CBCS SCHEME

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17MAT11

First Semester B.E. Degree Examination, Feb./Mar. 2022
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the n^{th} derivative of $y = \cos 2x \cos 3x$. **(06 Marks)**

b. Find the angle of intersection between the curves $r = a\,\text{cosec}^2\left(\frac{\theta}{2}\right)$ and $r = b\sec^2\left(\frac{\theta}{2}\right)$. **(07 Marks)**

c. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1). **(07 Marks)**

OR

2 a. If $y = \tan^{-1}x$, prove that $(1 + x^2)\, y_{n+2} + 2(n+1)\, xy_{n+1} + n(n+1)\, y_n = 0$. **(06 Marks)**

b. Derive $\tan\phi = r\frac{d\theta}{dr}$ with usual notations. **(07 Marks)**

c. Prove that the radius of curvature of the curve $r^n = a^n \cos n\theta$. **(07 Marks)**

Module-2

3 a. Expand $\tan^{-1}x$ upto and including x^5 using Maclaurin's series. **(06 Marks)**

b. If $u = \log_e\left(\frac{x^3 + y^3}{x^2 + y^2}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$ **(07 Marks)**

c. If $u = \frac{x_2 x_3}{x_1}$, $v = \frac{x_1 x_3}{x_2}$, $w = \frac{x_1 x_2}{x_3}$, prove that $J\left(\frac{u, v, w}{x_1, x_2, x_3}\right) = 4$. **(07 Marks)**

OR

4 a. Evaluate $\lim_{x \to 0}\left(\frac{xe^x - \log(1+x)}{x^2}\right)$ **(06 Marks)**

b. Expand $f(x) = \log_e x$ about $x = 1$ upto the term containing third degree terms using Taylor's series. **(07 Marks)**

c. If $u = f(r, s, t)$ and $r = \frac{x}{y}$, $s = \frac{y}{z}$, $t = \frac{z}{x}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ **(07 Marks)**

Module-3

5 a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = t + 5$, t – time, find the components of the velocity and acceleration at t = 2 in the direction of $\hat{i} + 3\hat{j} + 2\hat{k}$. **(06 Marks)**

b. Find div F and curl F if
$\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ **(07 Marks)**

c. Show that $\vec{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational field. Find ϕ such that $F = \nabla\phi$. **(07 Marks)**

OR

6 a. Find the value of a for which $f = (x+3y)i + (y-2z)j + (x+az)k$ is solenoidal. **(06 Marks)**

b. Prove that div(curl A) = 0. **(07 Marks)**

c. If $\vec{A} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$, find the value of curl (curl $\vec{A}$). **(07 Marks)**

Module-4

7 a. Obtain the reduction formula of $\int \cos^n x\,dx$ and hence evaluate $\int_0^{\pi/2} \cos^n x\,dx$. **(06 Marks)**

b. Solve $(xy + y^2)\,dx + (x + 2y - 1)\,dy = 0$. **(07 Marks)**

c. Find the orthogonal trajectories of the curve $r = 4a \sec\theta \tan\theta$, a is the parameter. **(07 Marks)**

OR

8 a. Evaluate $\int_0^1 x^{3/2}(1-x)^{3/2}dx$ **(06 Marks)**

b. Solve $(1 + xy^2)xy\frac{dy}{dx} = 1$ **(07 Marks)**

c. A body originally at 80°C cools down to 60°C in 20min. The temperature of the air being 40°C. What will be the temperature of the body after 40min from the original? **(07 Marks)**

Module-5

9 a. Solve by Gauss Elimination method the system of equations

$x + 2y = 3 - z$

$2x + 3y + 3z = 10$

$3x - y + 2z = 13$ **(06 Marks)**

b. Find the largest Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ by power method choosing $[1\ 0\ 0]^T$ as initial vector for obtaining 4 approximations. **(07 Marks)**

c. Reduce quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy + 4xz - 2yz$ to canonical form, using orthogonal transformation. **(07 Marks)**

OR

10 a. Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ **(06 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$ into diagonal form. **(07 Marks)**

c. Find the inverse transformation of

$u_1 = 9v_1 + 6v_2$

$u_2 = 10v_1 - 2v_2$. **(07 Marks)**

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