IAT-2 PHYSICS SCHEME

1a. [6]

To show that energy levels below Fermi energy are completely occupied:

For E **<** E_F, at T = 0,

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1} = 1$$

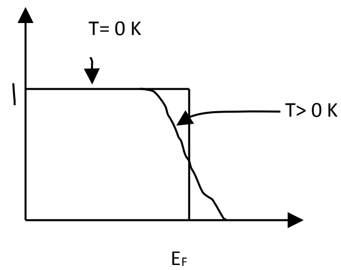
To show that energy levels above Fermi energy are empty:

For E > E_F , at T=0

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1} = 0$$

At ordinary temperatures, for E = E_F,

$$f(E) = \frac{1}{2}$$



1b [4]

<u>**Success of quantum theory:**</u>

1. Specific heat:

Classical theory predicted high values of specific heat for metals on the basis of theassumption that all the conduction electrons are capable of absorbing the heat energy as per Maxwell - Boltzmann distribution i.e.,

$$C_V = \frac{3}{2}R$$

But according to the quantum theory, only those electrons occupying energy levels close to Fermi energy (E_F) are capable of absorbing heat energy to get excited to higher energy levels. Thus only a small percentage of electrons are capable of receiving the thermal energy and specific heat value becomes small.

It can be shown that $C_V = 10^{-4} R$.

This is in conformity with the experimental values.

2. **Temperature dependence of electrical conductivity.** **2 Marks**

According to classical free electron theory,

Electrical conductivity $\propto \frac{1}{\sqrt{Temperature}}$

Where as from quantum theory

Electrical conductivity

$$\propto \frac{1}{\text{collisional area of crosssection of lattice atoms}} \alpha \frac{1}{\text{vibrational energy}} \alpha \frac{1}{\text{Temperature}}$$

This is in agreement with experimental values.

3. **Dependence of electrical conductivity on electron concentration:** **1 Marks**

According to classical theory,

$$\sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto n$$

But it has been experimentally found that Zinc which is having higher electron concentration

than copper has lower Electrical conductivity.

According to quantum free electron theory,

Electrical conductivity $\sigma = \frac{ne^2}{m}\left(\frac{\lambda}{V_F}\right)$ where V_F is the Fermi velocity.

Zinc possesses lesser conductivity because it has higher Fermi velocity.

Metal	n	σ
Cu	$8.45\times10^{28}/m^3$	$6\times10^7 (\Omega m)^{-1}$
Zn	$13\times10^{28}/m^3$	$1\times10^7 (\Omega m)^{-1}$

2a [6]

Let F = F_o Sinω_ft be the oscillating applied force

The equation of motion is given by

$$F = ma = -kx - bv + F_o \sin\omega_f t$$

$$m\frac{d^2x}{dt^2} + b.\frac{dx}{dt} + kx = F_o \sin\omega_f t$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_o}{m}\sin\omega_f t$$

$$Let\ \frac{b}{m} = 2R; \frac{k}{m} = \omega^2; \frac{F_o}{m} = F$$

$$\frac{d^2x}{dt^2} + 2R\frac{dx}{dt} + \omega_o{}^2 x = F\sin\omega_f t \ .....(1)$$ **3 Marks**

Let one particular solution be $x = A.\sin(\omega_f t - \phi)$

$$\frac{dx}{dt} = \omega_f A.\cos(\omega_f t - \phi)$$

$$\frac{d^2x}{dt^2} = -\omega_f^2 A.\sin(\omega_f t - \phi)$$

Also

$$F\sin\omega_f t = F.\sin(\omega_f t - \phi + \phi)$$

$$= F\sin(\omega_f t - \phi)\cos\phi + F\cos(\omega_f t - \phi)\sin\phi$$

Substituting in (1)

$$-\omega_f^2 A\sin(\omega_f t - \phi) + 2RA\omega_f\cos(\omega_f t - \phi) + \omega_o^2 A\sin(\omega_f t - \phi$$

Comparing coefficients of $\sin(\omega_f t - \phi)$ and $\cos(\omega_f t - \phi)$ on both sides

$$A(.\omega_o^2 - \omega_f^2) = F\cos\phi$$

$$2RA\omega_f = F\sin\phi$$

$$\therefore F^2 = A^2(\omega_o^2 - \omega_f^2)^2 + 4R^2A^2\omega_f^2$$

$$A = \frac{F}{\sqrt{(\omega_o^2 - \omega_f^2)^2 + 4R^2\omega_f^2}}$$

$$A = \frac{F_o / m}{\sqrt{(\omega_o^2 - \omega_f^2) + \frac{b^2}{m^2}\omega_f^2}}$$

$$\tan\phi = \frac{2R\omega_f}{\omega_o^2 - \omega_f^2}$$

2b. [4]

For peak amplitude, resonance condition to be applied.

$\omega_o = \omega_f$ = 2.pi.1000

$$A = \frac{F_o / m}{\sqrt{(\omega_o^2 - \omega_f^2) + \frac{b^2}{m^2}\omega_f^2}}$$

A=(F_o/m)/(b/m) 2.pi.f

= 0.099 m

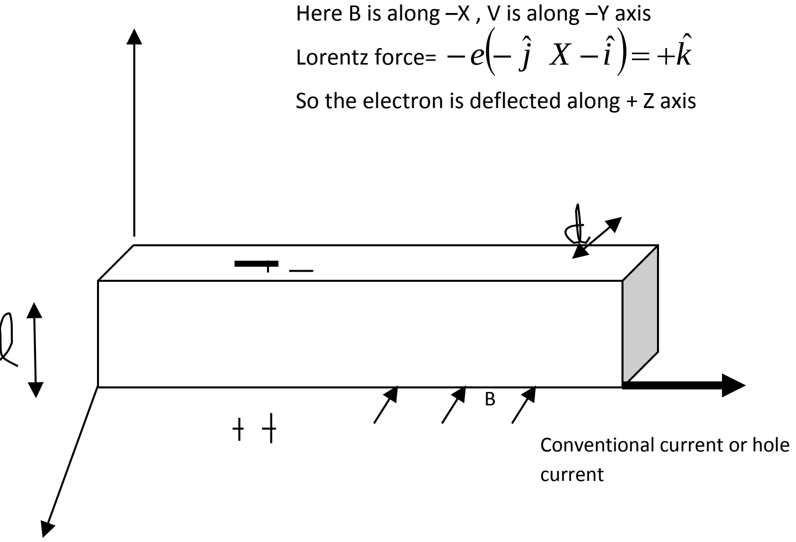
3a [6]

Hall effect: When a conductor carrying current is placed in transverse magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field.

Here B is along –X , V is along –Y axis

Lorentz force= $-e\left(-\hat{j}\ X - \hat{i}\right) = +\hat{k}$

So the electron is deflected along + Z axis



Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is applied along –Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. This develops a potential V_H-Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field E_Y is set up.

Expression for Hall Coefficient:

At equilibrium, Lorentz force is equal to force due to applied electric field

BeV = e E

Hall Field E_H = Bv

$$Current\ \ density\ \ J = -n_e ev$$

$$v = \frac{J}{n_e e}$$

$$E_H = B\frac{-J}{n_e e}$$

Hence $\frac{E_H}{JB} = -\frac{1}{n_e e} = R_H$

$$V_H = E_H.l = -R_H JBl$$

3b [4]

$$\rho = \frac{1}{\sigma} = \frac{1}{ne(\mu_h + \mu_e)} = 0.44\Omega m$$

$n_e = n_h = 2.4 \times 10^{19} /m^3$

4a [6]

CLAUSIUS – MOSOTTI RELATION:

This expression relates dielectric constant of an insulator (ε) to the polarization of individual atoms (α) comprising it.

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$$

where N is the number of atoms per unit volume

α is the polrisability of the atom

ε_r is the relative permittivity of the medium

ε_o is the permittivity of free space.

Proof:

If there are N atoms per unit volume, the electric dipole moment per unit volume – known as polarization is given by

$$P = N\alpha E_i$$

By the definition of polarization P, it can be shown that

$$P = \varepsilon_0 E_a(\varepsilon_r - 1) = N\alpha E_i$$

$$\varepsilon_0\varepsilon_r E_a - \varepsilon_0 E_a = N\alpha E_i$$

$$\varepsilon_r = 1 + \frac{N\alpha E_i}{\varepsilon_0 E_a} \quad \text{......................(1)} \textbf{ 2 Marks}$$

The internal field at an atom in a cubic structure (γ =1/3) is of the form

$$E_i = E_a + \frac{p}{3\varepsilon_0} = E_a + \frac{N\alpha E_i}{3\varepsilon_0}$$

$$\frac{E_i}{E_a} = \frac{1}{\left[1 - \left(\frac{N\alpha}{3\varepsilon_0}\right)\right]}$$

Substituting for $\frac{E_i}{E_a}$ in equation (1) **2 Marks**

$$\varepsilon_r = 1 + \frac{N\alpha}{\varepsilon_0}\left[\frac{1}{\left(1 - \frac{N\alpha}{3\varepsilon_0}\right)}\right] = \frac{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right] + \frac{N\alpha\varepsilon_0}{\varepsilon_0}}{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right]}$$

$$= \frac{1 + \frac{2}{3}\left(\frac{N\alpha}{\varepsilon_0}\right)}{1 - \frac{1}{3}\left[\frac{N\alpha}{\varepsilon_0}\right]}$$

$$\left[\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right] = \frac{\dfrac{1 + (2/3)\frac{N\varepsilon}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}} - 1}{\dfrac{1 - (2/3)\frac{N\alpha}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}} + 2} = \frac{N\alpha}{3\varepsilon_0}$$

4 b [4]

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$$

$\alpha = 7\times10^{-40}\,Fm^2$

5a [5]

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)} + 1} = 0.02$$

5b [5]

$$E_F = \frac{h^2}{8m}\left[\frac{3n}{\pi}\right]^{\frac{2}{3}}$$

E_F = 7.93 x10^{-19}J = 4.93 eV

6a [7]

For the oscillating mass in a medium with resistive coefficient b, the equation of motion is given by

$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

This is a homogeneous, linear differential equation of second order.

The auxiliary equation is $D^2 + \frac{b}{m}D + \frac{k}{m} = 0$

The roots are $D_1 = -\frac{b}{2m} + \frac{1}{2m}\sqrt{b^2 - 4mk}$ and $D_2 = -\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2 - 4mk}$

The solution can be derived as

$$x(t) = Ce^{-\left(\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2-4mk}\right)t} + De^{-\left(\frac{b}{2m} + \frac{1}{2m}\sqrt{b^2-4mk}\right)t}$$

…….(1)

Note: This can be expressed as x(t) = $Ae^{-\frac{b}{2m}t}\cos(\omega t - \phi)$

where $\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

$$A = \sqrt{C^2 + D^2} \quad \phi = \tan^{-1}(D/C)$$

Here, the term $Ae^{-\frac{b}{2m}t}$ represents the decreasing amplitude and (ωt-ϕ) represents phase

Applying following boundary conditions in (1) :

1. At t = 0 x = x_o 2. At t = 0 $\frac{dx}{dt} = 0$

Simpify

$$C = \frac{x_0}{2}\left(1 - \frac{b}{\sqrt{b^2 - 4mk}}\right)$$

$$D = \frac{x_0}{2}\left(1 + \frac{b}{\sqrt{b^2 - 4mk}}\right)$$

Case 1: $b^2 > 4mk$ OVER DAMPING

Case 2: $b^2 < 4mk$ UNDER DAMPING

Case 3: $b^2 = 4mk$ CRITICAL DAMPING

6b[3]

$$V_{\max} = \omega A = \frac{2\pi}{T} A$$

$$T = 1s$$

7a[6]

EXPRESSION FOR FERMI ENERGY

From Fermi –Dirac theory

Expression for N(E)dE **2 Marks**

$$n = \int_0^{E_F} g(E).f(E).dE = \int_0^{E_F} \frac{4\pi(2m)^{\frac{3}{2}}}{h^3} E^{\frac{1}{2}} dE.1$$

$$= \frac{4\pi(2m)^{3/2}}{h^3} \frac{E_F^{\frac{3}{2}}}{\frac{3}{2}}$$

$$E_F^{\frac{3}{2}} = \frac{h^3 3n}{8\pi(2m)^{\frac{3}{2}}}$$

$$E_F = \frac{h^2}{8m}\left[\frac{3n}{\pi}\right]^{\frac{2}{3}}$$

7b[4]

Expression for Fermi Level in Intrinsic Semiconductor

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}}$$

Hole density in valence band may be obtained from the result

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

For an intrinsic semiconductor, $n_e = n_h$

$$2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}} = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

$$\left(\frac{m_e^*}{m_h^*}\right)^{\frac{3}{2}} = e^{\frac{-E_f + E_v + E_c - E_f}{kT}}$$

$$\frac{3}{2}\ln\left(\frac{m_e^*}{m_h^*}\right) = \frac{-2E_f + E_v + E_c}{kT}$$

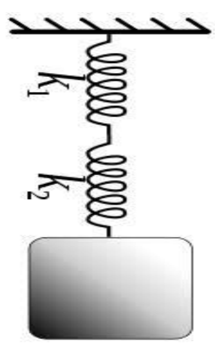
$$E_f = \frac{E_v + E_c}{2} - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*}\right)$$

For Intrinsic semiconductor, $m^*(e) = m^*(h)$

$$E_f = \frac{E_v + E_C}{2} = E_g / 2$$

8a [3+3]

Expression for Spring Constant for Series Combination



Consider a load suspended through two springs with spring constants k_1 and k_2 in series combination. Both the springs experience same stretching force. Let Δx_1 and Δx_2 be their elongation.

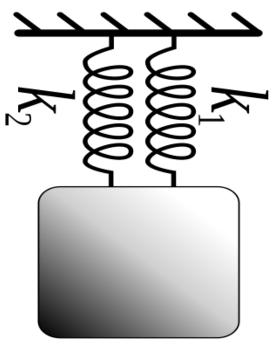
Total elongation is given by

$$\Delta X = \Delta X_1 + \Delta X_2 = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{F}{k_{eqv}} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{1}{k_{eqv}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Expression for Spring Constant forParallel Combination



Consider a load suspended through two springs with spring constants k_1 and k_2 in parallel combination. The two individual springs both elongate by x but experience the load nonuniformly.

Total load across the two springs is given by

$$F = F_1 + F_2$$

$$k_{eqv}.\Delta X = k_1.\Delta X + k_2.\Delta X$$

$$k_{eqv} = k_1 + k_2$$

8b [2+2]

Polar dielectrics: The atoms of these dielectrics are permanently polarized in nature and possess dipole moment. They show orientation polarisation

Ex: Water, Kcl, NH_3

Non Polar dielectrics:

The atoms of these are the materials do not possess permanent dipole moment.They get polarized only in the presence of external electric field**.**

Ex**:** O_2, N_2, He, Ne,CO_2