

# CBCS SCHEME



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18MAT31

## Third Semester B.E. Degree Examination, Feb./Mar. 2022 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-1

- 1 a. Evaluate (i)  $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$  (ii)  $L(t^2 e^{-3t} \sin 2t)$  (06 Marks)
- b. If  $f(t) = \begin{cases} t, & 0 \leq t \leq a \\ 2a - t, & a \leq t \leq 2a \end{cases}$ ,  $f(t + 2a) = f(t)$  then show that  $L(f(t)) = \frac{1}{s^2} \tanh\left(\frac{as}{2}\right)$  (07 Marks)
- c. Solve by using Laplace Transforms  
 $\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 4y = e^{-t}$ ,  $y(0) = 0$ ,  $y'(0) = 0$  (07 Marks)

OR

- 2 a. Evaluate  $L^{-1}\left(\frac{4s+5}{(s+1)^2(s+2)}\right)$  (06 Marks)
- b. Find  $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$  by using convolution theorem. (07 Marks)
- c. Express  $f(t) = \begin{cases} \sin t, & 0 \leq t < \pi \\ \sin 2t, & \pi \leq t < 2\pi \\ \sin 3t, & t \geq 2\pi \end{cases}$   
 in terms of unit step function and hence find its Laplace Transform. (07 Marks)

### Module-2

- 3 a. Obtain fourier series for the function  $f(x) = |x|$  in  $(-\pi, \pi)$  (06 Marks)
- b. Expand  $f(x) = \frac{(\pi-x)^2}{4}$  as a Fourier series in the interval  $(0, 2\pi)$  and hence deduce that  
 $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$  (07 Marks)
- c. Express y as a Fourier series upto the second harmonic given :

x:	0	60	120	180	240	300
y:	4	3	2	4	5	6

(07 Marks)

OR

- 4 a. Find the Half-Range sine series of  $\pi x - x^2$  in the interval  $(0, \pi)$  (06 Marks)
- b. Obtain fourier expansion of the function  $f(x) = 2x - x^2$  in the interval  $(0, 3)$ . (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- c. Obtain the Fourier expansion of  $y$  upto the first harmonic given :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(07 Marks)

**Module-3**

- 5 a. If  $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ , find the Fourier transform of  $f(x)$  and hence find the

value of  $\int_0^{\infty} \frac{\sin x}{x} dx$

(06 Marks)

- b. Find the infinite Fourier cosine transform of  $e^{-ax}$ . (07 Marks)  
 c. Solve using z-transform  $y_{n+2} - 4y_n = 0$  given that  $y_0 = 0, y_1 = 2$  (07 Marks)

**OR**

- 6 a. Find the Fourier sine transform of  $f(x) = e^{-|x|}$  and hence evaluate  $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx$ ;  $m > 0$ . (06 Marks)

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- b. Obtain the z-transform of  $\cos n\theta$  and  $\sin n\theta$ . (07 Marks)

- c. Find the inverse z-transform of

$$\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$$

(07 Marks)

**Module-4**

- 7 a. Solve  $\frac{dy}{dx} = x^3 + y$ ,  $y(1) = 1$  using Taylor's series method considering up to fourth degree terms and find  $y(1.1)$ . (06 Marks)  
 b. Given  $\frac{dy}{dx} = 3x + \frac{y}{2}$ ,  $y(0) = 1$  compute  $y(0.2)$  by taking  $h = 0.2$  using Runge - Kutta method of fourth order. (07 Marks)  
 c. If  $\frac{dy}{dx} = 2e^x - y$ ,  $y(0) = 2$ ,  $y(0.1) = 2.010$ ,  $y(0.2) = 2.040$  and  $y(0.3) = 2.090$ , find  $y(0.4)$  correct to 4 decimal places using Adams-Bashforth method. (07 Marks)

**OR**

- 8 a. Use fourth order Runge-Kutta method, to find  $y(0.8)$  with  $h = 0.4$ , given  $\frac{dy}{dx} = \sqrt{x+y}$ ,  $y(0.4) = 0.41$  (06 Marks)  
 b. Use modified Euler's method to compute  $y(20.2)$  and  $y(20.4)$  given that  $\frac{dy}{dx} = \log_{10} \left( \frac{x}{y} \right)$  with  $y(20) = 5$  Taking  $h = 0.2$ . (07 Marks)  
 c. Apply Milne's predictor-corrector formulae to compute  $y(2.0)$  given  $\frac{dy}{dx} = \frac{x+y}{2}$  with

x	0.0	0.5	1.0	1.5
y	2.000	2.6360	3.5950	4.9680

(07 Marks)

**Module-5**

- 9 a. Using Runge-Kutta method, solve

$$\frac{d^2y}{dx^2} = x \left( \frac{dy}{dx} \right)^2 - y^2, \text{ for } x = 0.2, \text{ correct to four decimal places, using initial conditions } y(0) = 1, y'(0) = 0 \quad (07 \text{ Marks})$$

- b. Derive Euler's equation in the standard form viz,
- $\frac{\partial f}{\partial y} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) = 0 \quad (07 \text{ Marks})$

- c. Find the extremal of the functional
- $\int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x) dx \quad (06 \text{ Marks})$

OR

- 10 a. Given the differential equation
- $2 \frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$
- and the following table of initial values:

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	2.0657

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- Compute  $y(1.4)$  by applying Milne's Predictor-corrector formula. (07 Marks)
- b. Prove that geodesics of a plane surface are straight lines. (07 Marks)
- c. On what curves can the functional  $\int_0^1 (y'^2 + 12xy) dx$  with  $y(0) = 0, y(1) = 1$  can be extremized? (06 Marks)

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