CBCS SCHEME

USN | | | | | | | | | | |

17MAT31

Third Semester B.E. Degree Examination, Feb./Mar.2022
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find a Fourier Series to represent $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. Hence prove that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} \ldots$ **(08 Marks)**

b. Obtain a Fourier series of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$. **(06 Marks)**

c. Find the half-range Fourier sine series of $f(x) = e^x$ in $0 < x < 1$. **(06 Marks)**

OR

2 a. Find the Fourier series expansion upto second harmonic using the following table of values:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(08 Marks)

b. Express $f(x) = (\pi - x)^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. **(06 Marks)**

c. Obtain the Half range cosine series of $f(x) = x^2$ in $0 \le x \le \pi$. **(06 Marks)**

Module-2

3 a. Find the Fourier transform of the function, $f(x) = \begin{cases} 1, & \text{for } |x| \le a \\ 0, & \text{for } |x| > a \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin ax}{x} dx$. **(08 Marks)**

b. Find the Fourier cosine transform of $f(x) = e^{-ax}$, $a > 0$ **(06 Marks)**

c. Solve $u_n + 3u_{n-1} - 4u_{n-2} = 0$ for $n \ge 2$ given $u_0 = 3$, $u_1 = -2$ using z-transform. **(06 Marks)**

OR

4 a. Find the Fourier sine transform of e^{-ax}, a>0, x>0 show that $\int_0^\infty \frac{x \sin mx}{a^2 + x^2} dx = \frac{\pi}{2} e^{-am}$, m>0. **(08 Marks)**

b. Find the z-transform of $\cosh\left(\frac{n\pi}{2} + \theta\right)$. **(06 Marks)**

c. Find the inverse z-transform of, $\frac{3z^2 + z}{(5z-1)(5z+2)}$. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

5 a. Find the correlation coefficient using the following table as values: (08 Marks)

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

b. Obtain an equation of the form $y = ax + b$ given that, (06 Marks)

x	0	5	10	15	20	25
y	12	15	17	22	24	30

c. Apply Regula-Falsi method to find the root of $xe^x = \cos x$ in four approximations with four decimals in (0, 1). (06 Marks)

OR

6 a. Obtain the regression line of y on x for the following table of values: (08 Marks)

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

b. Fit a parabola $y = a + bx + cx^2$ to the following data: (06 Marks)

x	20	40	60	80	100	120
y	5.5	9.1	14.9	22.8	33.3	46

c. Find the root of the equation $x^4 - x - 9 = 0$ by Newton-Raphson method in three approximations with three decimal places. (Take $x_0 = 2$) (06 Marks)

Module-4

7 a. Use Newton's forward interpolation formula to find y(8) from the table of values, (08 Marks)

x	0	5	10	15	20	25
y(x)	7	11	14	18	24	32

b. Determine y at x = 1 using Newton's general interpolation formula given that, (06 Marks)

x	-4	-1	0	2	5
y(x)	1245	33	5	9	1335

c. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Weddle's rule with h = 1. (06 Marks)

OR

8 a. Find f(4) using Newton's Backward interpolation formula given that,

x	0	1	2	3
y = f(x)	1	2	1	10

(08 Marks)

b. Apply Lagrange's interpolation formula to find y (x = 10) given that,

x	5	6	9	11
y(x)	12	13	14	16

(06 Marks)

c. Apply Simpson's $\frac{1}{3}^{rd}$ formula to evaluate $\int_0^{120} V(t)dt$ given that,

t	0	12	24	36	48	60	72	84	96	108	120
V(t)	0	3.60	10.08	18.90	21.60	18.54	10.26	5.40	4.5	5.4	9.0

(06 Marks)

Module-5

9 a. Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region defined by $x = 0$, $y = 0$, $x + y = 1$. **(08 Marks)**

b. Evaluate $\oint_C \vec{F}.\vec{dr}$ by Stoke's theorem with $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and C is the boundary of the triangle with vertices at, (0, 0, 0), (1, 0, 0) and (1, 1, 0). **(06 Marks)**

c. Show that the geodesies on a plane are straight lines. **(06 Marks)**

OR

10 a. Find $\iint_S \vec{F}.\vec{dS}$, where $F = (2x + 3z)\hat{i} - (xz + y)\hat{j} + (y^2 + 2z)\hat{k}$ and S is the surface of the sphere having center at (3, -1, 2) and radius 3. (Use Gauss divergence theorem). **(08 Marks)**

b. Derive Euler's equation with usual notations as, $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

c. Find the extremals of the functional,

$$\int_{x_0}^{x_1} \left(\frac{y'^2}{x^3}\right) dx.$$ **(06 Marks)**

* * * * *