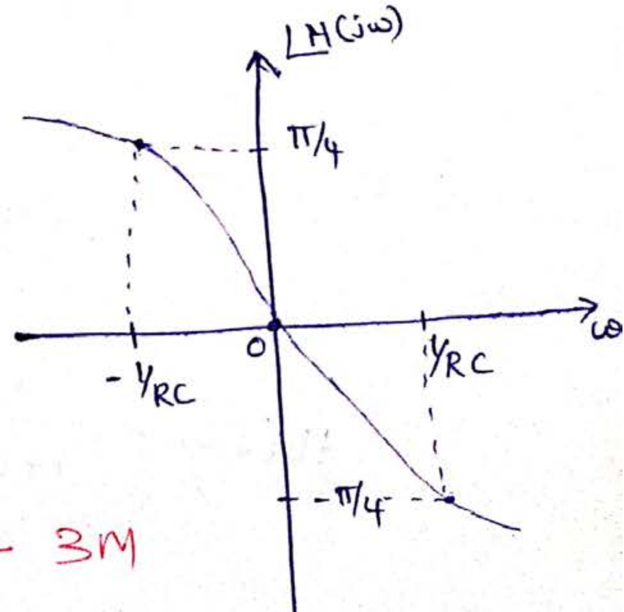
$$H(j\omega) = \frac{1/RC}{1/RC + j\omega} = \frac{1}{1 + j\omega RC}.$$

$$|H(j\omega)| = \frac{1/RC}{\sqrt{(1/RC)^2 + \omega^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}(\omega RC).$$

ω	$\|H(j\omega)\|$	$\angle H(j\omega)$
$\omega = 0$	1	$-\tan^{-1}(0) = 0°$
$\omega = -1/RC$	$\frac{1}{\sqrt{1+1}} = 1/\sqrt{2}$	$-\tan^{-1}(-1) = \frac{\pi}{4}$.
$\omega = 1/RC$	$\frac{1}{\sqrt{2}}$	$-\tan^{-1}(1) = -\pi/4$.





— 3M

6M

4 (a) Inverse fourier transform,

$$X(j\omega) = \frac{-j\omega}{(j\omega)^2 + 3j\omega + 2}$$

Sol:-

$$X(j\omega) = \frac{-j\omega}{(j\omega)^2 + 3j\omega + 2}$$

$$= \frac{-j\omega}{(j\omega)^2 + 2j\omega + j\omega + 2}$$

$$X(j\omega) = \frac{-j\omega}{(j\omega+1)(j\omega+2)} = \frac{A}{j\omega+1} + \frac{B}{j\omega+2}.$$ —— 1M

$-j\omega = A(j\omega+2) + B(j\omega+1)$

let $j\omega = -2$,

$2 = -B$

$\boxed{-2 = B}$

let $j\omega = -1$

$\boxed{1 = A}$

$$X(j\omega) = \frac{1}{j\omega+1} - \frac{2}{j\omega+2}$$ —— 1M

Apply Inverse fourier transform,

$$x(t) = e^{-t}u(t) - 2e^{-2t}u(t)$$

$$\boxed{x(t) = (e^{-t} - 2e^{-2t})\,u(t)}$$ —— 1M

$e^{-t}u(t) \xleftrightarrow{FT} \frac{1}{1+j\omega}$

$e^{-2t}u(t) \xleftrightarrow{FT} \frac{1}{2+j\omega}.$ —— 1M

4M

(4) (b) find frequency response and Impulse response.

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = -\frac{dx(t)}{dt}$$

Apply fourier transform,

By using time-differentiation property,

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$\frac{dx(t)}{dt} \xleftrightarrow{FT} j\omega X(j\omega)$$

$$y(t) \xleftrightarrow{FT} Y(j\omega)$$

$$\frac{dy(t)}{dt} \xleftrightarrow{FT} j\omega Y(j\omega)$$

$$\frac{d^2y(t)}{dt^2} \xleftrightarrow{FT} (j\omega)^2 Y(j\omega).$$

$$(j\omega)^2 Y(j\omega) + 5j\omega Y(j\omega) + 6Y(j\omega) = -j\omega X(j\omega)$$

$$Y(j\omega)\left[(j\omega)^2 + 5j\omega + 6\right] = -j\omega X(j\omega)$$

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{-j\omega}{(j\omega)^2 + 5j\omega + 6} \quad \text{--- 2M}$$

frequency response $H(j\omega) = \dfrac{-j\omega}{(j\omega)^2+5j\omega+6}$

$$H(j\omega) = \frac{-j\omega}{(j\omega)^2+5j\omega+6}$$

$$H(j\omega) = \frac{-j\omega}{(j\omega+2)(j\omega+3)} = \frac{A}{j\omega+2} + \frac{B}{j\omega+3}.$$

$$-j\omega = A(j\omega+3) + B(j\omega+2)$$

let $j\omega = -2$

$\boxed{2 = A}$

let $j\omega = -3$

$3 = -B$

$\boxed{-3 = B}$

$$H(j\omega) = \frac{2}{j\omega+2} - \frac{3}{j\omega+3} \quad \text{— 2M}$$

Apply Inverse fourier transform,

$$h(t) = 2e^{-2t}u(t) - 3e^{-3t}u(t)$$

$$h(t) = (2e^{-2t} - 3e^{-3t})u(t). \quad \text{— 2M}$$

$e^{-at}u(t) \xleftrightarrow{FT} \dfrac{1}{a+j\omega}$

$e^{-2t}u(t) \xleftrightarrow{FT} \dfrac{1}{2+j\omega}$

$2e^{-2t}u(t) \xleftrightarrow{FT} \dfrac{2}{2+j\omega}$

$e^{-3t}u(t) \xleftrightarrow{FT} \dfrac{1}{3+j\omega}$

$3e^{-3t}u(t) \xleftrightarrow{FT} \dfrac{3}{3+j\omega}.$

$$\therefore \boxed{\begin{aligned} H(j\omega) &= \frac{-j\omega}{(j\omega)^2+5j\omega+6} \\ h(t) &= (2e^{-2t} - 3e^{-3t})u(t) \end{aligned}} \quad \text{— 6M}$$

5 (a) state and prove (i) time-shifting (b) Parseval's theorem in discrete domain. (11)

(i) Time Shifting property :-

$$x(n) \xleftrightarrow{DTFT} X(e^{j\Omega})$$

$$y(n) = x(n-n_0) \xleftrightarrow{DTFT} e^{-j\Omega n_0} X(e^{j\Omega}) = Y(e^{j\Omega}).$$

Proof :-

$$y(n) \xleftrightarrow{DTFT} Y(e^{j\Omega})$$

$$\cancel{Y(e^{j\Omega})} = \cancel{\frac{1}{2\pi} \int_{-\pi}^{\pi} y(n)}$$

$$Y(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\Omega n}$$

$$Y(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n-n_0)\, e^{-j\Omega n}.$$

Let $n-n_0 = m$ | m limits also $-\infty$ to ∞.

$n = m+n_0$

$$Y(e^{j\Omega}) = \sum_{m=-\infty}^{\infty} x(m)\, e^{-j\Omega(m+n_0)}$$

$$= \sum_{m=-\infty}^{\infty} x(m)\, e^{-j\Omega m}\, e^{-j\Omega n_0}$$

$$Y(e^{j\Omega}) = e^{-j\Omega n_0} \sum_{m=-\infty}^{\infty} x(m)\, e^{-j\Omega m}.$$

$$\boxed{Y(e^{j\Omega}) = e^{-j\Omega n_0}\, X(e^{j\Omega}).}$$ — 3M

(ii) Parseval's theorem

$$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$$

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\Omega})|^2 \, d\Omega.$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega}) e^{j\Omega n} \, d\Omega$$

$$x^*(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{j\Omega}) e^{-j\Omega n} \, d\Omega$$

Proof :- $$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} x(n)\, x^*(n)$$

$$= \sum_{n=-\infty}^{\infty} x(n) \left[\frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{j\Omega}) e^{-j\Omega n} \, d\Omega\right]$$

$$= \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} x(n) \left[\int_{-\pi}^{\pi} X^*(e^{j\Omega}) e^{-j\Omega n} \, d\Omega\right]$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{j\Omega}) \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n} \, d\Omega.$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{j\Omega}) \, X(e^{j\Omega}) \, d\Omega.$$

Ref.

$$\boxed{X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\Omega})|^2 \, d\Omega.$$

$$\boxed{\therefore \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\Omega})|^2 \, d\Omega} \text{ — 3M}$$

6M

(5) (b) DTFT of $x[n] = \alpha^n u(n)$

$$x(n) \xleftrightarrow{DTFT} X(e^{j\Omega})$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}.$$

$$= \sum_{n=-\infty}^{\infty} \alpha^n u(n) e^{-j\Omega n}$$

$$= \sum_{n=0}^{\infty} \alpha^n e^{-j\Omega n}$$

$$= \sum_{n=0}^{\infty} (\alpha e^{-j\Omega})^n.$$

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} ; |\alpha| < 1$$

$$X(e^{j\Omega}) = \frac{1}{1-\alpha e^{-j\Omega}} \text{ — 1M}$$

$$|X(e^{j\Omega})| = \left| \frac{1}{1-\alpha(\cos\Omega - j\sin\Omega)} \right| = \left| \frac{1}{(1-\alpha\cos\Omega) + j\alpha\sin\Omega} \right|$$

$$|X(e^{j\Omega})| = \frac{1}{\sqrt{(1-\alpha\cos\Omega)^2 + (\alpha\sin\Omega)^2}} \text{ — 1M}$$

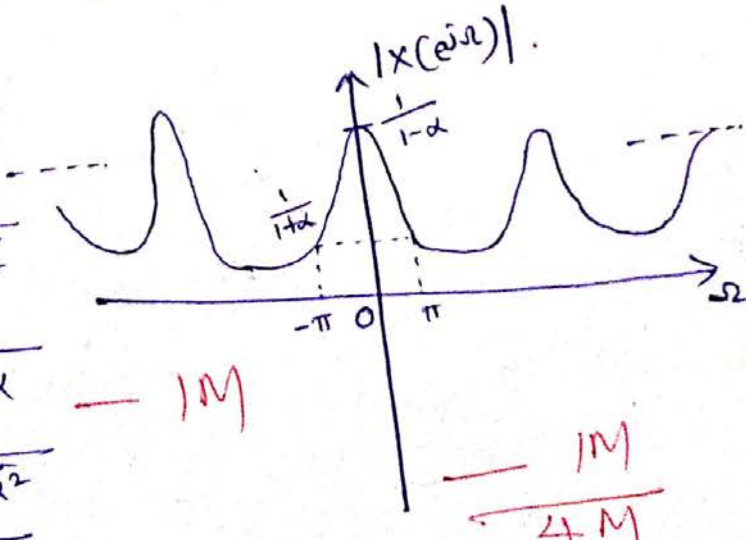
$\Omega = -\pi, \; |X(e^{j\Omega})| = \frac{1}{1+\alpha}$

$\Omega = -\pi/2, \; |X(e^{j\Omega})| = \frac{1}{\sqrt{1+\alpha^2}}$

$\Omega = 0, \; |X(e^{j\Omega})| = \frac{1}{1-\alpha}$ — 1M

$\Omega = \frac{\pi}{2}, \; |X(e^{j\Omega})| = \frac{1}{\sqrt{1+\alpha^2}}$

$\Omega = \pi, \; |X(e^{j\Omega})| = \frac{1}{1+\alpha}$.



— 1M

4M

6 obtain frequency response and impulse response.

Given

output $y[n] = \frac{1}{4}\left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{4}\right)^n u(n)$

input $x(n) = \left(\frac{1}{2}\right)^n u(n)$

frequency response $H(e^{j\Omega}) = \dfrac{Y(e^{j\Omega})}{X(e^{j\Omega})}$

$y[n] = \frac{1}{4}\left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{4}\right)^n u(n)$.

Apply DTFT,

$a^n u(n) \xleftrightarrow{DTFT} \dfrac{1}{1-ae^{-j\Omega}}$

$\left(\frac{1}{2}\right)^n u(n) \xleftrightarrow{DTFT} \dfrac{1}{1-\frac{1}{2}e^{-j\Omega}}$

$\frac{1}{4}\left(\frac{1}{2}\right)^n u(n) \xleftrightarrow{DTFT} \dfrac{1/4}{1-\frac{1}{2}e^{-j\Omega}}$.

$\left(\frac{1}{4}\right)^n u(n) \xleftrightarrow{DTFT} \dfrac{1}{1-\frac{1}{4}e^{-j\Omega}}$.

$$Y(e^{j\Omega}) = \frac{1/4}{(1-\frac{1}{2}e^{-j\Omega})} + \frac{1}{(1-1/4\,e^{-j\Omega})}$$ — 2 M

2 M

$x[n] = \left(\frac{1}{2}\right)^n u(n)$

Apply DTFT,

$$a^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-ae^{-j\Omega}}.$$

$$X(e^{j\Omega}) = \frac{1}{1-\frac{1}{2}e^{-j\Omega}} \text{ — 2M} \qquad \left(\frac{1}{2}\right)^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{2}e^{-j\Omega}}.$$

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$$

$$= \frac{\left(\frac{1/4}{1-\frac{1}{2}e^{-j\Omega}}\right) + \left(\frac{1}{1-\frac{1}{4}e^{-j\Omega}}\right)}{\frac{1}{\left(1-\frac{1}{2}e^{-j\Omega}\right)}}$$

$$= \frac{1}{4}\frac{\left(1-\frac{1}{2}e^{-j\Omega}\right)}{1-\frac{1}{2}e^{-j\Omega}} + \frac{1-\frac{1}{2}e^{-j\Omega}}{1-\frac{1}{4}e^{-j\Omega}}.$$

$$H(e^{j\Omega}) = \frac{1}{4} + \frac{1}{1-\frac{1}{4}e^{-j\Omega}} - \frac{1}{2}\frac{e^{-j\Omega}}{1-\frac{1}{4}e^{-j\Omega}}. \text{ — 2M}$$

Apply Inverser fourier transform,

$$IDTFT\left[H(e^{j\Omega})\right] = IDTFT\left[\frac{1}{4}\right] + IDTFT\left[\frac{1}{1-\frac{1}{4}e^{-j\Omega}}\right] - \frac{1}{2} IDTFT\left[\frac{e^{-j\Omega}}{1-\frac{1}{4}e^{-j\Omega}}\right]$$

$$\text{IDTFT}\left[\frac{1}{4}\right] = \frac{1}{4}\delta[n].$$

$$\text{IDTFT}\left[\frac{1}{1-\frac{1}{4}e^{-j\Omega}}\right] = \left(\frac{1}{4}\right)^{n} u(n)$$

$$\text{IDTFT}\left[\frac{e^{-j\Omega}}{1-\frac{1}{4}e^{-j\Omega}}\right] = \left(\frac{1}{4}\right)^{n-1} u(n-1).$$

$$\delta[n] \xleftrightarrow{\text{DTFT}} 1$$

$$\frac{1}{4}\delta(n) \longleftrightarrow \frac{1}{4}$$

$$a^{n}u(n) \xleftrightarrow{\text{DTFT}} \frac{1}{1-ae^{-j\Omega}}$$

$$\left(\frac{1}{4}\right)^{n}u(n) \xleftrightarrow{\text{DTFT}} \frac{1}{1-\frac{1}{4}e^{-j\Omega}}.$$

$$\left(\frac{1}{4}\right)^{n-1}u(n-1) \xleftrightarrow{\text{DTFT}} e^{-j\Omega}\left(\frac{1}{1-\frac{1}{4}e^{-j\Omega}}\right)$$

— 2M

$$\therefore\ h(n) = \frac{1}{4}\delta(n) + \left(\frac{1}{4}\right)^{n}u(n) - \frac{1}{2}\left(\frac{1}{4}\right)^{n-1}u(n-1).$$

$$\boxed{h(n) = \frac{1}{4}\delta(n) + \left(\frac{1}{4}\right)^{n}u(n) - 2\left(\frac{1}{4}\right)^{n}u(n-1).}$$

2M

10M.

7) (a) find frequency and impulse response.

$$y[n] + \frac{1}{2}y[n-1] = x(n] - 2x[n-1].$$

By definition of Convolution,

$$y(n) = x(n) * h(n)$$

Apply DTFT,

$$Y(e^{j\Omega}) = X(e^{j\Omega})\, H(e^{j\Omega})$$

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$$

$$y(n) + \frac{1}{2} y(n-1) = x(n) - 2x(n-1)$$

Apply DTFT,

$$Y(e^{j\Omega}) + \frac{1}{2} e^{-j\Omega} Y(e^{j\Omega}) = X(e^{j\Omega}) - 2e^{-j\Omega} X(e^{j\Omega})$$

$$Y(e^{j\Omega})\left[1 + \frac{1}{2} e^{-j\Omega}\right] = X(e^{j\Omega})\left[1 - 2e^{-j\Omega}\right]$$

$$y(n) \xleftrightarrow{DTFT} Y(e^{j\Omega})$$

$$y(n-1) \xleftrightarrow{DTFT} e^{-j\Omega} Y(e^{j\Omega})$$

$$x(n) \xleftrightarrow{DTFT} X(e^{j\Omega})$$

$$x(n-1) \xleftrightarrow{DTFT} e^{-j\Omega} X(e^{j\Omega})$$

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \frac{1 - 2e^{-j\Omega}}{1 + \frac{1}{2} e^{-j\Omega}}$$ — 3M

$$H(e^{j\Omega}) = \frac{1}{1 + \frac{1}{2} e^{-j\Omega}} - \frac{2e^{-j\Omega}}{1 + \frac{1}{2} e^{-j\Omega}}$$

$$\alpha^n u(n) \xleftrightarrow{DTFT} \frac{1}{1 - \alpha e^{-j\Omega}}$$

$$\left(-\frac{1}{2}\right)^n u(n) \xleftrightarrow{DTFT} \frac{1}{1 + \frac{1}{2} e^{-j\Omega}}$$

$$2\left(-\frac{1}{2}\right)^{n-1} u(n-1) \xleftrightarrow{DTFT} \frac{2e^{-j\Omega}}{1 - \left(-\frac{1}{2}\right)e^{-j\Omega}}$$

Apply IDTFT,

$$h(n) = \left(\frac{-1}{2}\right)^n u(n) - 2\left(\frac{-1}{2}\right)^{n-1} u(n-1)$$ — 2M

$$\left(\frac{-1}{2}\right)^{n-1} = \left(\frac{-1}{2}\right)^n \left(\frac{-1}{2}\right)^{-1} = \left(-\frac{1}{2}\right)^n (-2)$$

$$h(n) = \left(\frac{-1}{2}\right)^n u(n) + 4\left(\frac{-1}{2}\right)^n u(n-1)$$

5M

7(b) fourier transform of (18)

$$x[n] = \cos\left(\frac{\pi}{4}n\right)\left(\frac{1}{2}\right)^n u(n-2).$$

$$e^{j\pi/4 n} = \cos\frac{\pi}{4}n + j\sin\frac{\pi}{4}n$$

$$e^{-j\pi/4 n} = \cos\frac{\pi}{4}n - j\sin\frac{\pi}{4}n.$$

$$\left(e^{j\pi/4 n} + e^{-j\pi/4 n}\right) = 2\cos\frac{\pi}{4}n$$

$$\cos\left(\frac{\pi}{4}n\right) = \frac{1}{2}\left(e^{j\pi/4 n} + e^{-j\pi/4 n}\right)$$

$$x(n) = \cos\left(\frac{\pi}{4}n\right)\left(\frac{1}{2}\right)^n u(n-2)$$

$$x(n) = \cos\left(\frac{\pi}{4}n\right)\left(\frac{1}{2}\right)^{n-2+2} u(n-2)$$

$$= \cos\left(\frac{\pi}{4}n\right)\left(\frac{1}{2}\right)^{n-2} u(n-2)\left(\frac{1}{2}\right)^2$$

$$= \frac{1}{4}\cos\left(\frac{\pi}{4}n\right)\left(\frac{1}{2}\right)^{n-2} u(n-2)$$

$$x(n) = \frac{1}{8}\left(e^{j\pi/4 n} + e^{-j\pi/4 n}\right)\left(\frac{1}{2}\right)^{n-2} u(n-2)$$

$$x(n) = \frac{1}{8}\left[e^{j\pi/4 n}\left(\frac{1}{2}\right)^{n-2} u(n-2) + e^{-j\pi/4 n}\left(\frac{1}{2}\right)^{n-2} u(n-2)\right]$$

2M

$$\alpha^{n} u(n) \xleftrightarrow{DTFT} \frac{1}{1-\alpha e^{-j\Omega}}.$$

$$\left(\frac{1}{2}\right)^{n} u(n) \xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{2} e^{-j\Omega}}.$$

using time shifting property,

$$\boxed{\begin{array}{l} x(n) \xleftrightarrow{DTFT} X(e^{j\Omega}) \\ x(n-n_0) \xleftrightarrow{DTFT} e^{-j\Omega n_0} X(e^{j\Omega}) \end{array}}$$

$$\left(\frac{1}{2}\right)^{n-2} u(n-2) \xleftrightarrow{DTFT} \frac{e^{-2j\Omega}}{1-\frac{1}{2} e^{-j\Omega}}.$$ — 1M

using frequency shifting property.

$$\boxed{\begin{array}{l} x(n) \xleftrightarrow{DTFT} X(e^{j\Omega}) \\ e^{j\beta n}\, x(n) \xleftrightarrow{DTFT} X(e^{j(\Omega-\beta)}) \end{array}}$$

$$e^{j\pi/4\, n} \left(\frac{1}{2}\right)^{n-2} u(n-2) \xleftrightarrow{DTFT} \frac{e^{-2j(\Omega-\pi/4)}}{1-\frac{1}{2} e^{-j(\Omega-\pi/4)}}$$ — 1M

$$e^{-j\pi/4\, n} \left(\frac{1}{2}\right)^{n-2} u(n-2) \xleftrightarrow{DTFT} \frac{e^{-2j(\Omega+\pi/4)}}{1-\frac{1}{2} e^{-j(\Omega+\pi/4)}}.$$

— 1M

$$x(n) = \frac{1}{8}\left[\frac{e^{-2j(\Omega-\pi/4)}}{1-\frac{1}{2} e^{-j(\Omega-\pi/4)}} + \frac{e^{-2j(\Omega+\pi/4)}}{1-\frac{1}{2} e^{-j(\Omega+\pi/4)}}\right]$$

5M

$$\cancel{= \frac{1}{8}\left[\frac{}{\left(1-\frac{1}{2} e^{-j(\Omega-\pi/4)}\right)}\right]}$$

8 a find the DTFT

(i) $x(n) = \left(\frac{1}{4}\right)^n u(n-4)$

Standard fourier transform signal,

$$\alpha^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-\alpha e^{-j\Omega}}$$

$$\left(\frac{1}{4}\right)^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{4}e^{-j\Omega}}$$

By using time shifting property,

$$x(n) \xleftrightarrow{DTFT} X(e^{j\Omega})$$

$$x(n-n_0) \xleftrightarrow{DTFT} e^{-j\Omega n_0} X(e^{j\Omega})$$

$$\left(\frac{1}{4}\right)^{n-4} u(n-4) \xleftrightarrow{DTFT} \frac{e^{-4j\Omega}}{1-\frac{1}{4}e^{-j\Omega}}$$

$$\left(\frac{1}{4}\right)^{-4}\left(\frac{1}{4}\right)^n u(n-4) \xleftrightarrow{DTFT} \frac{e^{-4j\Omega}}{1-\frac{1}{4}e^{-j\Omega}}.$$

$$\left(\frac{1}{4}\right)^n u(n-4) \xleftrightarrow{DTFT} \frac{\left(\frac{1}{4}\right)^4 e^{-4j\Omega}}{1-\frac{1}{4}e^{-j\Omega}}.$$

$$\left(\frac{1}{4}\right)^n u(n-4) \xleftrightarrow{DTFT} \frac{\left(\frac{1}{4}e^{-j\Omega}\right)^4}{1-\frac{1}{4}e^{-j\Omega}} \quad //.$$

— 3M

(ii) $x(n) = \left(\frac{1}{3}\right)^n u(n) * \left(\frac{1}{4}\right)^n u(n)$.

Standard ~~[illegible]~~ fourier transfrom Signal,

$$\alpha^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-\alpha e^{-j\Omega}}.$$

$$\left(\frac{1}{3}\right)^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{3}e^{-j\Omega}}.$$

$$\left(\frac{1}{4}\right)^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{4}e^{-j\Omega}}.$$

By using Convolution in time-domain property,

$$x(n) \xleftrightarrow{DTFT} X(e^{j\Omega})$$

$$y(n) \xleftrightarrow{DTFT} Y(e^{j\Omega})$$

$$x(n) * y(n) \xleftrightarrow{DTFT} X(e^{j\Omega})\, Y(e^{j\Omega})$$

$$\therefore \left(\frac{1}{3}\right)^n u(n) * \left(\frac{1}{4}\right)^n u(n) \xleftrightarrow{DTFT} \frac{1}{\left(1-\frac{1}{3}e^{-j\Omega}\right)} \frac{1}{\left(1-\frac{1}{4}e^{-j\Omega}\right)}$$

$$\therefore X(e^{j\Omega}) = \frac{1}{\left(1-\frac{1}{3}e^{-j\Omega}\right)\left(1-\frac{1}{4}e^{-j\Omega}\right)}$$ — 2M $\quad \frac{1}{3}+\frac{1}{4}=\frac{7}{12}$

$$\boxed{X(e^{j\Omega}) = \frac{1}{\frac{1}{12}e^{-2j\Omega} - \frac{7}{12}e^{-j\Omega} + 1}}$$

5M

8 b

$$H(e^{j\Omega}) = 1 + \frac{e^{-j\Omega}}{(1-\frac{1}{2}e^{-j\Omega})(1+\frac{1}{4}e^{-j\Omega})}$$

$$H(e^{j\Omega}) = \frac{(1-\frac{1}{2}e^{-j\Omega})(1+\frac{1}{4}e^{-j\Omega}) + e^{-j\Omega}}{(1-\frac{1}{2}e^{-j\Omega})(1+\frac{1}{4}e^{-j\Omega})}$$

$$\frac{(1-\frac{1}{2}e^{-j\Omega})(1+\frac{1}{4}e^{-j\Omega}) + e^{-j\Omega}}{(1-\frac{1}{2}e^{-j\Omega})(1+\frac{1}{4}e^{-j\Omega})} = \frac{A}{(1-\frac{1}{2}e^{-j\Omega})} + \frac{B}{(1+\frac{1}{4}e^{-j\Omega})}$$

$$(1-\tfrac{1}{2}e^{-j\Omega})(1+\tfrac{1}{4}e^{-j\Omega}) + e^{-j\Omega} = A(1+\tfrac{1}{4}e^{-j\Omega}) + B(1-\tfrac{1}{2}e^{-j\Omega})$$

let $e^{-j\Omega} = 2$,

$$2 = A(3/2)$$

$$\frac{4}{3} = A$$

let $e^{-j\Omega} = -4$

$$-4 = B(1-\tfrac{1}{\cancel{2}}(-\cancel{4})^{2})$$

$$-\frac{4}{3} = B$$

$$H(e^{j\Omega}) = \frac{4/3}{(1-\frac{1}{2}e^{-j\Omega})} - \frac{4/3}{1+1/4\, e^{-j\Omega}}.$$

$a^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-ae^{-j\Omega}}.$

$(\frac{1}{2})^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{2}e^{-j\Omega}}.$

$(\frac{-1}{4})^n u(n) \xleftrightarrow{DTFT} \frac{1}{1+\frac{1}{4}e^{-j\Omega}}.$

Apply Inverse fourier transform (IDTFT),

$$h(n) = \frac{4}{3}\left(\frac{1}{2}\right)^n u(n) - \frac{4}{3}\left(\frac{-1}{4}\right)^n u(n).$$