

1(a) State and explain Kirchhoff's Laws, as applied to D.C. Circuit.

Ans. : There are two Kirchhoff's laws.

1. Kirchoff's Current Law (KCL)

- The total current flowing towards a junction point is equal to the total current flowing away from that junction point.
- Another way to state the law is,

The algebraic sum of all the current meeting at a junction point is always zero.

$$\sum I \text{ at junction point} = 0$$

Sign convention : Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.

- Consider a junction point in a complex network as shown in the Fig. Q.7.1. The currents I_1 and I_2 are positive as entering the junction while I_3 and I_4 are negative as leaving the junction.

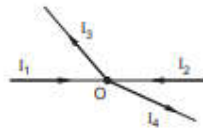


Fig. Q.7.1 Junction point

- Applying KCL, $\sum I \text{ at junction } O = 0$

$$I_1 + I_2 - I_3 - I_4 = 0$$

ie. $I_1 + I_2 = I_3 + I_4$

2. Kirchoff's Voltage Law (KVL)

"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f.s in the path"

In other words, "the algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."

$$\therefore \text{Around a closed path } \sum V = 0$$

- Sum of all the potential rises must be equal to sum of all the potential drops while tracing any closed path of the circuit. The total change in potential along a closed path is always zero.

1(b) Find equivalent resistance of the following circuit shown in fig 1.1

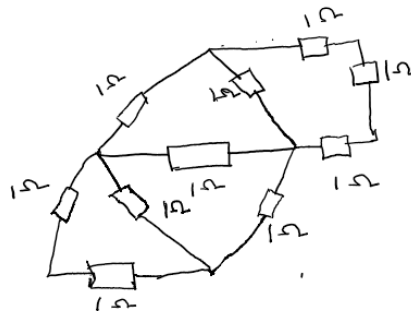
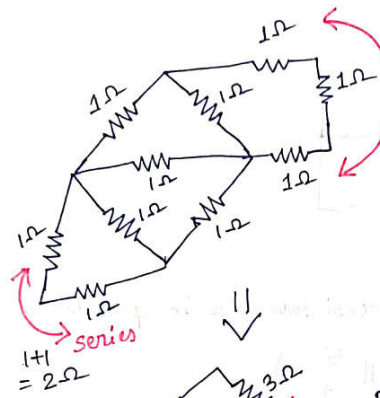
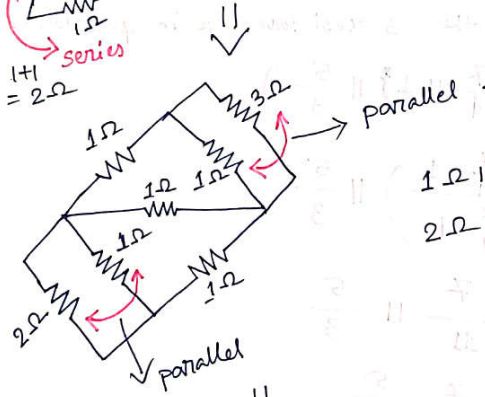


fig 1.1

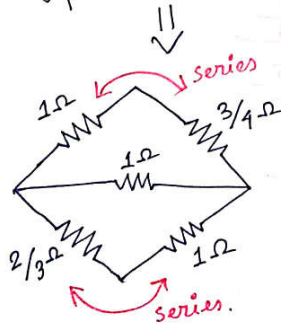


All 3 resistors are connected in series
 So, $R_{eq} = 1 + 1 + 1 = 3 \Omega$.



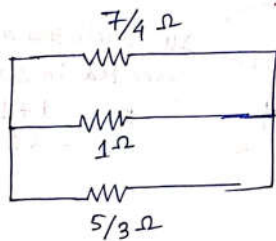
$$1 \Omega \parallel 3 \Omega = \frac{3}{4} \Omega$$

$$2 \Omega \parallel 1 \Omega = \frac{2}{3} \Omega$$



$$1 + \frac{3}{4} = \frac{7}{4} \Omega$$

$$1 + \frac{2}{3} = \frac{5}{3} \Omega$$



Now all the 3 resistors are in parallel.

$$\therefore R_{eq} = \left(\frac{7}{4} \parallel 1 \parallel \frac{5}{3} \right)$$

$$= \left(\frac{\frac{7}{4} \times 1}{\frac{7}{4} + 1} \right) \parallel \frac{5}{3}$$

$$= \frac{7}{11} \parallel \frac{5}{3}$$

$$= \frac{\frac{7}{11} \times \frac{5}{3}}{\frac{7}{11} + \frac{5}{3}}$$

$$= \frac{35}{76}$$

$$R_{eq} = 0.46 \Omega$$

2(a) Using series parallel reduction calculate the current supplied by the source for the circuit shown in fig 2.1

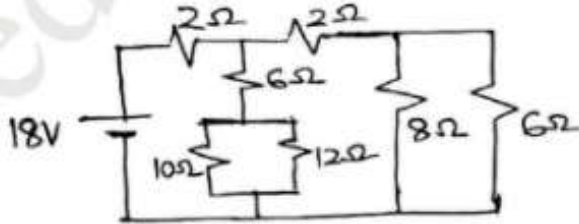


fig.2.1

Here, $10\Omega \parallel 12\Omega$ and $8\Omega \parallel 6\Omega$

$$\begin{aligned} 10 \parallel 12 &= \frac{10 \times 12}{10 + 12} \\ &= \frac{120}{22} \end{aligned}$$

$$\begin{aligned} 8 \parallel 6 &= \frac{8 \times 6}{8 + 6} \\ &= \frac{48}{14} \end{aligned}$$

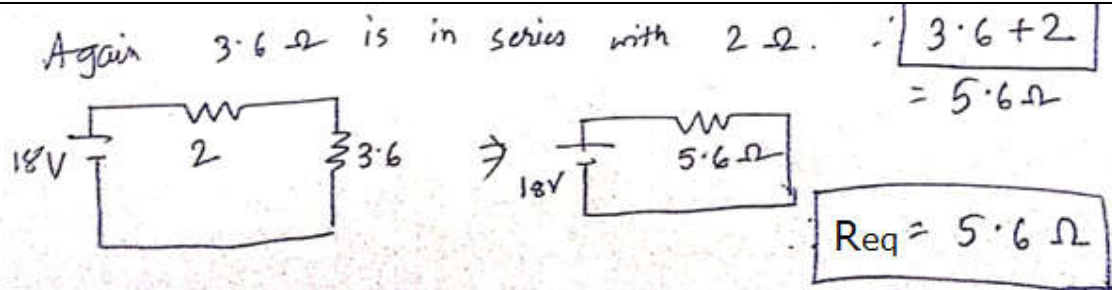
Now, $\frac{48}{14}$ is series with 2Ω .

$$\therefore \frac{48}{14} + 2 = \frac{76}{14} = \frac{38}{7} \Omega$$

Now, 6Ω and $\frac{120}{22} \Omega$ are in series.

$$\therefore 6 + \frac{120}{22} = \frac{252}{22} = \frac{126}{11} \Omega$$

Now, $\frac{126}{11} \parallel \frac{38}{7} = \frac{\frac{126}{11} \times \frac{38}{7}}{\frac{126}{11} + \frac{38}{7}} = 3.6 \Omega$



$$I = \frac{V}{R_{eq}}$$

$$I = \frac{18}{5.6} = 3.21 A$$

2(b) State and explain Ohm's law with an illustration. Also list its limitations

Ans. : • This law gives relationship between the potential difference (V), the current (I) and the resistance (R) of a d.c. circuit. Dr. Ohm in 1827 discovered a law called **Ohm's law**.

• It states that, the current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.

• Mathematically, $I \propto \frac{V}{R}$ where I is the current flowing in amperes, the V is the voltage applied and R is the resistance of the conductor, as shown in the Fig. Q.3.1.

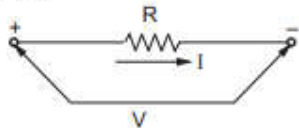


Fig. Q.3.1 Ohm's law

Now $I = \frac{V}{R}$

• The unit of potential difference is defined in such a way that the constant of proportionality is unity.

• Ohm's law is, $I = \frac{V}{R}$ amperes or $V = IR$ volts or $\frac{V}{I} = \text{Constant} = R$ ohms. These are the three forms of Ohm's law.

The limitations of Ohm's law are,

- 1) It is not applicable to the nonlinear devices such as diodes, zener diodes, voltage regulators etc.

- 2) It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by $V = k I^m$ where k, m are constants.
- 3) It is not applicable to electrolytes.
- 4) It is not applicable to discharge lamps and vacuum tubes.
- 5) It is valid only at constant temperature.

3(a) Find the power dissipated in 16Ω resistor in the circuit shown in fig3.1

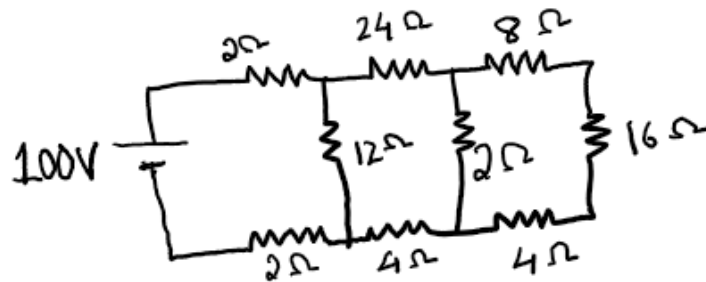
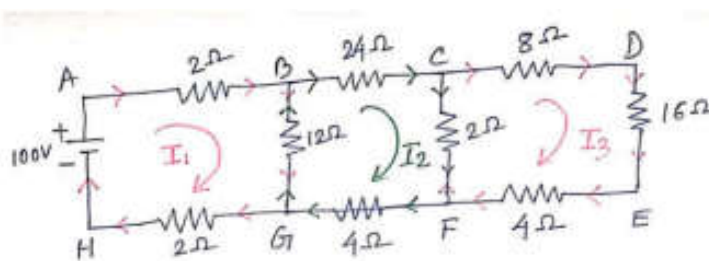


fig 3.1



Applying KVL in ABGHA loop,

$$-2I_1 - 12(I_1 - I_2) - 2I_1 + 100 = 0$$

$$-2I_1 - 12I_1 + 12I_2 - 2I_1 + 100 = 0$$

$$-16I_1 + 12I_2 = -100$$

$$16I_1 - 12I_2 = 100 \quad \text{--- (i)}$$

Applying KVL in BCFGB loop,

$$-24I_2 - 2(I_2 - I_3) - 4I_2 - 12(I_2 - I_1) = 0$$

$$-24I_2 - 2I_2 + 2I_3 - 4I_2 - 12I_2 + 12I_1 = 0$$

$$12I_1 - 42I_2 + 2I_3 = 0 \quad \text{--- (ii)}$$

Applying KVL in CDEFC loop,

$$-8I_3 - 16I_3 - 4I_3 - 2(I_3 - I_2) = 0$$

$$-8I_3 - 16I_3 - 4I_3 - 2I_3 + 2I_2 = 0$$

$$2I_2 - 30I_3 = 0 \quad \text{--- (iii)}$$

By solving equation (i), (ii) and (iii),

$$I_1 = 7.96 \text{ A}$$

$$I_2 = 2.28 \text{ A}$$

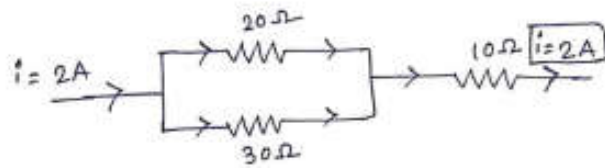
$$I_3 = 0.15 \text{ A}$$

$$\therefore I_{16\Omega} = I_3 = 0.15 \text{ A}$$

$$P_{16\Omega} = I_3^2 R = 0.15^2 \times 16 = 0.36 \text{ W}$$

- 3(b) A circuit consists of two parallel resistors having resistances of $20\ \Omega$ and $30\ \Omega$ respectively connected in series with a $10\ \Omega$ resistor if current through $10\ \Omega$ resistor is 2A . Find i) Current through the branches.
 ii) Voltage across whole circuit
 iii) Power consumed by $20\ \Omega$ and $10\ \Omega$ resistor.

The circuit arrangement can be drawn like:



(i) Total current = 2A
 $\therefore I_{20\Omega} = 2 \times \frac{30}{20+30}$ [By using current division rule]
 $= 1.2\text{A}$
 $I_{30\Omega} = 2 \times \frac{20}{20+30}$
 $= 0.8\text{A}$

(ii) Voltage across the whole circuit:-

$$V = R_{eq} \times i$$

$$R_{eq} = \left(\frac{20 \parallel 30} \right) + 10$$

$$= \frac{20 \times 30}{20 + 30} + 10$$

$$= 22\ \Omega$$

$$\therefore V = 22 \times 2 = 44\text{V}$$

(iii) $P_{20\Omega} = I_{20}^2 \times 20 = 28.8\text{Watt}$
 $P_{30\Omega} = I_{30}^2 \times 30 = 2 \times 30 = 60\text{Watt}$

- 4(a) Find the current in various branches of the given network shown in fig 4.1

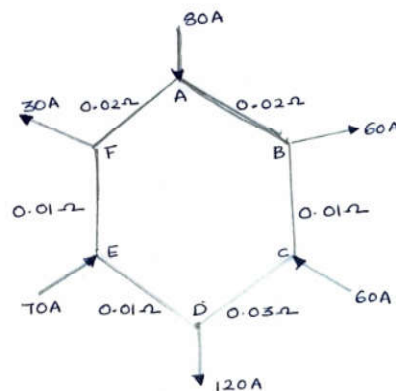
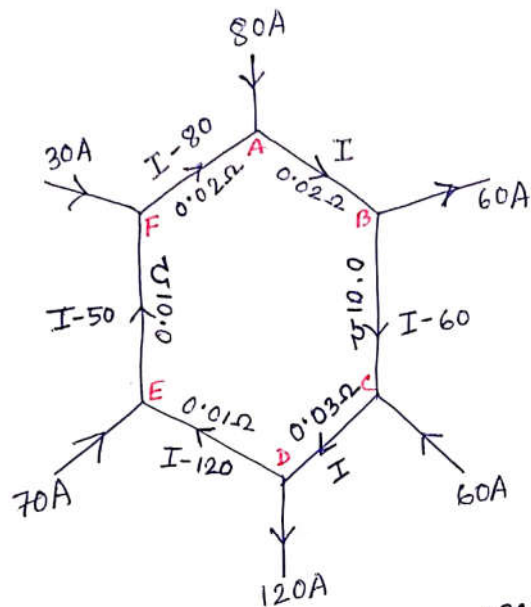


Fig 4.1



ABCDEFA

By applying KVL in the loop, we get,

$$-0.02I - 0.01(I-60) - 0.03I - 0.01(I-120) - 0.01(I-50) - 0.02(I-80) = 0$$

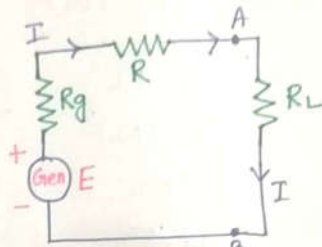
$$-0.02I - 0.01I + 0.6 - 0.03I - 0.01I + 1.2 - 0.01I + 0.5 - 0.02I + 1.6 = 0$$

$$-0.1I = -3.9$$

$$\therefore I = 39 \text{ A}$$

4 (b) Derive maximum power transfer theorem applied to the series circuit. Mention its applications.

⇒ "A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals, with all energy sources removed leaving behind their internal resistances."



In the above fig, R_L is connected across the terminals A and B. The circuit consists of a generator of emf E with internal resistance of R_g and a series resistance R .

Circuit current, $I = \frac{E}{R_L + R_i}$ where $R_i = R_g + R$.

power consumed by the load (R_L) is:-

$$P_L = I^2 R_L = \frac{E^2 R_L}{(R_L + R_i)^2} \quad \text{--- (i)}$$

For P_L to be maximum, $\frac{dP_L}{dR_L} = 0$

Differentiating Eq. (i), we have.

$$\begin{aligned} \frac{dP_L}{dR_L} &= E^2 \left[\frac{1}{(R_L + R_i)^2} + R_L \left(\frac{-2}{(R_L + R_i)^3} \right) \right] \\ &= E^2 \left[\frac{1}{(R_L + R_i)^2} - \frac{2R_L}{(R_L + R_i)^3} \right] \end{aligned}$$

As $\frac{dP_L}{dR_L} = 0$
 $\therefore 0 = E^2 \left[\frac{1}{(R_L + R_i)^2} - \frac{2R_L}{(R_L + R_i)^3} \right]$

or, $2R_L = R_L + R_i$

$\therefore R_L = R_i$ where $R_i = R_g + R$.

Maximum power (P_{max}) = $\frac{E^2 R_L}{(R_L + R_i)^2}$

$P_{max} = \frac{E^2 R_L}{(R_L + R_L)^2} \quad [\because R_L = R_i]$

$$\begin{aligned} P_{max} &= \frac{E^2 R_L}{4R_L^2} \\ &= \frac{E^2}{4R_L} \end{aligned}$$

$\therefore P_{max} = \frac{E^2}{4R_L} = \frac{E^2}{4R_i} \quad [\because R_L = R_i]$

Application:- In some applications, the purpose of a circuit is to provide maximum power to a load. Some examples are:-

- i) Stereo amplifier.
- ii) Radio transmitter.
- iii) Communication equipment.
- iv) Car engine.

5 Derive the equation for root-mean-square value and average value of an alternating quantity in terms of maximum value. Also define Form factor and Peak Factor

RMS Value:

Ans. : The effective or r.m.s. value of an alternating current is given by that steady current (D.C.) which, when flowing through a given circuit for a given time, produces the same amount of heat as produced by the alternating current, which when flowing through the same circuit for the same time.

Consider sinusoidally varying alternating current and square of this current as shown in the Fig. Q.5.1.

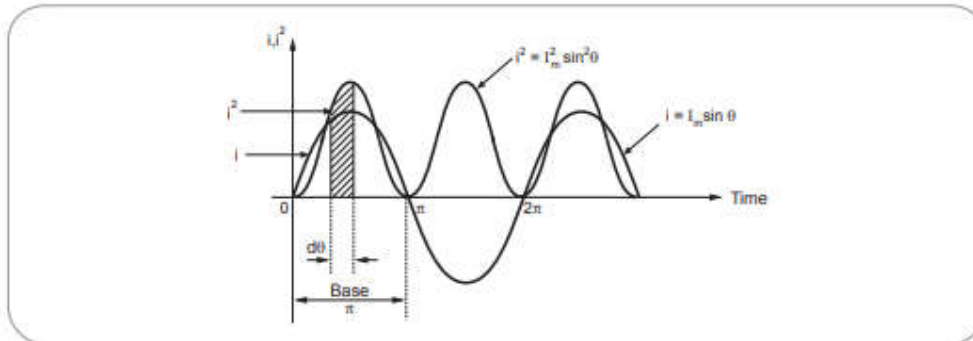


Fig. Q.5.1 Waveform of current and square of the current

Step 1 : The current, $i = I_m \sin \theta$

Step 2 : Square of current, $i^2 = I_m^2 \sin^2 \theta$

- The area of curve over half a cycle can be calculated by considering an interval $d\theta$ as shown.

Area of square curve over half cycle = $\int_0^{\pi} i^2 d\theta$ and length of the base is π .

Step 3 :

∴ Average value of square of the current over half cycle is,

$$\begin{aligned}
 &= \frac{\text{Area of curve over half cycle}}{\text{Length of base over half cycle}} = \frac{\int_0^{\pi} i^2 d\theta}{\pi} \\
 &= \frac{1}{\pi} \int_0^{\pi} i^2 d\theta = \frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2 \theta d\theta \\
 &= \frac{I_m^2}{\pi} \int_0^{\pi} \left[\frac{1 - \cos 2\theta}{2} \right] d\theta = \frac{I_m^2}{2\pi} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi} \\
 &= \frac{I_m^2}{2\pi} [\pi] = \frac{I_m^2}{2}
 \end{aligned}$$

Step 4 : Root mean square value i.e. r.m.s. value can be calculated as,

$$\begin{aligned}
 I_{r.m.s.} &= \sqrt{\text{Mean or average of square of current}} \\
 &= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}
 \end{aligned}$$

$$I_{r.m.s.} = \frac{I_m}{\sqrt{2}} = 0.707 I_m \quad \text{and} \quad V_{r.m.s.} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

- The r.m.s. value of the sinusoidal alternating current is 0.707 times the maximum or peak value or amplitude of that alternating current.

Average Value:

- Consider sinusoidally varying current, $i = I_m \sin \theta$
- Consider the elementary interval of instant 'dθ' as shown in the Fig. Q.6.1.
- The average instantaneous value of current in this interval is say, 'I' as shown.
- The average value can be obtained by taking ratio of area under curve over **half cycle** to length of the base for half cycle.

$$\therefore I_{av} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle}}$$

$$\begin{aligned} I_{av} &= \frac{\int_0^\pi i \, d\theta}{\pi} = \frac{1}{\pi} \int_0^\pi i \, d\theta = \frac{1}{\pi} \int_0^\pi I_m \sin \theta \, d\theta \\ &= \frac{I_m}{\pi} \int_0^\pi \sin \theta = \frac{I_m}{\pi} [-\cos \theta]_0^\pi \\ &= \frac{I_m}{\pi} [-\cos \pi + \cos 0] = \frac{I_m}{\pi} [2] \\ &= \frac{2I_m}{\pi} = 0.637 I_m \end{aligned}$$

- For a purely sinusoidal waveform, the average value is expressed in terms of its maximum value as,

$$\therefore I_{av} = 0.637 I_m \quad \text{and} \quad V_{av} = 0.637 V_m$$

Form Factor:

Ans. : • The form factor of an alternating quantity is defined as the ratio of r.m.s. value to the average value,

Form factor,

$$K_f = \frac{\text{r.m.s. value}}{\text{Average value}}$$

- The form factor for sinusoidal alternating currents or voltages can be obtained as,

$$\begin{aligned} K_f &= \frac{0.707 I_m}{0.637 I_m} \\ &= 1.11 \quad \text{for sinusoidally varying quantity} \end{aligned}$$

Peak Factor:

Ans. : • The peak factor of an alternating quantity is defined as ratio of maximum value to the r.m.s. value.

Peak factor

$$K_p = \frac{\text{maximum value}}{\text{r.m.s. value}}$$

- The peak factor for sinusoidally varying alternating currents and voltages can be obtained as,

$$K_p = \frac{I_m}{0.707 I_m} = 1.414 \quad \text{for sinusoidal waveform}$$

6 (a) Explain the generation of 1-φ AC induced emf with suitable diagram.

Ans. : The Fig. 1 shows single phase alternator used to explain the generation of single phase a.c. e.m.f.

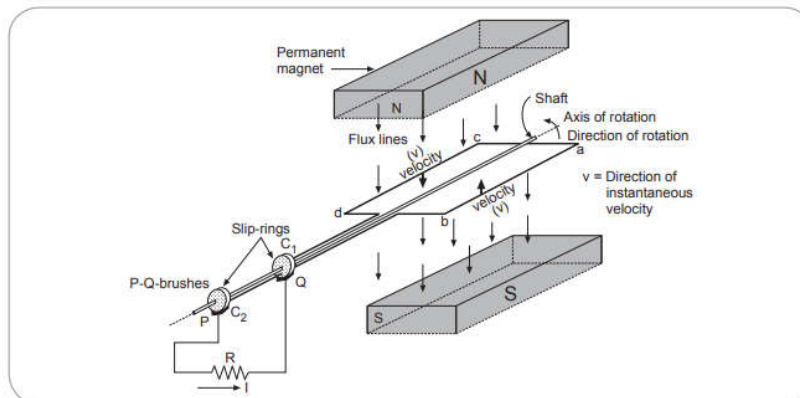


Fig. Q.1.1 Single turn alternator

- The coil is made up of two conductors ab and cd , which are connected at the end. The coil can be rotated about its axis.
- The brushes P and Q are used which are resting against slip rings C_1 and C_2 . The induced e.m.f. is made available to the external circuit using the brushes.

- The coil is rotated in the magnetic field produced by the permanent magnet.
- When coil is rotated, it cuts the magnetic flux. Due to Faraday's law of electromagnetic induction, the e.m.f. gets induced in the coil, proportional to the rate of change of flux associated with the coil.
- Let the initial position of coil is such that the velocity component is parallel to the flux and there is no cutting of flux. Thus no e.m.f. is induced in the coil as shown in the Fig. Q.1.2 (a). In this position, $\theta = 0^\circ$ where θ is the measured with respect to axis of magnetic field i.e. vertical and $e = 0$ V.

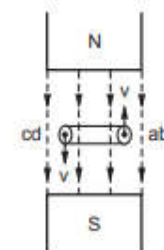


Fig. Q.1.2 (a) $\theta = 0^\circ$

- As coil rotates, θ changes as shown in the Fig. Q.1.2(b). Then $v \sin \theta$ component of velocity cuts the flux and responsible for the induced e.m.f.
- At $\theta = 90^\circ$, entire velocity is at right angles to flux and maximum e.m.f. gets induced in the coil.
- As coil rotates further, again e.m.f. decreases and becomes zero at $\theta = 180^\circ$.
- Between $\theta = 180^\circ$ to 360° , the conductors reverse their positions and e.m.f. behaves in similar fashion as before but with opposite direction.

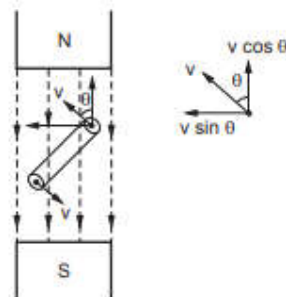


Fig. Q.1.2 (b) $0^\circ < \theta < 90^\circ$

- The waveform of e.m.f. induced for $\theta = 0^\circ$ to 360° i.e. one rotation of coil is shown in the Fig. Q.1.2(c).

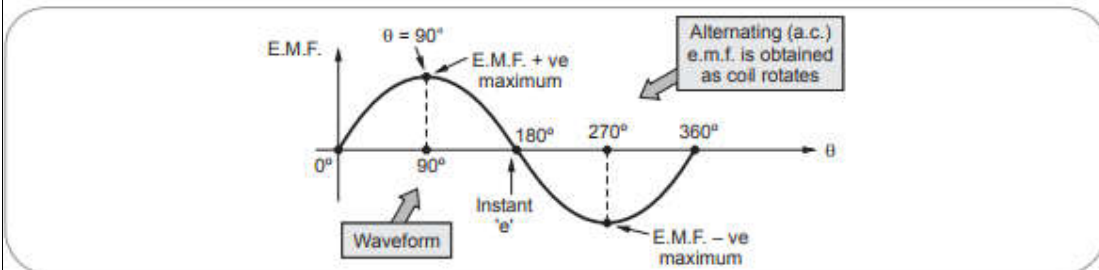


Fig. Q.1.2 (c)

- Thus a.c. e.m.f. gets generated.

6 (b) For the figure shown below (fig 6.1) calculate current through 2Ω resistor.

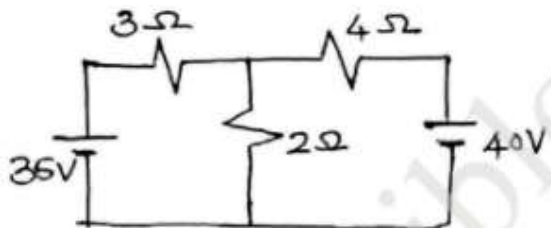
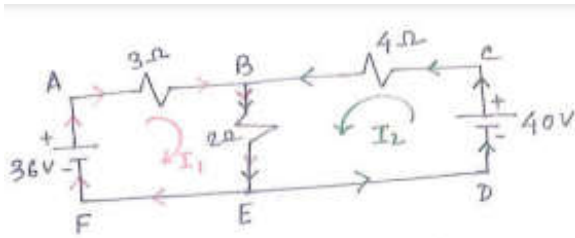


fig 6.1



Applying KVL in ABEFA loop,

$$\begin{aligned}
 -3I_1 - 2(I_1 + I_2) + 36 &= 0 \\
 -3I_1 - 2I_1 - 2I_2 &= -36 \\
 5I_1 + 2I_2 &= 36 \quad \text{--- (i)}
 \end{aligned}$$

Applying KVL in BCDEB loop,

$$\begin{aligned}
 -4I_2 - 2(I_1 + I_2) + 40 &= 0 \\
 -4I_2 - 2I_1 - 2I_2 &= -40 \\
 2I_1 + 6I_2 &= 40 \quad \text{--- (ii)}
 \end{aligned}$$

By solving equation (i) and (ii),

$$I_1 = 5.23 \text{ A}$$

$$I_2 = 4.92 \text{ A}$$

$$\begin{aligned}
 \therefore I_{2\Omega} &= (I_1 + I_2) \\
 &= (5.23 + 4.92) \text{ A}
 \end{aligned}$$

$$I_{2\Omega} = 10.15 \text{ A}$$

7. (a) Explain the terms (i) Time period (ii) Cycle (iii) Instantaneous value (iv) Frequency

Time period (T): The time taken by an alternating quantity to complete its one cycle is known as its **time period** denoted by **T seconds**.

- After every T seconds, the cycle of an alternating quantity repeats.

Cycle: Each repetition of a set of positive and negative instantaneous values of the alternating quantity is called a **cycle**.

Instantaneous value: The value of an alternating quantity at a particular instant is known as its instantaneous value. e.g. e_1 and $-e_2$ are the instantaneous values of an alternating e.m.f. at the instants t_1 and t_2 respectively shown in the Fig. Q.2.1.

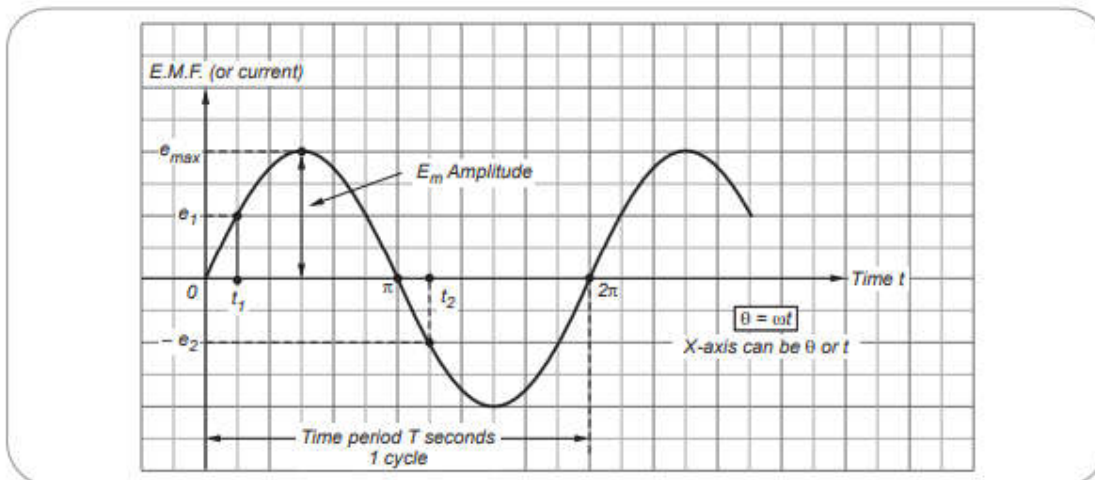


Fig. Q.2.1 Graphical representation of an alternating quantity

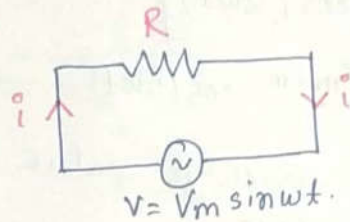
Frequency (f): The number of cycles completed by an alternating quantity per second is known as its **frequency**. It is denoted by **f** and it is measured in **cycles/second** which is known as **Hertz**, denoted as **Hz**.

- Frequency is reciprocal of the time period.

$$f = \frac{1}{T} \text{ Hz}$$

7.(b) Show that voltage and current in pure resistive circuit are in phase and power consumed in circuit is equal to product of rms voltage and current. The circuit is excited by AC source.

\Rightarrow Let's consider a simple circuit consisting of a pure resistance 'R' ohms connected across a voltage $V = V_m \sin \omega t$



According to Ohm's Law,

$$i = \frac{V}{R} = \frac{V_m \sin \omega t}{R}$$

$$\therefore i = \left(\frac{V_m}{R}\right) \sin \omega t \quad \text{--- (i)}$$

The standard equation of current is:-

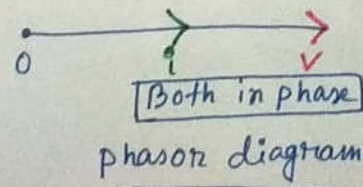
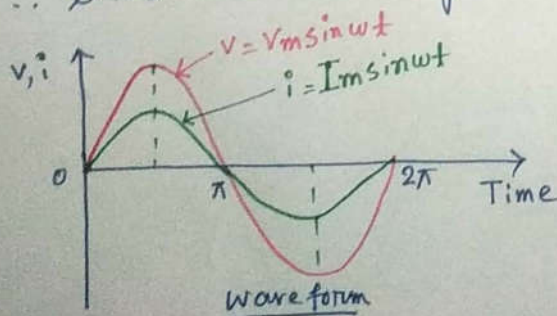
$$i = I_m \sin(\omega t + \phi) \quad \text{--- (ii)} \quad [\phi \text{ is phase}]$$

By comparing equation (i) and (ii), we get,

$$I_m = \frac{V_m}{R}$$

$$\phi = 0^\circ$$

$\therefore \phi = 0^\circ$ means voltage and current are in phase.



power calculation

$$P = VI$$

$$= V_m \sin(\omega t) \times I_m \sin(\omega t)$$

$$= V_m I_m \sin^2(\omega t)$$

$$= \frac{V_m I_m}{2} (2 \sin^2 \omega t)$$

$$= \frac{V_m I_m}{2} [1 - \cos(2\omega t)]$$

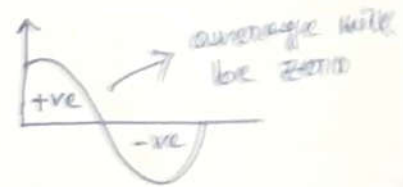
$$\therefore P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos(2\omega t) \quad \text{--- (iii)}$$

In equation (iii), the average value of the cosine part will be zero.

$$\therefore P_{avg} = \frac{V_m I_m}{2}$$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$\therefore P_{avg} = V_{rms} I_{rms}$$



- 8(a) For the current wave $i = 140 \sin 314t$. Find (i) Peak current (ii) Average value (iii) Frequency (iv) Time period (v) RMS value (vi) instantaneous value at 3s (vii) Form factor (viii) Peak factor.

$$i = 140 \sin 314t$$

By comparing the above equation with standard form $I = I_m \sin \omega t$, we get,

(i) Peak current $I = I_m = 140 \text{ A}$

(ii) RMS value = $\frac{I_m}{\sqrt{2}}$
 $= \frac{140}{\sqrt{2}}$
 $= 98.99 \text{ A}$

(iii) Average value = $0.637 \times I_m = 0.637 \times 140 = 89.18 \text{ A}$.

(iv) Frequency, $\omega = 314$
 $\therefore 2\pi f = 314$
 $\therefore f = \frac{314}{2\pi} = 50 \text{ Hz}$

(v) Time period (T) = $\frac{1}{f} = \frac{1}{50} = 0.02 \text{ sec.}$

(vi) Instantaneous value at $t = 3 \text{ msec.}$

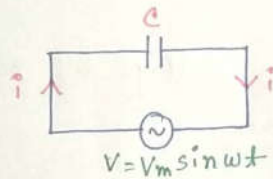
$i = 140 \sin(314 \times 3 \times 10^{-3}) = 113.22 \text{ A}$
 ~~$140 \sin(314 \times 3)$~~ (calculation will be done in radian mode)

(vii) Form factor = $\frac{\text{RMS value}}{\text{average value}} = \frac{98.99 \text{ A}}{89.18 \text{ A}} = 1.11$

(viii) Peak factor = $\frac{\text{max}^m \text{ value}}{\text{RMS value}} = \frac{140}{98.99} = 1.41$

8(b) Show that Power consumed by pure capacitor is zero. Draw the voltage current and power waveform.

\Rightarrow Consider a pure capacitor of C Farads is connected across a voltage $v = V_m \sin \omega t$.



The instantaneous charge, $q = CV$
 $\therefore q = CV_m \sin \omega t$.

Also, current is rate of flow of charge,

$$\begin{aligned} \therefore i &= \frac{dq}{dt} \\ &= \frac{d}{dt} (CV_m \sin \omega t) \\ &= CV_m \frac{d}{dt} (\sin \omega t) \\ &= \omega CV_m \cos(\omega t) \end{aligned}$$

$$i = \omega CV_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\text{or, } i = \frac{V_m}{X_c} \sin\left(\omega t + \frac{\pi}{2}\right)$$

—(i)

where, $X_c = \frac{1}{\omega C}$
 X_c is called capacitive reactance.

The standard equation of current is,

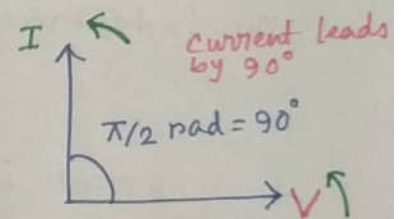
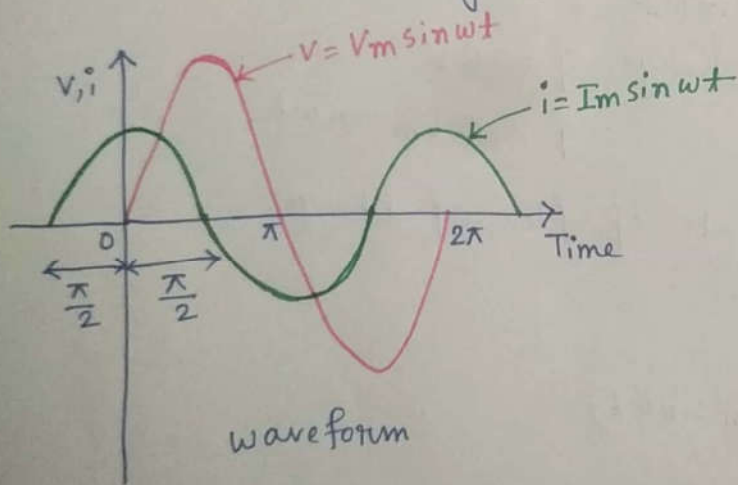
$$i = I_m \sin(\omega t + \phi) \text{ --- (ii)}$$

By comparing equation (i) and (ii),

$$I_m = \frac{V_m}{X_c}$$

$$\phi = \frac{\pi}{2} \text{ or } 90^\circ$$

↑
This means current leads voltage by 90° . +ve sign indicates leading phase.



phasor diagram

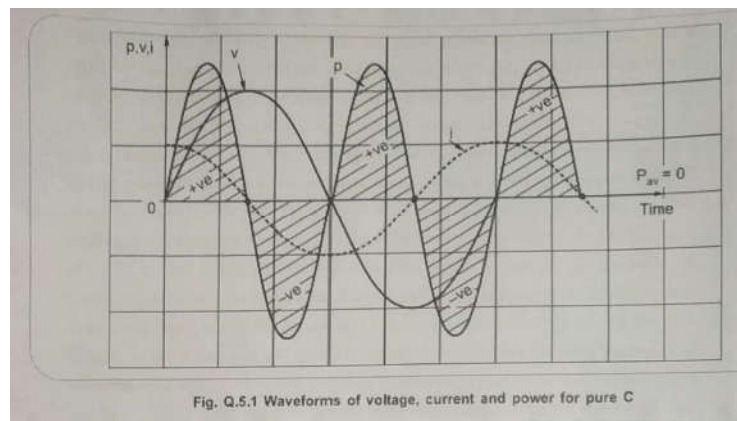


Fig. Q.5.1 Waveforms of voltage, current and power for pure C