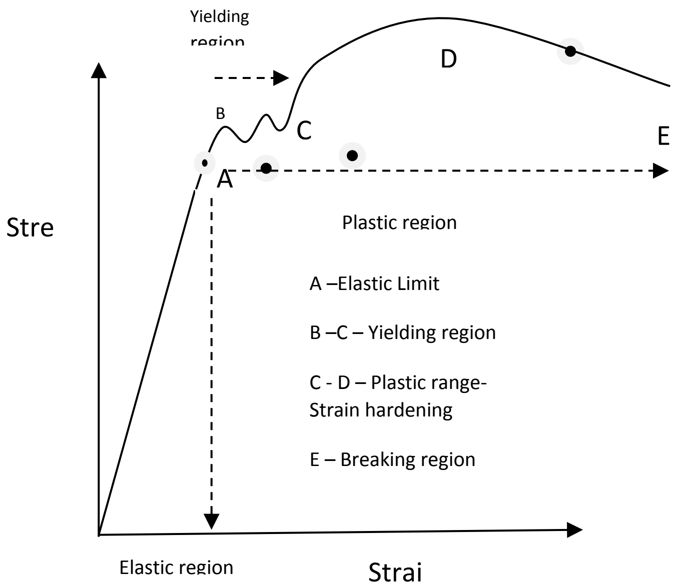
IAT-3 PHYSICS SCHEME

1.a

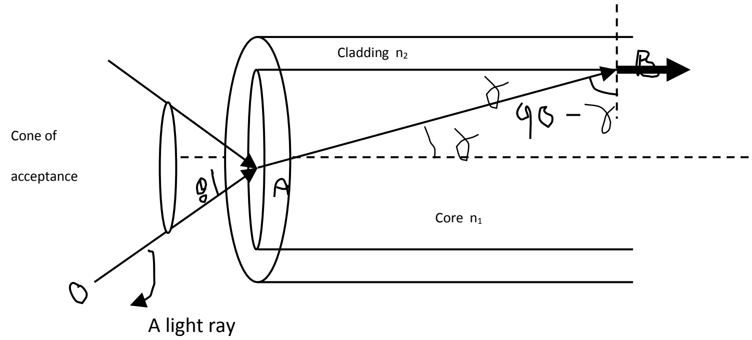
Hooke's Law: For sufficiently small stresses, strain is proportional to stress; the constant of proportionality known as modulus of elasticity depends on the material being deformed and on the nature of the deformation.



Strain is proportional to stress at smaller magnitudes. As the stress is increased to large magnitudes strain increases more rapidly and the linear relationship between stress and strain ceases to hold. This is referred as elastic limit (A). Increase in stress beyond elastic limit causes permanent deformation. This is yielding region. Small increase in stress causes large change in strain. Near the yield point C, the specimen will continue to elongate (strain) without any increase in load. When the material is in this state, it is often referred to as being **perfectly plastic**. After the yielding point (C), the strain randomly increases. This may lead to strain softening/ strain hardening in some materials. C onwards the material attains permanent status (Plastic) and is known as strain hardening region. After D, the material undergoes fracture and breaks.

1.B

Expression for condition for propagation :



Consider a light ray falling in to the optical fibre at an angle of incidence θ_0 equal to acceptance angle. Let n_0 be the refractive index of the surrounding medium .

Let n_1 be the refractive index of the core.

Let n_2 be the refractive index of the cladding.

From Snell's Law:

For the ray OA $n_0 \sin\theta_0 = n_1 \sin r$

$= n_1(1-\cos^2\theta_1)^{\frac{1}{2}}$ ………………….(1)

For the ray AB $n_1 \sin(90 - r) = n_2 \sin 90$

[here the angle of incidence is (90 - r) for which angle of refraction is 90^0].

$n_1 \cos r = n_2$

Substituting for $\cos\theta_1$ in equation (1)

$n_0 \sin\theta_0 = n_1\sqrt{1-\frac{n_2^2}{n_1^2}}$

$\sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

If the medium surrounding the fiber is air then $n_0 = 1$,

Numerical aperture $= \sin\theta_0 = \sqrt{n_1^2 - n_2^2}$

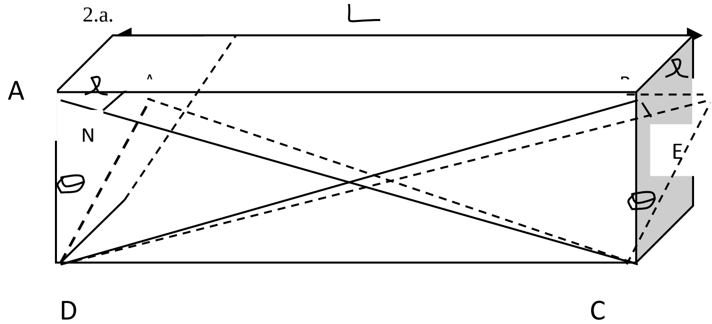
The total internal reflection will take place only if the angle of incidence $\theta_i < \theta_0$

$\therefore \sin\theta_i < \sin\theta_0$

$\mathbf{\sin\theta_i} < \sqrt{n_1^2 - n_2^2}$

This is the condition for propagation.

2.a.



From the triangle BEB[1], $B\hat{B}^1E = 45^0$

Here EB is the perpendicular from B to DB[1] . $\therefore DB = DE$

$$\cos 45 = \frac{EB^1}{BB^1} \Rightarrow EB^1 = \frac{l}{\sqrt{2}}$$

$$DE = \sqrt{2}L$$

Extension strain along DE = $\frac{EB^1}{DE} = \frac{l}{2L} = \frac{\theta}{2}$ where θ is the shearing strain

From the triangle AA^1N, $AA^1N = 45^0$

$$\cos 45 = \frac{AN}{AA^1} \Rightarrow AN = \frac{l}{\sqrt{2}}$$

$$CN = \sqrt{2}L$$

Compression strain along AC = $\frac{AN}{CN} = \frac{l}{2L} = \frac{\theta}{2}$

Elongation strain + Compression strain = ϴ/2 + ϴ/2 = ϴ

RELATION BETWEEN K - n - σ

$$K = \frac{1}{3(\alpha - 2\beta)} \quad n = \frac{1}{2(\alpha + \beta)}$$

$$K = \frac{1}{3\alpha(1-2\sigma)} \quad n = \frac{1}{2\alpha(1+\sigma)}$$

$$K = \frac{Y}{3(1-2\sigma)} \quad n = \frac{Y}{2(1+\sigma)}$$

$$\sigma = \frac{3K - 2n}{6K + 2n}$$

2.b

$$Torque = -\frac{\pi\eta\theta}{2l}R^4$$

$$\eta = 8.27X10^{10} N/m^2$$

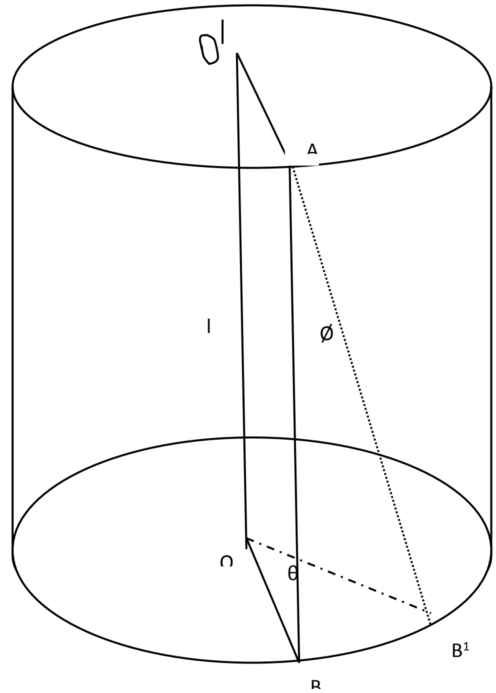
3a.

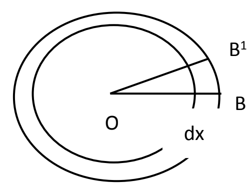
Consider a cylindrical rod of rigidity modulus n, length l, radius r fixed at one end and twisted at the other end through an angle θ by a couple. Imagine the cylinder to be made of large number of coaxial cylinders of increasing radius. Consider a cylinder of radius x and thickness dx. For a given couple, the displacement at its rim is maximum. On twisting, the point B shifts to B^1.

$$BB^1 = l\phi = x\theta$$

$$\phi = \frac{x\theta}{l}$$

$$n = \frac{S}{\phi} \Rightarrow Stress\ S = \frac{nx\theta}{l}$$





This stress is acting on the area 2.πx.dx .

Total force is $F = \frac{nx\theta}{l}2\pi x.dx$

Moment of force along OO^1 $= couple = F.x = \frac{nx^3\theta}{l}2\pi.dx$

Total twisting couple C

$$= \int_0^R \frac{nx^3\theta}{l}2\pi.dx = \frac{2\pi n\theta}{l}\frac{R^4}{4}$$

Couple per unit twist $\frac{C}{\theta} = \frac{\pi n}{l}\frac{R^4}{2}$

3.b. For a square, b = d

$$I = ak^2 = b \times d \times \frac{d^2}{12} = 0.01 \times \frac{0.1^2}{12}$$

$$y = \frac{Wl^3}{3IY} = \frac{mg \times 1^3}{3 \times \left(\frac{0.0001}{12}\right) \times 9.78 \times 10^{10}} = 4 \times 10^{-6} m$$

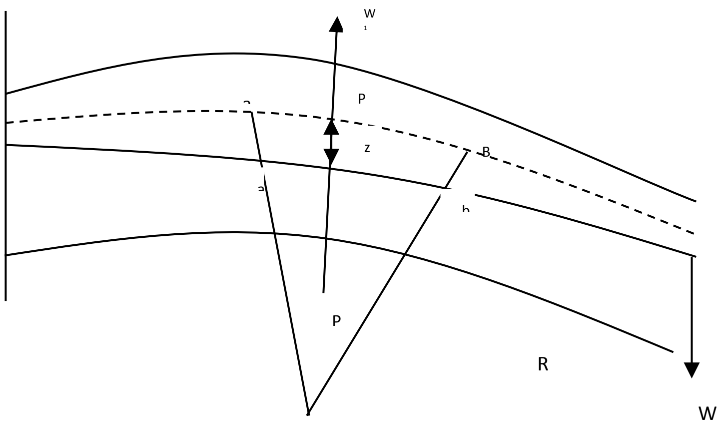
4.a.

Let the beam be bent in the form of circular arc subtending angle θ at the centre of curvature O. Let a^1b^1 be an element at a distance Z from the neutral axis.

$$a^1b^1 = (R+Z)\theta$$

Increase in length of the filament $= a^1b^1 - ab = (R+Z)\theta - R\theta$

$$\text{Strain} = \frac{Z\theta}{R\theta} = \frac{Z}{R}$$



Let LMNT be the rectangular cross section perpendicular to length. EF is the neutral surface. The restoring force on upper half acts inwards and outwards on the lower half.

Consider a small area da at a distance z from the Neutral surface.

$$\text{Strain produced in the filament} = \frac{Z}{R}$$

$$\text{Force on area da} = Y.da.\frac{Z}{R} \qquad \because Y = \frac{\frac{F}{da}}{\frac{Z}{R}}.$$

$$\text{Moment of this force about the neutral surface} = F.Z = Y.da.\frac{Z^2}{R}$$

$$\text{Total moment of forces in LMNT } M = Y.\frac{\sum Z^2.da}{R} = \frac{Y}{R}I$$

Here I is Geometrical Moment of Inertia.

4.b

Let AB be the neutral axis of the cantilever of length L fixed at A and loaded at B. Consider a section P of the beam at a distance x from A.

$$\text{Bending moment} = W.PC = W(L-X) = Y\frac{I}{R} = Ya\frac{k^2}{R}$$

Here R is the radius of curvature of neutral axis at P. The moment of the load increases towards the point A, the radius of curvature is different at different points and decreases towards A. For a point Q at a distance dx from P, it is same as at P.

PQ = dx = R.dΘ

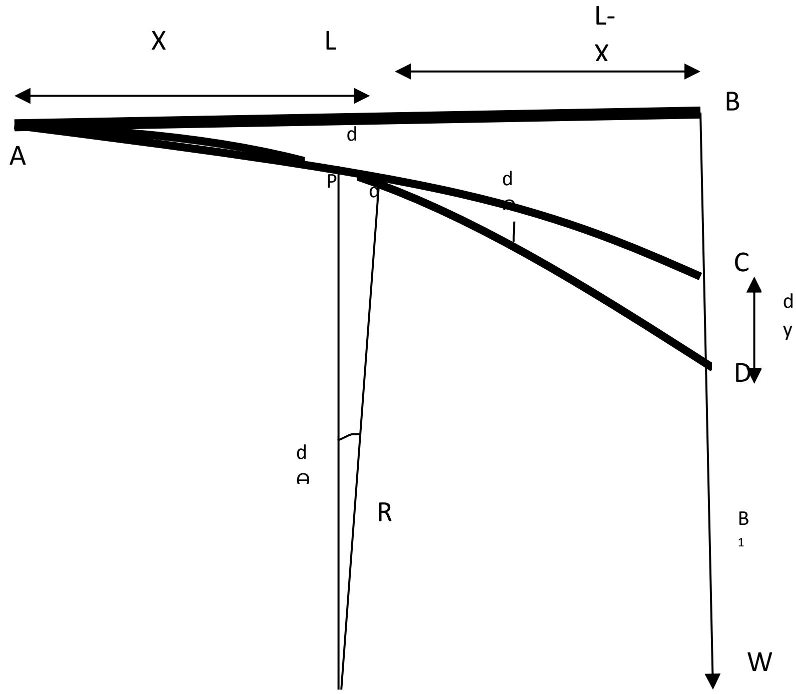
$$\text{Bending moment} \quad W(L-X) = Y\frac{ak^2 d\theta}{dx}$$

Draw tangents to the neutral axis at P and Q meeting the vertical line at C and D. The angle subtended by them is dΘ. The depression of Q below P is given by

$$dy = (L-X)d\theta = W\frac{(L-X)^2}{Yak^2}.dx$$

Total depression BB^1 of the loaded end

$$\int dy = \int_0^L W\frac{(L-X)^2}{Yak^2}.dx = W\frac{L^3}{3YI}$$



5. a

Divergence: It represents the magnitude of a physical quantity emerging or converging at a point. For example tip of a fountain head is a source of divergence. Electric fields are said to be divergent in nature. Mathematically it is obtained by differentiating components of a vector function F (F_x, F_y, F_z) with respect to position coordinates x,y,z respectively.

$$\nabla \bullet \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Ex: Volume charge density enclosed in a closed surface is expressed as

$$\nabla \bullet \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

Right hand side in the above expression is a scalar. Divergence operation on vector yields a scalar function. Divergence of vector is zero if there is no outflow or inflow. Magnetic fields form closed loops and their divergence is zero. $\nabla \bullet B = 0$

Diverging electric field lines at a positive charge is an example for Positive divergence. converging electric field lines at a negative charge is an example of negative divergence.

Curl : This operation is a measure of degree of rotation per unit area. It yields a vector.

$$\nabla X\vec{a} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix}$$

Ex: Magnetic field around a straight conductor carrying current is expressed as

$$\nabla XH = J$$

Curl of a irrotational vector is zero. Static electric fields possesses no curl.

$$\nabla XE = 0$$

5.b.

The following are the four Maxwell's equations for **time varying fields**.

1. $\nabla \bullet \vec{D} = \rho$

2. $\nabla \bullet \vec{B} = 0$

3. $\nabla x \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ **(Modified amperes law)**

4. $\nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ **(Faradays law)**

Displacement current density

From the equation of continuity

$\nabla \bullet \vec{J} = -\frac{\partial \rho}{\partial t}$ where ρ is the charge density

$$\nabla \bullet \vec{J} = -\frac{\partial}{\partial t}\left(\nabla \bullet \vec{D}\right) = \nabla \bullet \frac{\partial \vec{D}}{dt}$$

$$\Rightarrow \nabla \bullet \left(\vec{J} + \frac{\partial \vec{D}}{dt}\right) = 0$$

Thus although $\nabla \bullet \vec{J}$ is not zero but the divergence of $\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$ is always zero .Hence Maxwell made the assumption that the term $\vec{J}$ in amperes law must be replaced by $\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$.

$\therefore$ **For time varying fields** $\nabla x \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

The term $\left(\frac{\partial \vec{D}}{\partial t}\right)$ is known as displacement current density

6.a.

Gauss divergence theorem:

Statement: The volume integral of the divergence of a vector function 'F' over a volume 'V' is equal to the surface integral of the normal component of the vector function 'F' over the surface enclosing the volume V.

Explanation:

Consider a Gaussian surface enclosing a charge Q with a charge density ρ_v.

Then $Q = \iiint_v \rho dv$

From Gauss law, total charge enclosed is

$$\therefore Q = \iint \vec{D} \bullet ds$$

From differential form of Gauss law, $\nabla \bullet \vec{D} = \rho_v$

$$\therefore Q = \iint \int_v \nabla \bullet \vec{D} dv = \int_s \vec{D} \bullet ds$$

$$\boxed{\therefore \int_v \nabla \bullet \vec{D} dv = \int_s \vec{D} \bullet ds}$$

6.b.

$$\nabla \times A = 0$$
$$-j[1-c] = 0$$
$$\Rightarrow C = 1$$

7.a.

Wave equation for electric field:

Consider the equation $\nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$= -\mu \frac{\partial \vec{H}}{\partial t} \qquad \because \vec{B} = \mu \vec{H}$$

Taking curl on both sides

$$\nabla x(\nabla x \vec{E}) = -\mu \frac{\partial}{\partial t}\left(\nabla x \vec{H}\right) \qquad \text{............(1)}$$

Also $\nabla x(\nabla x \vec{E}) = \nabla\left(\nabla \bullet \vec{E}\right) - \nabla^2 \vec{E} = \nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2 \vec{E}$

............(2)

From (1) and (2) ,

$$\nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t}\left(\nabla x \vec{H}\right)$$

$$\because \nabla \bullet \vec{D} = \rho$$

$$\nabla \bullet \varepsilon \vec{E} = \rho$$

$$\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon}$$

$$\nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t}\left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \mu \frac{\partial J}{\partial t} + \nabla\left(\frac{\rho}{\varepsilon}\right)$$

For a free space where there are no charges (ρ=0), no currents (J=0).

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \qquad \text{................(3)}$$

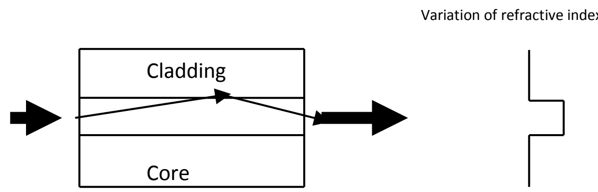
This is the characteristic form of a wave equation .The solution to this equation represents a wave .

7.b.

Types:

1. Single mode fiber:

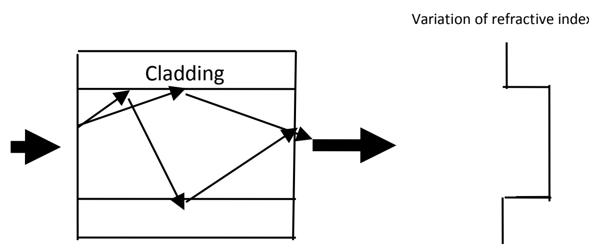
Core diameter is around 5-10 µm. The core is narrow and hence it can guide just a single mode.



- No modal dispersion
- Difference between n_1& n_2 is less.Critical angle is high. Low numerical aperture.
- Low Attenuation -0.35db/km
- Bandwidth -100GHz
- Preferred for short range

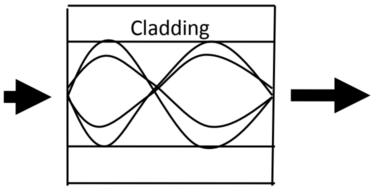
Step index multimode fibre :

- Here the diameter of core is larger so that large number of rays can propagate. Core diameter is around 50. µm.
- High modal dispersion
- Difference between n_1& n_2 is high. Low Critical angle. Large numerical aperture.
- Losses high
- Bandwidth -500MHz
- Allows several modes to propagate
- Preferred for Long range

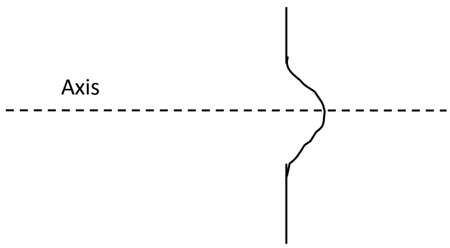


Graded index multimode fiber:

In this type, the refractive index decreases in the radially outward direction from the axis and becomes equal to that of the cladding at the interface. Modes travelling close to the axis move slower where as the modes close to the cladding move faster.As a result the delay between the modes is reduced. This reduces modal dispersion.



Variation of refractive index



- Low modal dispersion
- High data carrying capacity.
- High cost
- Many modes propagate
- Bandwidth -10GHz

8a

Attenuation

Attenuation is the loss of power suffered by the optical signal as it propagates through the fiber.If P_{in} is the incident energy and P_O is the energy at a distance L, then

$$P_O = P_{in} e^{-\alpha L}$$

$$-\alpha z \ln_e e = \ln \frac{P_O}{P_{in}}$$

$$\alpha = \frac{1}{z} \ln\left(\frac{P_{in}}{P_O}\right) \text{bel/km}$$

$$\alpha = \frac{1}{z} \ln\left(\frac{P_{in}}{P_O}\right) \text{ per km}$$

For easier representation of loss percentage, the following expression is used

$$\text{Attenuation constant } \alpha = \frac{10}{z} \log\left(\frac{P_{in}}{P_{out}}\right) \text{dB/Km}$$

Different loss mechanisms:

1. Absorption losses:

In this case, the loss of signal power occurs due to absorption of photons by the impurities and defects present in glass .Impurities such as Ge- 0 , B-O, absorb in the range of 1-2 µm, chromium and copper ions absorb at 0.6µm.,Fe ions absorb at 1.1µm. Hydoxy ions absorb at 1.38µm. Better techniques of making glass with reduced water content can minimize these losses.Tominimise the absorption loss, impurity content has to be less than 1 part in 10^9.

2. Scattering losses:

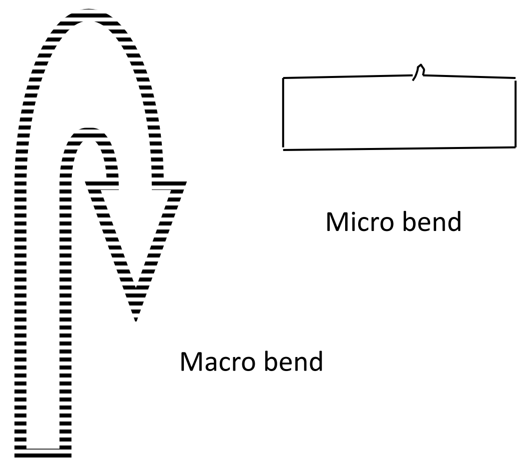
This occurs due to the Rayleigh scattering of the signal caused by variations in refractive index of the glass due to changes in composition, defects, presence of air bubbles, strains etc. The scattered light moves in random direction and escapes from the fiber reducing the intensity. These losses decrease at higher wavelengths.Hence, this loss is minimized by operating at high wavelengths.

$$\text{Scattered Intensity } \alpha \frac{1}{\lambda^4}$$

3. Radiation losses:

Radiative losses occur due to bending of fiber.

Macroscopic Bends:This refers to the bends having radii that are large compared to the fibre diameter.These losses are reduced by using lower wavelength and lower numerical aperture. This loss is high at 1550nm.



Microscopic bends:

These are repetitive small scale fluctuations in the linearity of the fibre axis.

8b

$$V = \frac{2\pi r}{\lambda} NA$$

$$dia = 2r = \frac{V\lambda}{\pi . NA} = 3.98x10^{-5} m$$