USN

Internal Assessment Test II – February 2022

Sub:	**Calculus and Differential Equations**					Sub Code:	**21MAT11**	
Date:	**28/02/2022**	Duration:	**90 mins**	Max.marks	**50**	Sem / Sec:	**A to G(Physics cycle)**	OBE

	Question 1 is compulsory and answer any SIX questions from the rest.	MARKS	CO	RBT
1 .	Show that the radius of curvature for the curve $x^3 + y^3 = 3\,axy$, at the point $(a/2, a/2)$ is **a/4√2**	[08]	CO1	L3
2 .	With usual notation, prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}(\frac{dr}{d\theta})^2$.	[07]	CO1	L3
3 .	Obtain the Maclaurin's expansion of $\sqrt{(1 + sin2x)}$ upto the fourth degree term.	[07]	CO1	L3

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5.	Find the pedal equation of the polar curve: $r^n sec\, n\theta = a^n$.	[07]	CO1	L3
6.	Show that for the polar curve $r\,(1 + cos\theta) = a, \rho^2 varies\ as\ r^3$ is a constant.	[07]	CO1	L3
7.	Derive the expression to find the radius of curvature for a Cartesian curve.	[07]	CO2	L3
8.	Evaluate the following limits: (A) $\lim_{x \to 1} (2-x)^{\tan(\frac{\pi x}{2})}$ (B) $\lim_{x \to 0} (\frac{1^x+2^x+3^x}{3})^{1/x}$	[07]	CO2	L3

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1. The given curve is,

$$x^3+y^3=3axy.$$

$\Rightarrow$ We know, Radius of curvature ρ,

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}, \text{ where, } y_1=\frac{dy}{dx} \ \& \ y_2=\frac{d^2y}{dx^2}$$

$\Rightarrow$ Taking log on both sides of the curve.

$$3\log x + 3\log y = \log(3axy).$$

$\Rightarrow$ Differentiating the given curve w.r.t x,

$$3x^2+3y^2y_1 = 3ay+3axy_1$$

$$y_1(3y^2-3ax) = 3(ay-x^2)$$

$$y_1 = \frac{ay-x^2}{y^2-ax}$$

$$y_1\Big|_{\left(\frac{a}{2},\frac{a}{2}\right)} = \frac{a\left(\frac{a}{2}\right)-\left(\frac{a}{2}\right)^2}{\left(\frac{a}{2}\right)^2-a\left(\frac{a}{2}\right)} = \frac{\frac{a^2}{2}-\frac{a^2}{4}}{\frac{a^2}{4}-\frac{a^2}{2}}$$

$$= \frac{\frac{a^2}{4}}{\frac{-a^2}{4}} = -1$$

$\Rightarrow$ $$y_1\Big|_{\left(\frac{a}{2},\frac{a}{2}\right)} = -1$$

$\Rightarrow$ $$y_2 = \frac{(y^2-ax)(ay_1-2x)-(2yy_1-a)(ay-x^2)}{(y^2-ax)^2}$$

$$y_2\Big|_{\left(\frac{a}{2},\frac{a}{2}\right)} = \frac{\left(\frac{a^2}{4}-\frac{a^2}{2}\right)(-a-a)-(-a-a)\left(\frac{a^2}{2}-\frac{a^2}{4}\right)}{\left(\frac{a^2}{4}-\frac{a^2}{2}\right)^2}$$

$$= \frac{\left(\frac{-a^2}{4}\right)(-2a)-(-2a)\left(\frac{a^2}{4}\right)}{\frac{a^4}{16}} = \frac{\frac{a^3}{2}+\frac{a^3}{2}}{\frac{a^4}{16}}$$

$$= 16/a$$

$\Rightarrow \quad y_2\Big|_{\left(\frac{a}{2},\frac{a}{2}\right)} = \dfrac{16}{a}$

$\Rightarrow \quad \rho = \dfrac{(1+y_1^2)^{3/2}}{y_2}$

$= \dfrac{(1+(-1)^2)^{3/2}}{16/a}$

$= \dfrac{a}{16} \times \left((2\times2\times2)\right)^{1/2}$.

$= \dfrac{a\sqrt{2}\times 2}{16} = \dfrac{a\sqrt{2}}{8}$

$= \dfrac{a\sqrt{2}\times\sqrt{2}}{8\times\sqrt{2}} = \dfrac{2a}{8\sqrt{2}}$

$= \dfrac{a}{4\sqrt{2}}$.

$\Rightarrow \quad \rho = \dfrac{a}{4\sqrt{2}}$

$\Rightarrow$ Hence proved.

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2. To prove : $\dfrac{1}{p^2} = \dfrac{1}{r^2} + \dfrac{1}{r^4}\left(\dfrac{dr}{d\theta}\right)^2$

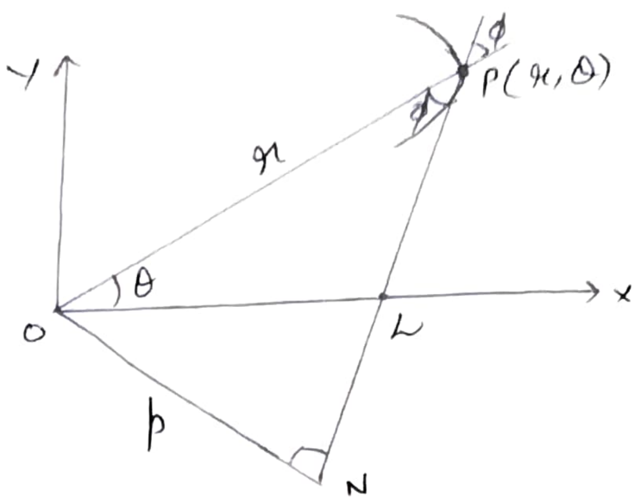
proof:

$\Rightarrow$ Let a curve $r=f(\theta)$, be in the x-y plane.

$\Rightarrow$ Let PL be a tangent to the curve.

$\Rightarrow$ Drop a perpendicular $ON=p$ from the pole, O to the tangent PL.

$\Rightarrow$ Let $\angle L\hat{O}P=\theta$ and $O\hat{P}L=\phi$



$\Rightarrow$ OP is radius vector, r.

$\Rightarrow$ Now, $\angle ONL = 90^\circ$, so,
Triangle ONP is Right Angled at N.

$$\Rightarrow \quad \sin\phi = \frac{ON}{OP} = \frac{p}{r}$$

$$\Rightarrow \quad p = r\sin\phi \quad \text{—— (1)}$$

$\Rightarrow$ squaring the equation (1), we get, $p^2 = r^2\sin^2\phi$

$$\frac{1}{p^2} = \frac{1}{r^2}(\operatorname{cosec}^2\phi) \quad \text{[reciprocal of squared Equation]}$$

$\Rightarrow$ We know,

$$\operatorname{cosec}^2\phi = 1 + \cot^2\phi$$

$$\text{and} \quad \cot\phi = \frac{1}{r}\frac{dr}{d\theta} = \frac{r'}{r}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{r^2}\left[1 + \cot^2\phi\right] = \frac{1}{r^2}\left[1 + \left(\frac{1}{r}\right)^2\left(\frac{dr}{d\theta}\right)^2\right]$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$$

$\Rightarrow$ Hence proved.

3) The maclaurin's series is given ~~by~~ at $x=0$ by

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{IV}(0) + \cdots$$

is $f(x) = \sqrt{1+\sin 2x}$

$$f(x) = \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}$$
$$= \sqrt{(\sin x + \cos x)^2}$$
$$f(x) = \sin x + \cos x$$
$$f(0) = \sin 0 + \cos 0$$
$$f(0) = 0 + 1 = 1$$
$$f'(x) = \cos x - \sin x$$
$$f'(0) = \cos 0 - \sin 0$$
$$= 1$$
$$f''(x) = -\sin x - \cos x$$
$$f''(0) = -\sin 0 - \cos 0$$
$$= -0 - 1 = -1$$
$$f'''(x) = -\cos x + \sin x$$
$$f'''(0) = -\cos 0 + \sin 0$$
$$= -1$$
$$f^{iv}(x) = \sin x + \cos x$$
$$f^{iv}(0) = \sin 0 + \cos 0 = 1$$

then,

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0) + \dots$$
$$= 1 + x(1) + \frac{x^2}{2!}(-1) + \frac{x^3}{3!}(-1) + \frac{x^4}{4!}(1) + \dots$$
$$f(x) = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

4. The given curves are,

$r = a(1+\sin\theta)$ —— ①

$r = a(1-\sin\theta)$ —— ②

$\Rightarrow \cot\phi = \frac{1}{r}\frac{dr}{d\theta}$

$\Rightarrow$ Let ϕ_1 & ϕ_2 be the angles for curve ① & ② respectively

$\Rightarrow$ For ①,

Taking logarithm on both sides,

$\log r = \log a + \log(1+\sin\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{(\cos\theta)}{1+\sin\theta}$ [differentiating wrt θ]

$\cot\phi_1 = \frac{1}{r}\frac{dr}{d\theta} = \frac{\cos\theta}{1+\sin\theta}$

$= \frac{\cos^2\theta/2 - \sin^2\theta/2}{\cos^2\theta/2 + \sin^2\theta/2 + 2\sin\theta/2\cos\theta/2}$ [$\because \cos\theta = \cos^2\theta/2 - \sin^2\theta/2$] [$\because \sin\theta = 2\sin\theta/2\cos\theta/2$]

$= \frac{(\cos\theta/2 + \sin\theta/2)(\cos\theta/2 - \sin\theta/2)}{(\cos\theta/2 + \sin\theta/2)^2}$

$= \frac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}$

$= \frac{1 - \tan\theta/2}{1 + \tan\theta/2}$ [divide N'r & D'r by $\cos\theta/2$]

$= \cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

$\therefore \phi_1 = \frac{\pi}{4} + \frac{\theta}{2}$

$\Rightarrow$ For (2),

Taking logarithms on both sides,

$$\log r = \log a + \log(1-\sin\theta)$$

Differentiating w.r.t θ,

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{-\cos\theta}{1-\sin\theta}$$

$$\cot\phi_2 = \frac{1}{r}\frac{dr}{d\theta} = \frac{-\cos\theta}{1-\sin\theta}$$

$$= \frac{-(\cos^2\theta/2 - \sin^2\theta/2)}{(\cos^2\theta/2 + \sin^2\theta/2 - 2\sin\theta/2\cos\theta/2)}$$

$$= \frac{(\sin^2\theta/2 - \cos^2\theta/2)}{(-\cos\theta/2 + \sin\theta/2)^2}$$

$$= \frac{(\sin\theta/2 + \cos\theta/2)}{(\sin\theta/2 - \cos\theta/2)}$$

$$= \frac{\tan\theta/2 + 1}{\tan\theta/2 - 1}$$

$$= -\left(\frac{\tan\theta/2 + 1}{1-\tan\theta/2}\right)$$

$$= -\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$$

$$\phi_2 = \frac{\pi}{2} + \frac{\pi}{4} + \frac{\theta}{2} = \frac{3\pi}{4} + \frac{\theta}{2}$$

$$\therefore \phi_2 = \frac{3\pi}{4} + \frac{\theta}{2}$$

$\Rightarrow$ Angle between the curves $= |\phi_1 - \phi_2|$

$$= \left|\frac{\pi}{4} + \frac{\theta}{2} - \frac{3\pi}{4} - \frac{\theta}{2}\right|$$

$$= \left|-\frac{2\pi}{4}\right|$$

$$= \frac{\pi}{2} = 90^\circ$$

$\Rightarrow$ $\therefore$ The angle between the curves is 90° and the two curves are orthogonal.

5. The given curve is,

$$r^n \sec n\theta = a^n$$

$\Rightarrow$ Taking logarithm on both sides,

$$r^n = \frac{a^n}{\sec n\theta}$$

$$\log r^n = \log(a^n) - \log(\sec n\theta)$$

$$n \log r = n \log a - \log(\sec n\theta) \quad \text{—} \; (1)$$

$\Rightarrow$ Differentiating (1) wrt θ,

$$\frac{n}{r}\frac{dr}{d\theta} = 0 - \frac{n \sec n\theta \tan n\theta}{\sec n\theta}$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{n \tan n\theta}{n} = \tan(n\theta)$$

$\Rightarrow$ We know,

$$\cot\phi = \frac{1}{r}\frac{dr}{d\theta}$$

$\Rightarrow$ So,

$$\cot\phi = \tan(n\theta)$$

$$\phi = \left(\frac{\pi}{2} - n.\theta\right)$$

$\Rightarrow$ The pedal equation of the polar curve is,

$$P = r\sin\phi$$

$$P = r\sin\left(\frac{\pi}{2} - n\theta\right) \qquad \left[\because \phi = \left(\frac{\pi}{2} - n\theta\right)\right]$$

$\Rightarrow$ $\sin\left(\frac{\pi}{2} - n\theta\right) = \cos(n\theta)$

$\Rightarrow$ $P = r\cos n\theta$ —— (2)

$\Rightarrow$ From $r^n \sec n\theta = a^n$,

$$\cos n\theta = \left(\frac{r}{a}\right)^n \quad \text{--- (3)}$$

$\Rightarrow$ Substituting (3) in (2),

$$p = r\left(\frac{r}{a}\right)^n$$

$$= \frac{r^{n+1}}{a^n}$$

$\Rightarrow$ $\therefore$ The pedal Equation of the given curve is,

$$pa^n = r^{n+1}.$$

6. The given curve is,

$$r(1+\cos\theta) = a$$

$$r = \frac{a}{1+\cos\theta}$$

$\Rightarrow$ Taking logarithm on both sides,

$$\log r = \log a - \log(1+\cos\theta) \quad \text{--- (1)}$$

$\Rightarrow$ Differentiating (1) wrt θ,

$$\frac{1}{r}\frac{dr}{d\theta} = 0 - \frac{(-\sin\theta)}{1+\cos\theta}$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{\sin\theta}{1+\cos\theta}$$

$\Rightarrow$ we know,

$$\cot\phi = \frac{1}{r}\frac{dr}{d\theta}$$

$\Rightarrow$ so,

$$\cot\phi = \frac{\sin\theta}{1+\cos\theta} = \frac{2\sin\theta/2\cos\theta/2}{2\cos^2\theta/2} = \tan(\theta/2)$$

$\Rightarrow$ $\therefore$ $\phi = \left(\frac{\pi}{2} - \frac{\theta}{2}\right)$

$\Rightarrow$ The radius of curvature,

$$\rho = \frac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2-rr_2} \quad ; \text{where } r_1 = \frac{dr}{d\theta} \ \& \ r_2 = \frac{d^2r}{d\theta^2}$$

$\Rightarrow$ $$r = \frac{a}{\cos\theta+1} \quad \cancel{= a\sec\theta}$$

$\Rightarrow$ $$\frac{dr}{d\theta} = r_1 = -\frac{(-\sin\theta)(a)}{(1+\cos\theta)^2}$$

$$\frac{dr}{d\theta} = r_1 = r\tan\theta/2.$$

$\Rightarrow$ $$r_2 = \frac{d^2r}{d\theta^2} = \frac{r}{2}\sec^2\left(\frac{\theta}{2}\right) + r_1\tan\theta/2.$$

$$= \frac{r}{2}\sec^2\left(\frac{\theta}{2}\right) + r\tan^2\left(\frac{\theta}{2}\right)$$

$\Rightarrow$ $$r^2 = \frac{a^2}{(1+\cos\theta)^2} \quad ; \quad r_1^2 = r^2\tan^2(\theta/2)$$

$$r_2^2 = \left(\frac{r}{2}\sec^2\frac{\theta}{2} + r\tan^2\frac{\theta}{2}\right)^2$$

$\Rightarrow$ $$r^2+r_1^2 = r^2 + r^2\tan^2\theta/2 = r^2\sec^2\theta/2.$$

$\Rightarrow$ $$r^2+2r_1^2 = r^2 + 2r^2\tan^2\theta/2.$$

$\Rightarrow$ $$rr_2 = r^2\left[\frac{1}{2}(1+\tan^2\theta/2) + \tan^2\theta/2\right]$$

$$= r^2\left[\frac{1}{2} + \frac{3}{2}\tan^2\theta/2\right] = \frac{r^2}{2}\left[1+3\tan^2\theta/2\right]$$

$\Rightarrow$ $$r^2+2r_1^2-rr_2 = r^2\left[1+2\tan^2\theta/2 - \frac{1}{2} - \frac{3}{2}\tan^2\theta/2\right]$$

$$= r^2\left[\frac{1}{2}+\frac{1}{2}\tan^2\theta/2\right] = \frac{r^2}{2}\sec^2\frac{\theta}{2}$$

$$\Rightarrow \rho = \frac{\left(r^2 \sec^2\theta/2\right)^{3/2}}{\left(\frac{r^2}{2}\sec^2\theta/2\right)}$$

$$= \frac{2\left(r \sec\theta/2\right)^3}{\left(r\sec\theta/2\right)^2}$$

$$\Rightarrow \rho = 2\left(r\sec\theta/2\right)$$

$$\Rightarrow \rho^2 = 4\left(r\sec\theta/2\right)^2$$

$$= 4\cdot r^2\left(\sec^2\theta/2\right)$$

$$\Rightarrow \quad r = \frac{a}{1+\cos\theta}$$

$$= \frac{a}{2\cos^2\theta/2}$$

$$= \frac{a\sec^2\theta/2}{2}$$

$$\Rightarrow \quad \sec^2\theta/2 = \frac{2r}{a}$$

$\Rightarrow$ substituting $\sec^2\theta/2 = \frac{2r}{a}$ in $\rho^2 = 4r^2(\sec^2\theta/2)$,

$$\rho^2 = 4r^2 \times \frac{2r}{a}$$

$$= \frac{8r^3}{a}$$

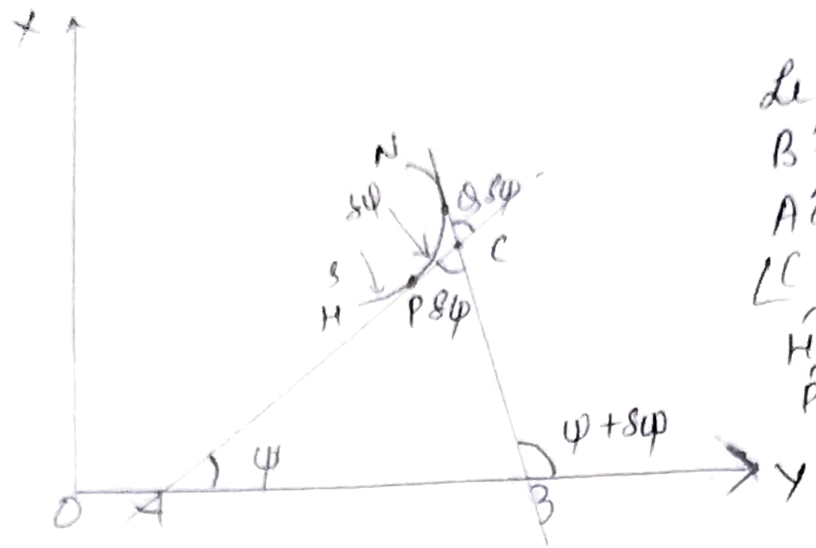
$$\Rightarrow \boxed{\rho^2 \propto r^3}$$

$$\Rightarrow \frac{\rho^2}{r^3} = \frac{8}{a}$$

$\Rightarrow$ $\therefore$ ρ^2 varies as r^3 is a constant for the given polar curve,

$$r = \frac{(a)}{1+\cos\theta}.$$

7. The Expression of Radius of Curvature in cartesian form, -



Let
$B\hat{A}C = (\psi)$
$A\hat{C}B = \delta\psi$
$\angle CBY = \psi + \delta\psi$
$\overset{\frown}{HP} = s$
$\overset{\frown}{PQ} = \delta s$

$\Rightarrow$ Let a curve $r = f(\theta)$ have two tangents at point P & Q.

$\Rightarrow$ $\dfrac{dy}{dx} = \tan\psi$ [is the slope of AC]

$\Rightarrow$ differentiate, $\dfrac{dy}{dx} = \tan\psi$ w.r.t to s,

$$\sec^2\psi \frac{d\psi}{ds} = \frac{d}{ds}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{ds}$$

$$\sec^2\psi \frac{d\psi}{ds} = \frac{d^2y}{dx^2} \cdot \frac{dx}{ds}$$

$\Rightarrow$ $\sec^2\psi = 1 + \tan^2\psi = 1 + \left(\dfrac{dy}{dx}\right)^2$

Let $\dfrac{dy}{dx} = y_1$ & $\dfrac{d^2y}{dx^2} = y_2$

$\Rightarrow$ Now, $\dfrac{d\psi}{ds} = \dfrac{y_2}{1+\tan^2\psi}$

$\Rightarrow$ $y_1^2 = \tan^2\varphi$

$\Rightarrow$ $\frac{ds}{dx} = \sqrt{1+y_1^2}$

$\Rightarrow$ $\frac{d\varphi}{ds} = \frac{y_2}{(1+y_1^2)} \times \frac{1}{\sqrt{1+y_1^2}} = \frac{y_2}{(1+y_1^2)^{3/2}}$

$\Rightarrow$ Now, Radius of curvature, $\rho = \frac{ds}{d\varphi}$

$\Rightarrow$ So,

$$\rho = \frac{ds}{d\varphi} = \frac{(1+y_1^2)^{3/2}}{y_2}.$$

$\Rightarrow$ Hence proved.

8.

(A) $\lim_{x\to 1} (2-x)^{\tan\left(\frac{\pi x}{2}\right)}$

$\Rightarrow$ Let $L = \lim_{x\to 1} (2-x)^{\tan\left(\frac{\pi x}{2}\right)}$

$= (2-1)^{\tan(\pi/2)}$

$= 1^\infty$

L is indeterminate form.

$\Rightarrow$ We use L'Hospital Rule for L.

$\Rightarrow$ Taking Logarithm on both sides,

$$\log L = \lim_{x\to 1}\left[\tan\left(\frac{\pi x}{2}\right)\log(2-x)\right]$$

$$= \lim_{x\to 1}\left[\frac{\log(2-x)}{\cot\left(\frac{\pi x}{2}\right)}\right]$$

$\Rightarrow$ Apply L Hospital Rule,

$$\log L = \lim_{x\to 1}\left[\frac{-1\times 2}{(2-x)\left(-\operatorname{cosec}^2\left(\frac{\pi x}{2}\right)\right)\times\pi}\right]$$

$$\log L = \frac{+2}{\pi(2-1)\,\cancel{\text{cosec}}^2\left(\frac{\pi\times 1}{2}\right)}$$

$$= \frac{2}{\pi\times 1\times 1}$$

$$= \frac{2}{\pi}$$

$$L = e^{2/\pi}$$

$$\therefore \lim_{x\to 1} (2-x)^{\tan\left(\frac{\pi x}{2}\right)} = e^{(2/\pi)}$$

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8. (B) $\lim_{x\to 0}\left(\frac{1^x+2^x+3^x}{3}\right)^{1/x}$

$\Rightarrow$ Let $L = \lim_{x\to 0}\left(\frac{1^x+2^x+3^x}{3}\right)^{1/x}$

$$= \left(\frac{1^0+2^0+3^0}{3}\right)^{1/0}$$

$$= 1^{\infty}$$

It is indeterminate form.

$\Rightarrow$ Taking logarithms on both sides,

$$\log L = \lim_{x\to 0}\frac{1}{x}\times\log\left(\frac{1^x+2^x+3^x}{3}\right)$$

$$= \lim_{x\to 0}\frac{\log(1^x+2^x+3^x)-\log 3}{x}$$

$$= \frac{\log(1^0+2^0+3^0)-\log 3}{0} = \frac{0}{0}\text{ form}$$

$\Rightarrow$ Apply L' Hospital Rule,

$$\log l = \lim_{x \to 0} \frac{(2^x \log 2 + 3^x \log 3)}{(1^x + 2^x + 3^x)}$$

$$= \frac{2^0 \log 2 + 3^0 \log 3}{1^0 + 2^0 + 3^0}$$

$$\log l = \frac{\log 2 + \log 3}{3}$$

$$= \frac{\log 6}{3}$$

$$= \frac{1}{3} \log 6$$

$$\log L = \log (6)^{1/3}$$

$$\boxed{L = (6)^{1/3}}$$