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Internal Assessment Test I – January 2022

Sub:	Calculus and Differential Equations					Sub Code:	21MAT11		
Date:	**24/01/2022**	Duration:	**90 mins**	Max.marks	**50**	Sem / Sec:	**I to O (CHEM CYCLE)**	OBE	
	Question 1 is compulsory and answer any SIX questions from the rest.						MARKS	CO	RBT
1 .	Diagonalize: $\boldsymbol{A} = \begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$.						[08]	CO6	L3
2 .	Find the rank of the matrix: $\boldsymbol{A} = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$.						[07]	CO6	L3
3 .	Investigate the values of $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ such that the following system may have (i) unique solution, (ii) infinitely many solutions and (iii) no solution. $$2x+3y+5z=9$$ $$7x+3y-2z=8$$ $$2x+3y+\lambda z=\mu$$						[07]	CO6	L3

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4. Solve using Gauss Jordan Method: [07] CO6 L3

$$\begin{aligned} \boldsymbol{x+y+z} &= \mathbf{9} \\ \mathbf{2}\boldsymbol{x}-\mathbf{3}\boldsymbol{y}+\mathbf{4}\boldsymbol{z} &= \mathbf{13} \\ \mathbf{3}\boldsymbol{x}+\mathbf{4}\boldsymbol{y}+\mathbf{5}\boldsymbol{z} &= \mathbf{40}. \end{aligned}$$

5. Find the dominant eigenvalue and the corresponding eigenvector of the matrix $\boldsymbol{A} = \begin{pmatrix} -\mathbf{2} & \mathbf{0} & -\mathbf{1} \\ \mathbf{1} & -\mathbf{1} & \mathbf{1} \\ \mathbf{2} & \mathbf{2} & \mathbf{0} \end{pmatrix}$ by power method taking the initial vector as $[1,1,1]^T$ **(perform only 4 iterations)**. [07] CO6 L3

6. Solve using Gauss Seidel method **(perform only 3 iterations)**: [07] CO6 L3

$$\begin{aligned} \mathbf{10}\boldsymbol{x}+\mathbf{2}\boldsymbol{y}+\boldsymbol{z} &= \mathbf{9} \\ \boldsymbol{x}+\mathbf{10}\boldsymbol{y}-\boldsymbol{z} &= -\mathbf{22} \\ -\mathbf{2}\boldsymbol{x}+\mathbf{3}\boldsymbol{y}+\mathbf{10}\boldsymbol{z} &= \mathbf{22}. \end{aligned}$$

7. Solve using Gauss Elimination Method: [07] CO6 L3

$$\begin{aligned} \boldsymbol{x}-\mathbf{2}\boldsymbol{y}+\mathbf{3}\boldsymbol{z} &= \mathbf{2} \\ \mathbf{3}\boldsymbol{x}-\boldsymbol{y}+\mathbf{4}\boldsymbol{z} &= \mathbf{4} \\ \mathbf{2}\boldsymbol{x}+\boldsymbol{y}-\mathbf{2}\boldsymbol{z} &= \mathbf{5}. \end{aligned}$$

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$$\begin{aligned} \boldsymbol{x}-\mathbf{2}\boldsymbol{y}+\mathbf{3}\boldsymbol{z} &= \mathbf{2} \\ \mathbf{3}\boldsymbol{x}-\boldsymbol{y}+\mathbf{4}\boldsymbol{z} &= \mathbf{4} \\ \mathbf{2}\boldsymbol{x}+\boldsymbol{y}-\mathbf{2}\boldsymbol{z} &= \mathbf{5}. \end{aligned}$$

Internal Assessment Test - 1 (Solutions)

Q1. Given $A = \begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$

Characteristic equation is given by $|A - \lambda I| = 0$

$$\begin{vmatrix} -1-\lambda & 3 \\ -2 & 4-\lambda \end{vmatrix} = 0$$

$\Rightarrow (-1-\lambda)(4-\lambda) + 6 = 0$

$\Rightarrow -4 + \lambda - 4\lambda + \lambda^2 + 6 = 0$

$\Rightarrow \lambda^2 - 3\lambda + 2 = 0$

$\Rightarrow \lambda^2 - 2\lambda - \lambda + 2 = 0$

$\Rightarrow \lambda(\lambda - 2) - 1(\lambda - 2) = 0$

$\Rightarrow (\lambda - 1)(\lambda - 2) = 0$

$\Rightarrow \lambda = 1, 2$ are the eigenvalues of A.

To find the eigenvectors,
consider the matrix equation $(A - \lambda I)X = 0$

$$\Rightarrow \begin{pmatrix} -1-\lambda & 3 \\ -2 & 4-\lambda \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \left.\begin{array}{r} (-1-\lambda)x + 3y = 0 \\ -2x + (4-\lambda)y = 0 \end{array}\right\} \text{—— } (*)$$

Case (i) :- Let $\lambda = 1$

$\Rightarrow -2x + 3y = 0$

$-2x + 3y = 0$, a homogeneous system.

$$\begin{pmatrix} -2 & 3 \\ -2 & 3 \end{pmatrix}$$

$R_2 \to R_2 - R_1 \sim \begin{pmatrix} -2 & 3 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow$ Rank, $r = 1 <$ no. of variables.

$\Rightarrow$ Non-trivial solution exists.

$\therefore$ Let $y = k$, $\quad -2x + 3y = 0$

$\Rightarrow -2x + 3k = 0$

$\Rightarrow \quad x = \frac{3k}{2}$.

$\therefore$ The eigenvector corresponding to $\lambda = 1$ is

$X_1 = \begin{pmatrix} 3k/2 \\ k \end{pmatrix}$, $k \neq 0$.

We let $k = 2$,

$X_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Case (ii) :- Let $\lambda = 2$

$\Rightarrow \quad -3x + 3y = 0$

$-2x + 2y = 0$, a homogeneous system.

$$\begin{pmatrix} -3 & 3 \\ -2 & 2 \end{pmatrix}$$

$R_1 \to -\frac{1}{3} R_1 \quad \sim \begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix}$

$R_2 \to R_2 + 2R_1 \quad \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow$ rank $= 1 <$ no. of variables.

$\Rightarrow$ Non-trivial solution exists.

Let $y = k_1$, $\quad x - y = 0$

$\Rightarrow x - k_1 = 0$

$\Rightarrow x = k_1$.

$\therefore$ The eigenvector corresponding to $\lambda = 2$ is

$$X_2 = \begin{pmatrix} k_1 \\ k_1 \end{pmatrix}$$

We let $k_1 = 1$

$$X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Modal matrix, $P = (X_1 \; X_2)$

$$= \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$$

$$P^{-1} = \frac{1}{3-2} \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1+2 & 3-4 \\ 2-6 & -6+12 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 \\ -4 & 6 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3-2 & 1-1 \\ -12+12 & -4+6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = D$$

$$= diag(1, 2).$$

Q2. $A = \begin{pmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{pmatrix}$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 4R_1$

$R_4 \to R_4 - 4R_1$ $\sim \begin{pmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -15 & -21 \end{pmatrix}$

$R_3 \to R_3 - R_2$

$R_4 \to R_4 - 3R_2$

$$\sim \begin{pmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \text{echelon form.}$$

$\therefore$ Rank, $\rho(A) = 2$.

Q3. Given $2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$2x + 3y + \lambda z = \mu$

The augmented matrix $(A:B)$

$$= \left(\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right)$$

$R_2 \rightarrow 2R_2 - 7R_1$

$R_3 \rightarrow R_3 - R_1$

$$\sim \left(\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -15 & -39 & -47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right)$$

which is in echelon form.

The given system has a unique solution if and only if $\rho(A:B) = \rho(A) = 3$. This is possible only when $\lambda \neq 5$.

If $\lambda = 5$, then $\rho(A) = 2$ and the given system is consistent only if the $\rho(A:B) = 2$. This is possible only if $\mu = 9$. Thus, the system has infinitely many solutions.

If $\lambda = 5$ and $\mu \neq 9$, then $\rho(A) = 2$ and $\rho(A:B) = 3$ which are unequal, and the system has no solution.

Thus, the given system has

(i) a unique soln when $\lambda \neq 5$ (and for any μ)

(ii) infinitely many solutions for $\lambda = 5$ and $\mu = 9$

(iii) no solution for $\lambda = 5$ and $\mu \neq 9$.

Q4. $x + y + z = 9$

$2x - 3y + 4z = 13$

$3x + 4y + 5z = 40.$

$$(A:B) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & -3 & 4 & 13 \\ 3 & 4 & 5 & 40 \end{array}\right)$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 1 & 2 & 13 \end{array}\right)$$

$R_3 \to R_3 + \frac{1}{5} R_2$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 0 & 12/5 & 12 \end{array}\right)$$

$R_3 \to \frac{5}{12} R_3$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

$R_2 \to R_2 - 2R_3$

$R_1 \to R_1 - R_3$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & -5 & 0 & -15 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

$R_2 \to -\frac{1}{5} R_2$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

$R_1 \to R_1 - R_2$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

We get $x=1$, $y=3$ and $z=5$.

Q5. Given $A = \begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix}$

$$X_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Iteration 1 :-

$$AX_0 = \begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}$$

$$= 4 \begin{pmatrix} -0.75 \\ 0.25 \\ 1 \end{pmatrix}$$

$$= \lambda_1 X_1$$

Iteration 2 :-

$$AX_1 = \begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} -0.75 \\ 0.25 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \\ -1 \end{pmatrix} = 1 \begin{pmatrix} 0.5 \\ 0 \\ -1 \end{pmatrix}$$

$$= \lambda_2 X_2$$

Iteration 3 :-

$$AX_2 = \begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -0.5 \\ 1 \end{pmatrix} = 1 \begin{pmatrix} 0 \\ -0.5 \\ 1 \end{pmatrix}$$

$$= \lambda_3 X_3$$

Iteration 4 :-

$$AX_3 = \begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -0.5 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1.5 \\ -1 \end{pmatrix} = 1.5 \begin{pmatrix} -0.6667 \\ 1 \\ -0.6667 \end{pmatrix}$$

$$= \lambda_4 X_4$$

$\therefore$ The dominant eigenvalue is 1.5 and the corresponding eigenvector is $\begin{pmatrix} -0.6667 \\ 1 \\ -0.6667 \end{pmatrix}$

Q6. $10x + 2y + z = 9$ $\quad |10| > |2| + |1|$

$x + 10y - z = -22$ $\quad |10| > |1| + |-1|$

$-2x + 3y + 10z = 22.$ $\quad |10| > |-2| + |3|$

The given system is diagonally dominant.

$$x = \frac{1}{10}(9 - 2y - z)$$

$$y = \frac{1}{10}(-22 - x + z)$$

$$z = \frac{1}{10}(22 + 2x - 3y)$$

Let $(x_0, y_0, z_0) = (0, 0, 0)$

Iteration 1 :-

$$x_1 = \frac{1}{10}\left[9 - 2(0) - 0\right] = 0.9$$

$$y_1 = \frac{1}{10}\left[-22 - 0.9 + 0\right] = -2.29$$

$$z_1 = \frac{1}{10}\left[22 + 2(0.9) - 3(-2.29)\right] = 3.067$$

Iteration 2 :-

$$x_2 = \frac{1}{10}\left[9 - 2(-2.29) - 3.067\right] = 1.0513$$

$$y_2 = \frac{1}{10}\left[-22 - 1.0513 + 3.067\right] = -1.9984$$

$$z_2 = \frac{1}{10}\left[22 + 2(1.0513) - 3(-1.9984)\right] = 3.0098$$

Iteration 3 :-

$$x_3 = \frac{1}{10}\left[9 - 2(-1.9984) - 3.0098\right] = 0.9987$$

$$y_3 = \frac{1}{10}\left[-22 - 0.9987 + 3.0098\right] = -1.9989$$

$$z_3 = \frac{1}{10}\left[22 + 2(0.9987) - 3(-1.9989)\right] = 2.9994$$

$\therefore$ $x = 0.9987$, $y = -1.9989$, $z = 2.9994$

Q7. $x - 2y + 3z = 2$

$3x - y + 4z = 4$

$2x + y - 2z = 5$

The augmented matrix is

$$(A:B) = \left(\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 3 & -1 & 4 & 4 \\ 2 & 1 & -2 & 5 \end{array}\right)$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 5 & -5 & -2 \\ 0 & 5 & -8 & 1 \end{array}\right)$$

$R_3 \to R_2 - R_3$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 5 & -5 & -2 \\ 0 & 0 & 3 & -3 \end{array}\right)$$

We get $x - 2y + 3z = 2$

$5y - 5z = -2$

$3z = -3 \Rightarrow z = -\frac{3}{3} = -1.$

By back substitution,

$$5y - 5z = -2$$

$$\Rightarrow 5y - 5(-1) = -2$$

$$\Rightarrow 5y + 5 = -2$$

$$\Rightarrow 5y = -7 \Rightarrow y = -7/5$$

$$x - 2y + 3z = 2$$

$$\Rightarrow x - 2\left(-\frac{7}{5}\right) + 3(-1) = 2$$

$$\Rightarrow x = 2 + 3 - \frac{14}{5}$$

$$\Rightarrow x = \frac{11}{5}$$

$$\therefore x = \frac{11}{5}, \quad y = -\frac{7}{5}, \quad z = -1$$