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Internal Assessment Test I – January 2022

IAT-1 Scheme

Sub:	Calculus and Differential Equations					Sub Code:	21MAT11	
Date:	24/01/2022	Duration:	90 mins	Max Marks:	50	Sem / Sec:	I / A to G (PHY CYCLE)	OBE

Answer all the Questions — MARKS | CO | RBT

1. Diagonalize the matrix $A = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$ **(Finding eigenvalues-2m, Finding eigenvectors- 4m, Finding the diagonal matrix-2m).** [08] CO6 L3

2. Find the rank of the matrix: $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ **(Reducing to the echelon form using valid row operations- 6m, Final answer- 1m).** [07] CO6 L3

3. Investigate the values of λ and μ such that the following system may have (i) unique solution, (ii) infinitely many solutions and (iii) no solution. [07] CO6 L3

$$x + 2y + 3z = 6$$
$$x + 3y + 5z = 9$$
$$2x + 5y + \lambda z = \mu$$

(Echelon form-1m, correct values for each case: 2+2+2 = 6m).

4. Solve using Gauss Jordan Method: [07] CO6 L3

$$x + 2y + z = 3$$
$$2x + 3y + 3z = 10$$
$$3x - y + 2z = 13$$

(Reducing to diagonal form- 4m, correct values of each variable- 3m).

5. Find the dominant eigenvalue and the corresponding eigenvector of the matrix [07] CO6 L3

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

by power method taking the initial vector as $[1,0,0]^T$ **(perform only 6 iterations)**.

(For each iteration- 1m, totally 6 marks, final answer-1m).

6. Solve using Gauss Seidel method **(perform only 3 iterations)**: [07] CO6 L3

$$10x + 2y + z = 9$$
$$x + 10y - z = -22$$
$$-2x + 3y + 10z = 22$$

(Checking for diagonally dominance- 1m, 3m for iterations, final answer- 3m).

7. Solve using Gauss Elimination Method: [07] CO6 L3

$$x + y + z = 9$$
$$x - 2y + 3z = 8$$
$$2x + y - z = 3$$

(Reducing to upper triangular form- 4m, finding the correct values of the variables by back substitution- 3m).

Internal Assessment Test - 1 (Solutions)

January - 2022

Calculus and Differential Equations - 21MAT11

Q1. $A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$

The characteristic equation $|A - \lambda I| = 0$

$$\begin{vmatrix} -1-\lambda & 2 \\ 2 & -1-\lambda \end{vmatrix} = 0 \Rightarrow (-1-\lambda)^2 - 4 = 0$$

$$\Rightarrow 1 + 2\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 + 2\lambda - 3 = 0 \Rightarrow (\lambda+3)(\lambda-1) = 0 \Rightarrow \lambda = 1, -3$$

The eigen vectors are given by $[A - \lambda I] X = 0$

$$\begin{bmatrix} -1-\lambda & 2 \\ 2 & -1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{——— } 2$$

(i) Let $X_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ for $\lambda = 1$

$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow -x_1 + y_1 = 0 \Rightarrow x_1 = y_1$$

$$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(ii) Let $X_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$ for $\lambda = -3$

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_2 + y_2 = 0 \Rightarrow x_2 = -y_2$$

$$X_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Let Modal Matrix $P = [X_1 \ X_2] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$P^{-1} = \frac{adj(P)}{|P|}, \quad adj(P) = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}, \quad |P| = -1-1 = -2$$

$$D = P^{-1}AP = \frac{-1}{2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 1 & 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & 0 \\ 0 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}$$

Q2. $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$, $R_1 \leftrightarrow R_2$, $R_3 \to R_3 - 3R_2$, $R_4 \to R_4 - R_2$

$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$, $R_3 \to R_3 - R_2$, $R_4 \to R_4 - R_2$

$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\rho(A) = 2$

Q3. $C = [A : B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 1 & 3 & 5 & 9 \\ 2 & 5 & \lambda & \mu \end{array}\right]$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_1$

$$C \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & \lambda-6 & \mu-12 \end{array}\right], \quad R_3 \to R_3 - R_2$$

$$C \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & \lambda-8 & \mu-15 \end{array}\right]$$

i) for unique solution : $\rho(A) = \rho(C) = 3$

$\Rightarrow \lambda - 8 \neq 0 \Rightarrow \lambda \neq 8, \ \forall \mu$

ii) for infinite solution, $\rho(A) = \rho(C) < 3$

$\lambda - 8 = 0, \ \mu - 15 = 0 \Rightarrow \mu = 15, \lambda = 8$

iii) for no solution, $\rho(A) \neq \rho(C)$

$\lambda - 8 = 0, \ \mu - 15 \neq 0$

$\Rightarrow \lambda = 8, \ \mu \neq 15$

Q4 $C = [A : B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{array}\right]$

$R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - 3R_1$

$$C \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & -1 & 4 \end{array}\right] \qquad R_3 \to R_3 - 7R_2$$

$$C \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right] \qquad \begin{array}{l} R_1 \to R_1 + 2R_2 \\ R_2 \to 8R_2 + R_3 \\ R_3 \to -R_3/8 \end{array}$$

$$C \sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 11 \\ 0 & -8 & 0 & 8 \\ 0 & 0 & 1 & 3 \end{array}\right], \qquad \begin{array}{l} R_1 \to R_1 - 3R_3 \\ R_2 \to R_2/8 \end{array}$$

$$C \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right] \qquad \left.\begin{array}{l} x = 2 \\ y = -1 \\ z = 3 \end{array}\right\} \text{Ans}$$

Q5. $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}, \quad X^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$AX^{(0)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \lambda^{(1)} X^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 2 \end{bmatrix} = 2.5 \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \lambda^{(2)} X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 0 \\ 2.6 \end{bmatrix} = 2.8 \begin{bmatrix} 1 \\ 0 \\ 0.928 \end{bmatrix} = \lambda^{(3)} X^{(3)}$$

$$AX^{(3)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.928 \end{bmatrix} = \begin{bmatrix} 2.928 \\ 0 \\ 2.856 \end{bmatrix} = 2.928 \begin{bmatrix} 1 \\ 0 \\ 0.975 \end{bmatrix} = \lambda^{(4)} X^{(4)}$$

$$AX^{(4)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.975 \end{bmatrix} = \begin{bmatrix} 2.975 \\ 0 \\ 2.950 \end{bmatrix} = 2.975 \begin{bmatrix} 1 \\ 0 \\ 0.991 \end{bmatrix} = \lambda^{(5)} X^{(5)}$$

$$AX^{(5)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.991 \end{bmatrix} = \begin{bmatrix} 2.991 \\ 0 \\ 2.982 \end{bmatrix} = 2.991 \begin{bmatrix} 1 \\ 0 \\ 0.997 \end{bmatrix}$$

Thus the dominant eigen value is 2.991 and eigen vector is $(1, 0, 0.997)^T$

Q6. $10x+2y+z=9$; $x+10y-z=-22$, $-2x+3y+10z=22$

The given system of equations is diagonally dominant.

$$x=\frac{1}{10}[9-2y-z] \quad \text{—— (1)}$$

$$y=\frac{1}{10}[-22-x+z] \quad \text{—— (2)}$$

$$z=\frac{1}{10}[22+2x-3y] \quad \text{—— (3)}$$

Let $X^{(0)}=(0,0,0)^T$

I Iteration

$$x^{(1)}=\frac{1}{10}(9-0-0)=0.9$$

$$y^{(1)}=\frac{1}{10}[-22-0.9+0]=-2.29$$

$$z^{(1)}=\frac{1}{10}[22+1.8+6.87]=3.067$$

II Iteration

$$x^{(2)}=\frac{1}{10}[9-2(-2.29)-(3.067)]=1.0513$$

$$y^{(2)}=\frac{1}{10}[-22-(1.0513)+(3.067)]=-1.9984$$

$$z^{(2)}=\frac{1}{10}[22+2(1.0513)-3(-1.9984)]$$

$$=3.0098$$

III Iteration

$$x^{(3)} = \frac{1}{10}\left[9 - 2(-1.9984) - (3.0093)\right]$$
$$= 0.9987$$

$$y^{(3)} = \frac{1}{10}\left[-22 - 1.0513 + 3.0098\right] \cancel{= 0.9987}$$
$$\cancel{= 1.99} \qquad = -2.0041$$

$$z^{(3)} = \frac{1}{10}\left[22 + 2(0.9987) - 3(-2.0041)\right]$$
$$= 3.0009$$

Thus the approx solution is $(0.9987, -2.0041, 3.0009)$ Ans

Q7. $C = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - 2R_1$

$$C \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$$

$R_3 \to 3R_3 - R_2$

$$C \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & -11 & -44 \end{array}\right]$$

By Gauss Elimination

$$x + y + z = 9$$

$$-3y + 2z = -1 \quad \Rightarrow$$

$$-11z = -44 \quad \Rightarrow z = 4,\ y = 3,\ x = 2$$

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x
