USN

18CV52

Fifth Semester B.E. Degree Examination, Feb./Mar. 2022
Analysis of Indeterminate Structures

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Assume missing data suitably.

Module-1

1 Analyze the continuous beam shown in Fig.Q.1 by slope deflection method. Draw BMD and SFD. **(20 Marks)**

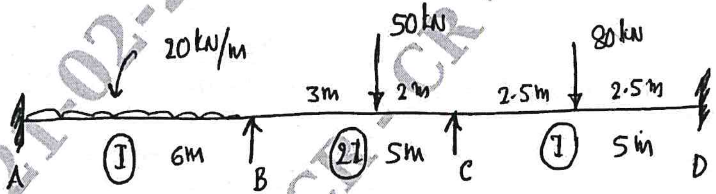

Fig.Q.1

OR

2 Analyze the portal frame shown in Fig.Q.2 by slope deflection method. Draw BMD. **(20 Marks)**

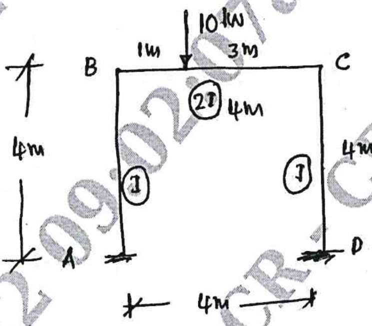

Fig.Q.2

Module-2

3 Analyze the beam shown in Fig.Q.3 by moment distribution method. Draw BMD EI is constant. **(20 Marks)**

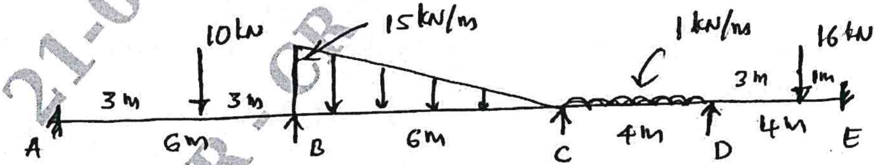

Fig.Q.3

OR

4 Analyze the portal frame by moment-distribution method draw BMD. **(20 Marks)**

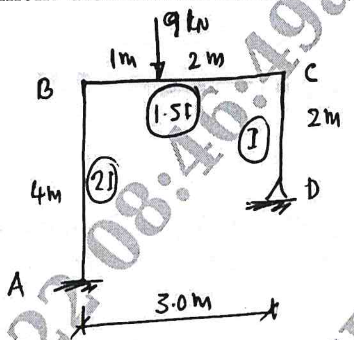

Fig.Q.4

Module-3

5 Analyze the continuous beam loaded shown in Fig.Q.5 by Kani's rotation method. Draw BMD. **(20 Marks)**

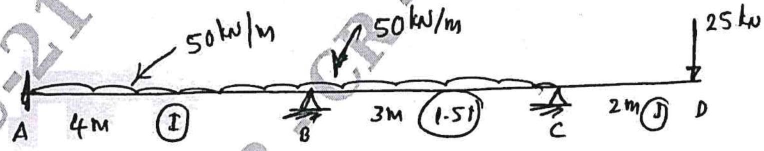

Fig.Q.5

OR

6 Analyze the frame shown in Fig.Q.6 by Kani's method. Take the advantage of symmetry. **(20 Marks)**

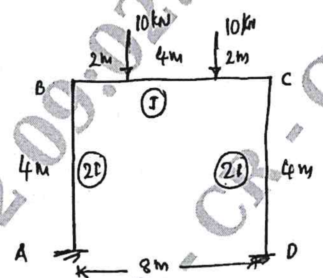

Fig.Q.6

Module-4

7 Analyze the continuous beam by flexibility matrix method (system approach). Draw BMD. (Fig.Q.7). **(20 Marks)**

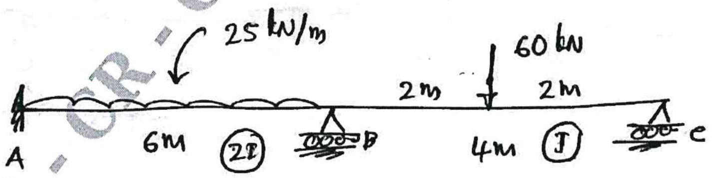

Fig.Q.7

OR

8 Analyze the L-frame shown in Fig.Q.8 by flexibility matrix method. Draw BMD (system approach). **(20 Marks)**

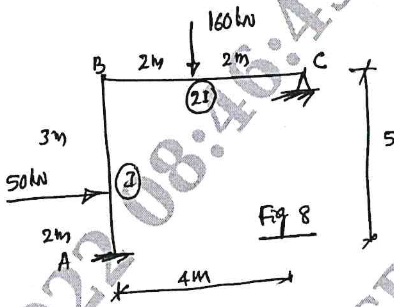

Fig.Q.8

Module-5

9 Analyze the continuous beam by stiffness matrix method (system approach) shown in Fig.Q.9. Draw BMD EI is constant. **(20 Marks)**

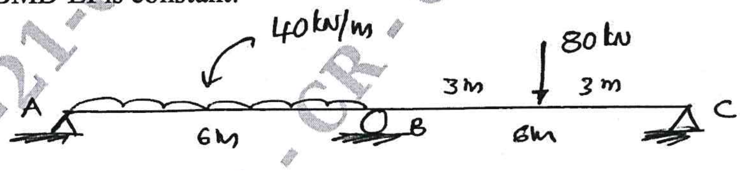

Fig.Q.9

OR

10 Find the forces in the members of a joint 'O' shown in Fig.Q.10 by stiffness matrix method. (system approach). **(20 Marks)**

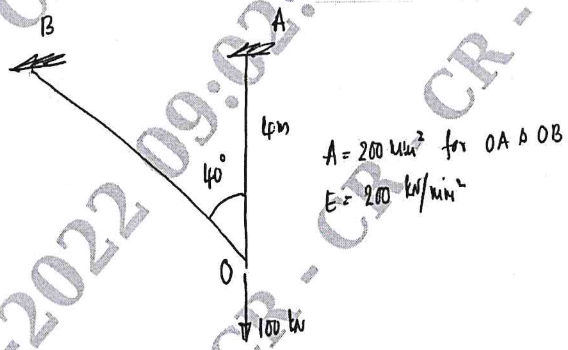

Fig.Q.10

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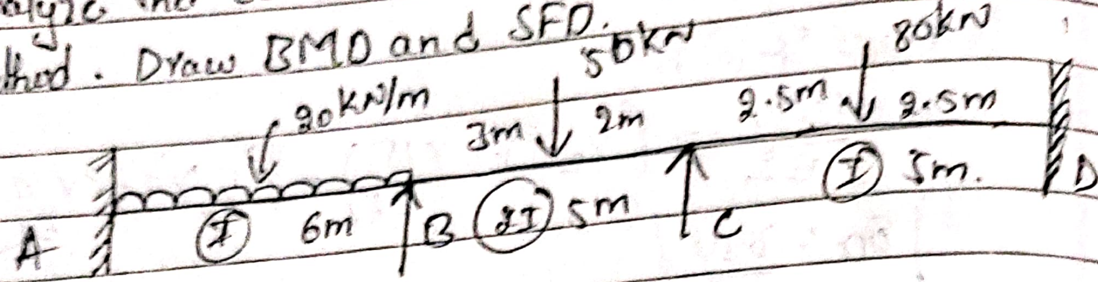
Module-1

1. Analyze the continuous beam shown in Fig. by slope deflection method. Draw BMD and SFD.



Step 1 F.E.M

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-20\times6^2}{12} = -60 \text{ kN-m}$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{20\times6^2}{12} = 60 \text{ kN-m}$$

$$M_{FBC} = \frac{-wab^2}{l^2} = -\frac{50\times3\times2^2}{5^2} = -24 \text{ kN-m}$$

$$M_{FCB} = \frac{wa^2b}{l^2} = \frac{50\times3^2\times2}{5^2} = 36 \text{ kN-m}$$

$$M_{FCD} = -\frac{wl}{8} = -\frac{80\times5}{8} = -50 \text{ kN-m}$$

$$M_{FDC} = \frac{wl}{8} = 50 \text{ kN-m}$$

Step 2: Slope deflection equation.

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[\cancel{2\theta_A}^{0} + \theta_B - \cancel{\frac{3\Delta}{l}}^{0}\right]$$

$$= -60 + \frac{2EI}{6}[\theta_B]$$

$$= -60 + 0.333 EI\theta_B \rightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \cancel{\theta_A}^{0} - \cancel{\frac{3\Delta}{l}}^{0}\right]$$

$$= 60 + \frac{2EI}{6}[2\theta_B]$$

$$= 60 + 0.667\theta_B EI \longrightarrow (2)$$

$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \cancel{\frac{3\Delta}{l}}\right]$

$= -24 + \frac{2EI}{5} \times 2\left[2\theta_B + \theta_C\right]$

$= -24 + 1.6 EI\theta_B + 0.8 EI\theta_C \longrightarrow (3)$

$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \cancel{\frac{3\Delta}{l}}\right]$

$= 36 + \frac{2EI \times 2}{5}\left[2\theta_C + \theta_B\right]$

$= 36 + 1.6 EI\theta_C + 0.8 EI\theta_B \longrightarrow (4)$

$M_{CD} = M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \cancel{\theta_D} - \cancel{\frac{3\Delta}{l}}\right]$

$= -50 + \frac{2EI}{5}\left[2\theta_C\right]$

$= -50 + 0.8EI\theta_C \longrightarrow (5)$

$M_{DC} = M_{FDC} + \frac{2EI}{l}\left[\cancel{2\theta_D} + \theta_C - \cancel{\frac{3\Delta}{l}}\right]$

$= 50 + \frac{2EI}{5}\left[\theta_C\right] \Rightarrow = 50 + 0.4 EI\theta_C \longrightarrow (6)$

Step (3) Applying condition of Equilibrium

$\Sigma m @ B$

$M_{BA} + M_{BC} = 0$

$\Rightarrow 60 + 0.667\theta_B EI - 24 + 1.6 EI\theta_B + 0.8 EI\theta_C = 0$

$2.267\theta_B EI + 0.8EI\theta_C = -36 \longrightarrow (7)$

$\Sigma m @ C$, $M_{CB} + M_{CD} = 0$

$36 + 1.6 EI\theta_C + 0.8 EI\theta_B - 50 + 0.8 EI\theta_C = 0$

$2.4 EI\theta_C + 0.8 EI\theta_B = 14 \longrightarrow (8)$

by solving (7) & (8) we get

$\boxed{\theta_B = \frac{-20.32}{EI}}$, $\boxed{\theta_C = \frac{12.60}{EI}}$

step (4) Final moments

$M_{AB} = -60 + 0.333 EI\theta_B$

$= -60 + 0.333 \times EI \times \frac{-20.32}{EI}$

$= -66.76$ kN-m

$M_{BA} = 60 + 0.667 \times \theta_B EI$

$= 60 + 0.667 \times \frac{-20.32}{EI} \times EI$

$= 46.44$ kN-m

$M_{BC} = -24 + 1.6 EI\theta_B + 0.8 EI\theta_C$

$= -24 + 1.6 EI \times \frac{-20.32}{EI} + 0.8 \times EI \times \frac{12.60}{EI}$

$= -46.43$ kN-m.

$M_{CB} = 36 + 1.6 EI\theta_C + 0.8 EI\theta_B$

$= 36 + 1.6 EI \times \frac{12.62}{EI} + 0.8 EI \times \frac{-20.32}{EI}$

$= 39.90$ kN-m

$M_{CD} = -50 + 0.8 EI\theta_C$

$= -50 + 0.8 \times \frac{12.60}{EI} \times EI$

$= -39.92$ kN-m

$M_{DC} = 50 + 0.4 EI\theta_C$

$= 50 + 0.4 \times EI \times \frac{12.60}{EI}$

$= 55.04$ kN-m

step (5) calculation of shear force

$\Sigma V = 0$

$V_A + V_B + V_C + V_D = 20 \times 6 + 50 + 80$

$V_A + V_B + V_C + V_D = 250 \longrightarrow (1)$

$\Sigma m @ B \text{ LHS} = 0$

$V_A \times 6 - 20 \times 6 \times 3 + M_{AB} + M_{BA} = 0$

$V_A \times 6 = 380.3$

$\boxed{V_A = 63.38 kN}$

$\Sigma m @ C \text{ RHS} = 0$

$-V_D \times 5 + 80 \times 2.5 + M_{CD} + M_{DC} = 0$

$-V_D \times 5 = -215.12$

$\boxed{V_D = 43.02 kN}$

$\Sigma m @ B \text{ RHS} = 0$

$-V_C \times 5 + 50 \times 3 + 80 \times 7.5 - V_D \times 10 + M_{BC} + M_{CB} + M_{CD} + M_{DC} = 0$

$-V_C \times 5 = -328.39$

$\boxed{V_C = 65.67 kN}$

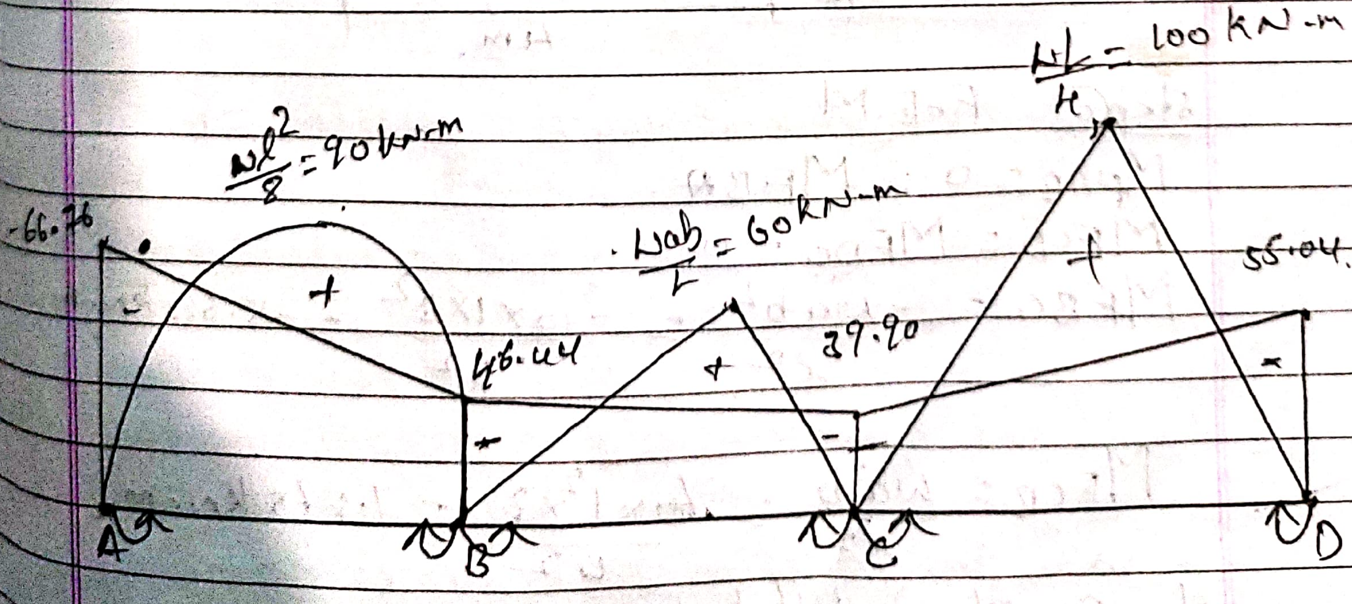
from eqn ①

$V_A + V_B + V_C + V_D = 250$

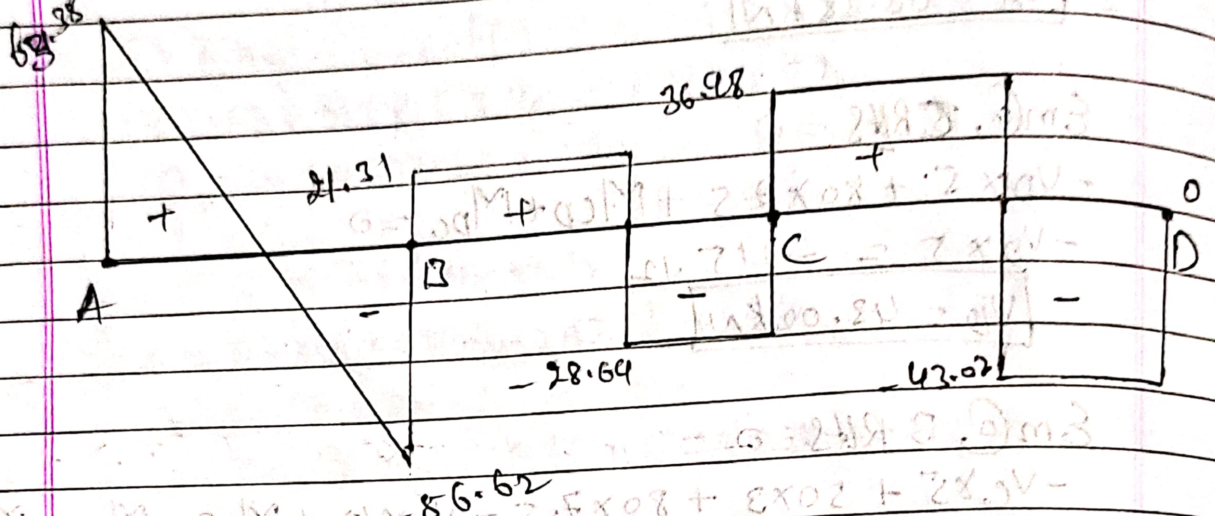
$63.38 + V_B + 43.02 + 65.67 = 250$

$\boxed{V_B = 77.93 kN}$

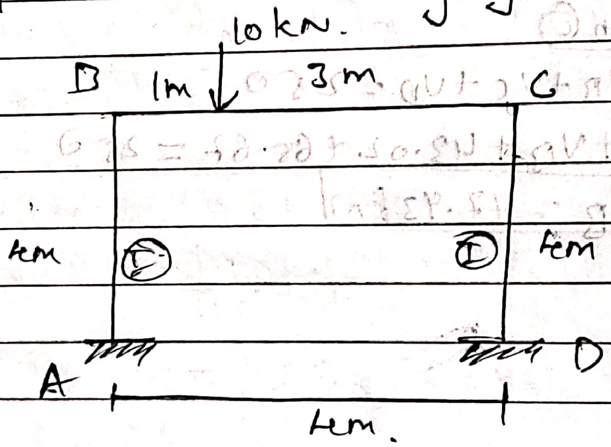
BMD



SFD



(2) Analyze the portal frame shown in fig by slope deflection method
Draw BMD



Step (1) F.E.M

$M_{FAB} = 0 = M_{FBA}$

$M_{FCD} = M_{FDC} = 0$

$M_{FBC} = -\frac{wab^2}{l^2} = -\frac{10 \times 1 \times 3^2}{4^2} = -5.625 \text{ kN-m}$

$M_{FCB} = \frac{wa^2b}{l^2} = \frac{10 \times 1^2 \times 3}{4^2} = 1.875 \text{ kN-m}$

Step (2) Slope deflection equation

$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\cancel{\theta_A}^{0} + \theta_B - \frac{3\Delta}{l}\right]$

$$= 0 + \frac{2EI}{4}\left[\theta_B - \frac{3\Delta}{l_1}\right]$$

$$= 0.5EI\theta_B - 0.375\Delta EI \longrightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \cancel{\theta_A}^{0} - \frac{3\Delta}{L}\right]$$

$$= 0 + \frac{2EI}{4}\left[2\theta_B - \frac{3\Delta}{4}\right]$$

$$= EI\theta_B - 0.375\Delta EI \longrightarrow (2)$$

$$M_{BC} = -5.625 + \frac{2EI\times 2}{L}\left[2\theta_B + \theta_C - \cancel{\frac{3\Delta}{l}}^{0}\right]$$

$$= -5.625 + \frac{2EI\times 2}{4}\left[2\theta_B + \theta_C\right]$$

$$= -5.625 + 2EI\theta_B + EI\theta_C \longrightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left[2\theta_C + \theta_B - \cancel{\frac{3\Delta}{l}}^{0}\right]$$

$$= 1.875 + \frac{2EI\times 2}{4}\left[2\theta_C + \theta_B\right]$$

$$= 1.875 + 2EI\theta_C + EI\theta_B \longrightarrow (4)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \cancel{\theta_D}^{0} - \frac{3\Delta}{l}\right]$$

$$= 0 + \frac{2EI}{4}\left[2\theta_C - \frac{3\Delta}{l_1}\right]$$

$$= EI\theta_C - 0.375EI\Delta \longrightarrow (5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left[\cancel{2\theta_D}^{0} + \theta_C - \frac{3\Delta}{l}\right]$$

$$= 0 + \frac{2EI}{4}\left[\theta_C - \frac{3\Delta}{4}\right]$$

$$= 0.5EI\theta_C - 0.375EI\Delta \longrightarrow (6)$$

Step ⑤ ~~eqn~~ Applying equilibrium equation

$\Sigma m @ B = 0$

$M_{BA} + M_{BC} = 0$

$EI\theta_B - 0.375\Delta EI - 5.625 + 2EI\theta_B + EI\theta_C = 0$

$3EI\theta_B + EI\theta_C - 0.375\Delta EI = 5.625 \rightarrow (7)$

$\Sigma m @ C = 0$

$M_{CB} + M_{CD} = 0$

$1.875 + EI\theta_B + 2EI\theta_C + EI\theta_C - 0.75 EI\Delta = 0$

$EI\theta_B + 3EI\theta_C - 0.375\Delta EI = -1.875 \rightarrow (8)$

Calculation of horizontal reaction

$H_A + H_D = 0$

$\Sigma m_B = H_A \times 4 + M_{BA} + M_{AB} = 0$

$- H_A = \frac{M_{AB} + M_{BA}}{4}$

$\Sigma m_C = H_D \times 4 + M_{DC} + M_{CD}$

$- H_D = \frac{M_{DC} + M_{CD}}{4}$

$\frac{M_{AB} + M_{BA} + M_{DC} + M_{CD}}{4} = 0$

$M_{AB} + M_{BA} + M_{DC} + M_{CD} = 0$

$0.5EI\theta_B - 0.375\Delta EI + EI\theta_B - 0.375 EI\Delta + EI\theta_C - 0.375EI\Delta + 0.5EI\theta_C - 0.375EI\Delta$

$= 1.5EI\theta_B - 0.75 EI\Delta + 1.5EI\theta_C - 0.75EI\Delta \rightarrow (9)$

on solving (7) & (8) & (9) we get.

$\theta_B = \frac{2.45}{EI}$, $\theta_C = \frac{-1.29}{EI}$, $\Delta = \frac{1.15}{EI}$

Step (4) final moment

$M_{AB} = 0.5\,EI\theta_B - 0.375\,EI\Delta$

$= 0.5 \times EI \times \frac{2.45}{EI} - 0.375 \times EI \times \frac{1.15}{EI}$

$= 0.792$ kN-m.

$M_{BA} = EI\theta_B - 0.375\,\Delta EI$

$= \frac{2.45}{EI} \times EI - 0.375 \times \frac{1.15}{EI} \times EI$

$= \cancel{20.186}$ 2.018 kN-m

$M_{BC} = -5.625 + 2EI\theta_B + EI\theta_C$

$= -5.625 + 2EI \times \frac{2.45}{EI} + EI \times \frac{-1.29}{EI}$

$= -2.021$ kN-m

$M_{CB} = 1.875 + 2EI\theta_C + EI\theta_B$

$= 1.875 + 2 \times EI \times \frac{-1.29}{EI} + EI \times \frac{2.45}{E}$

$= 1.73$ kN-m

$M_{CD} = EI\theta_C - 0.375\,EI\Delta$

$= EI \times \frac{-1.29}{EI} - 0.375 \times EI \times \frac{1.15}{EI}$

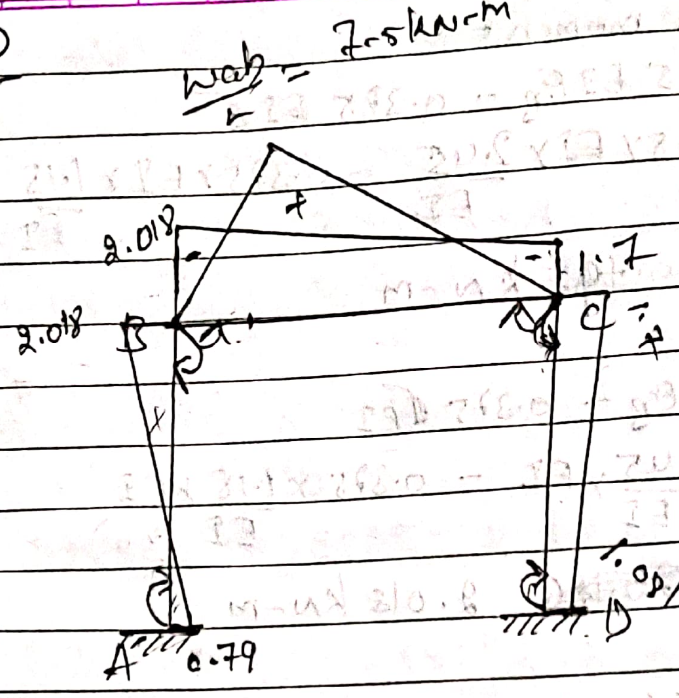
$= -1.73$ kN-m.

$M_{DC} = 0.5\,EI\theta_C - 0.375\,EI\Delta$

$= 0.5 \times \frac{-1.29}{EI} \times EI - 0.375 \times EI \times \frac{1.15}{EI}$

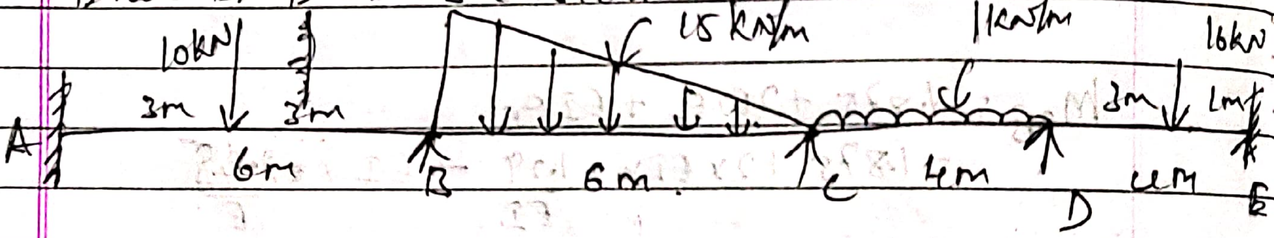
$= 1.081$ kN-m

BMD



Module - 2

(3) Analyze the beam shown in Fig by moment distribution method. Draw BMD EI is constant.



Step ① FEM

$$M_{FAB} = \frac{-Wl}{8} = \frac{-10\times 6}{8} = -7.5 \text{ kN-m}$$

$$M_{FBA} = \frac{Wl}{8} = \frac{10\times 6}{8} = 7.5 \text{ kN-m}$$

$$M_{FBC} = \frac{-wl^2}{20} = \frac{-15\times 6^2}{20} = -27 \text{ kN-m}$$

$$M_{FCB} = \frac{wl^2}{30} = \frac{15\times 6^2}{30} = 18 \text{ kN-m}$$

$$M_{FCD} = \frac{-wl^2}{12} = \frac{-1\times 4^2}{12} = -1.33 \text{ kN-m}$$

$$M_{FDC} = \frac{wl^2}{12} = \frac{1\times 4^2}{12} = 1.33 \text{ kN-m}$$

$$M_{FDE} = \frac{-wab^2}{l^2} = -\frac{16 \times 3 \times 1^2}{4^2} = -3\ kN-m$$

$$M_{FED} = \frac{wa^2 b}{l^2} = \frac{16 \times 3^2 \times 1}{4^2} = 9\ kN-m$$

Step ② :- Distribution factor (DF)

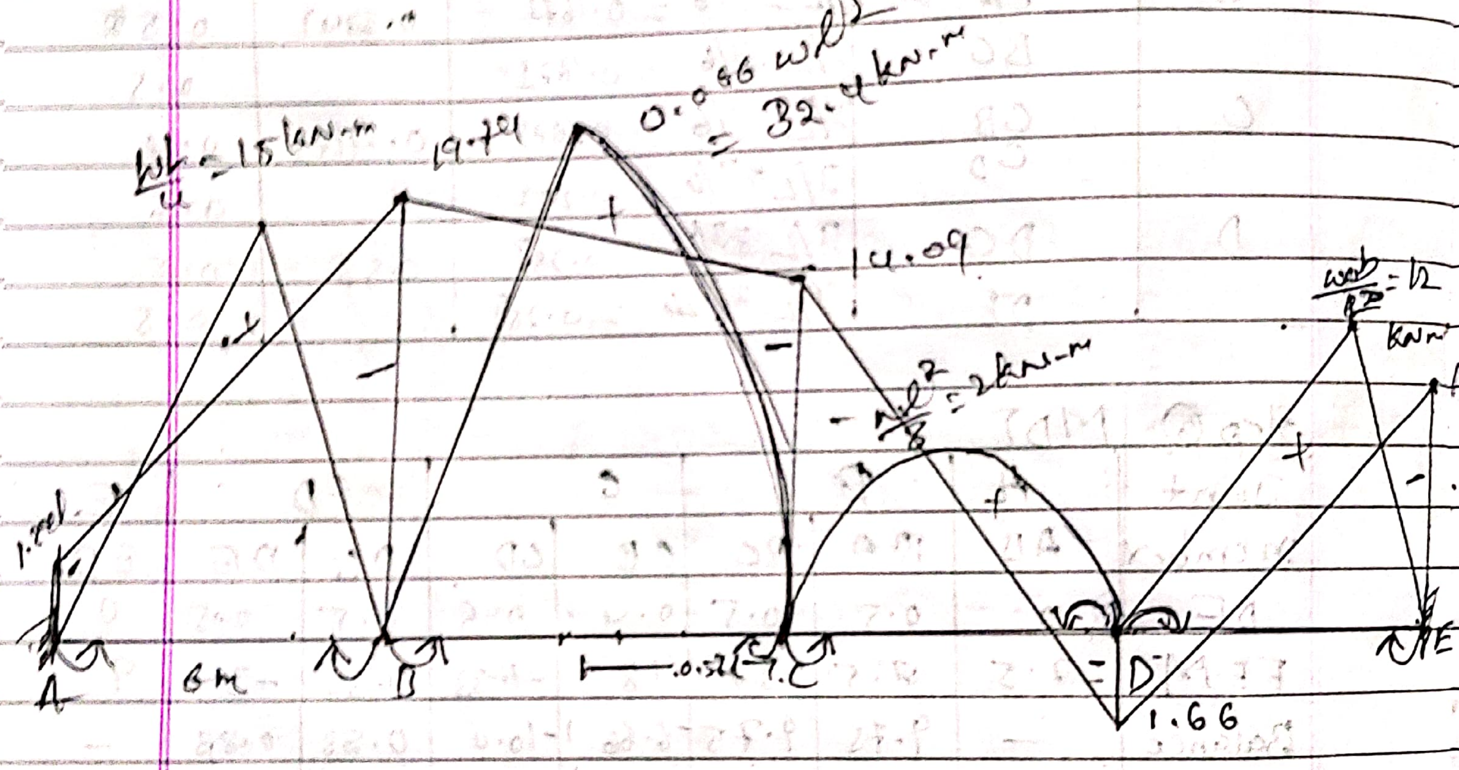
Joint	member	K	ΣK	DF = K/ΣK
B	BA	I/L = I/6 = 0.167I	0.334I	0.5
	BC	I/L = I/6 = 0.167I		0.5
C	CB	I/L = I/6 = 0.167I	0.417I	0.4
	CD	I/L = I/4 = 0.25I		0.6
D	DC	I/L = I/4 = 0.25I	0.5I	0.5
	DE	I/L = I/4 = 0.25I		0.5

Step ③ MDT

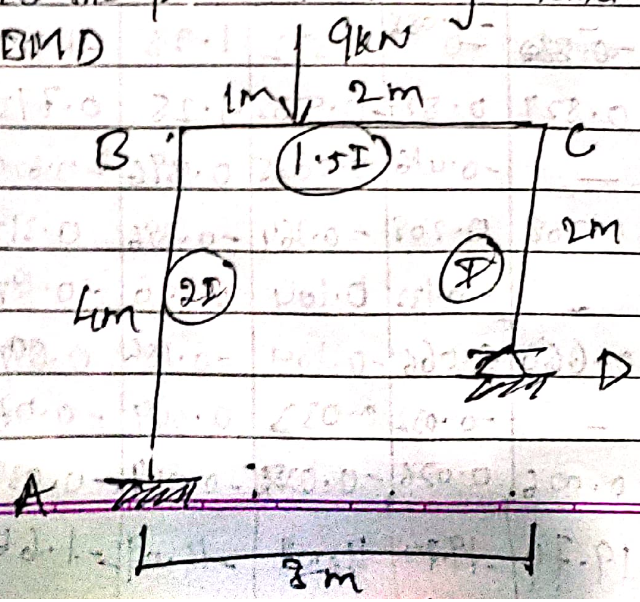
Joint	A	B		C		D		E
member	AB	BA	BC	CB	CD	DC	DE	ED
DF	0	0.5	0.5	0.4	0.6	0.5	0.5	0
FEM	-7.5	7.5	-27	18	-1.33	1.33	-3	9
Balance	—	9.75	9.75	-6.66	-10.0	0.83	0.83	—
Carry over	4.875	—	-3.33	4.875	0.415	-5	—	0.415
Balance	—	1.665	1.665	-2.116	-3.174	2.5	2.5	—
Carryover	0.832	~~0.832~~	-1.058	0.832	1.25	-1.587	—	1.25
Balance	—	0.529	0.529	-0.832	-1.25	0.793	0.793	—
Carryover	0.2645	—	-0.416	0.2645	0.396	-0.625	—	0.396
Balance	—	0.208	0.208	-0.264	-0.396	0.312	0.312	—
Carryover	0.104	—	-0.132	0.104	0.156	-0.128	—	0.156
Balance		0.066	0.066	-0.104	-0.156	0.099	0.099	—
Carryover	0.033	—	-0.052	0.033	0.049	-0.078	—	0.049
Balance	—	0.026	0.026	-0.033	-0.049	-0.039	0.039	—
Final moment	-1.391	19.74	-19.74	14.09	-14.09	-1.66	1.56	11.26

Final moments

$M_{AB} = -1.391$ kN-m
$M_{BA} = 19.74$ kN-m
$M_{BC} = -19.74$ kN-m
$M_{CB} = 14.09$ kN-m
$M_{CD} = -14.09$ kN-m
$M_{DC} = -1.66$ kN-m
$M_{DE} = 1.86$ kN-m
$M_{ED} = 11.26$ kN-m



(16) Analyze the portal frame by moment-distribution method draw BMD



Step ① F.E.M

$M_{FAB} = M_{FBA} = 0$

$M_{FCD} = M_{FDC} = 0$

$M_{FBC} = -\frac{wab^2}{l^2} = -\frac{9\times1\times2^2}{3^2} = -4 kN\text{-}m$

$M_{FCB} = \frac{wa^2b}{l^2} = \frac{9\times1^2\times2}{3^2} = 2 kN\text{-}m$

Step ② DF

Joint	member	K	ΣK	DF = K/ΣK
B	BA	$I/L = I/4 = 0.25I$	0.583I	0.42
	BC	$I/L = I/3 = 0.33I$		0.58
C	CB	$I/L = I/3 = 0.33I$	0.705I	0.46
	CD	$3/4 \times I/L = \frac{3}{4}\times\frac{I}{2} = 0.375I$		0.54

Non-Sway moments

Joint	A	B		C		D
member	AB	BA	BC	CB	CD	DC
DF	–	0.42	0.58	0.46	0.54	–
F.E.M	0	0	-4	2	0	0
Balance	–	1.68	2.32	-0.92	-1.08	–
Carry over	0.84	–	-0.46	1.16	–	-0.54
Balance	–	0.193	0.2668	-0.533	-0.626	–
Carry over	0.096	–	-0.266	0.1334	–	-0.313
Balance	–	0.111	0.154	-0.061	-0.072	–
Carry over	0.055	–	-0.0305	0.077	–	-0.036
Balance	–	0.0128	0.0176	-0.0354	-0.0415	–
Final moments	0.991	1.996	-1.998	1.821	-1.819	0

Sway Analysis
Assumed sway moments are in joint ratio

$$\frac{M_{BA}}{M_{CD}} = \frac{-6EI\Delta/l^2}{-3EI\Delta/l^2} = \frac{6EI\Delta \times 2}{4^2} \times \frac{2^2}{-3EI\Delta}$$

$$= \frac{-1}{-2}$$

Take sway moment are -10 for AB & BA
Take sway moment are -20 for CD & DC.

MDT for sway

Joint	A	B		C		D
member	AB	BA	BC	CB	CD	DC
DF	–	0.42	0.58	0.46	0.54	–
F.E.M	-10	-10	0	0	-20	-20
Balance	–	4.2	5.8	9.2	~~8~~ 10	+20
carryover	2.1	–	4.6	2.9	–	5.4
Balance	–	-1.93	-2.668	-1.334	-1.566	–
carryover	-0.965	–	-0.667	-1.334	–	-0.783
Balance	–	0.280	0.386	0.613	0.720	–
carr						

Joint	A	B		C		D
member	AB	BA	BC	CB	CD	DC
DF	–	0.42	0.58	0.46	0.54	–
F.E.M	-10	-10	0	0	-20	-20
Release joint					10	+20
Total	-10	-10	0	0	-10	0
Balance	–	4.2	5.8	4.6	5.4	–
carryover	2.1	–	2.3	2.9	–	2.7
Balance	–	-0.966	-1.334	-1.334	-1.566	–

Carry over	0.483	~~0.266~~ –	~~1.334~~ −0.667	−0.667	~~1.5~~ –	−0.763
Balance	–	0.280	0.386	0.306	0.360	–
Carry over	0.14	–	0.153	0.153	–	0.18
Balance	–	−0.064	−0.088	−0.070	−0.082	–
Final moment ~~Carry over~~	−8.243	−6.55	6.55	5.888	−5.888	0

~~$\Sigma M_B = 0$~~

$\Sigma H = H_A + H_D = 0$

$\Sigma M_B = 0$

$H_A \times 4 + m'_{AB} + m'_{BA} + m''_{AB} \times k + m''_{BA} \times k = 0$

$-H_A = \frac{1}{4}\left(0.991 + 1.996 + (-8.243 \times k) + (-6.55) \times k\right)$

~~$-H_A = \frac{1}{4}\left(2.987 - 14.79k\right)$~~

$-H_A = 0.746 - 3.69\,k \Rightarrow H_A = -0.746 + 3.69k$

$\Sigma M_C = 0$

$H_C \times 2 + m'_{CD} + m''_{CD} \times k + m'_{DC} + m''_{DC}\,k = 0$

$-H_C = \frac{1}{2}\left(-1.819 + 0 + (-5.888k) + 0\right)$

$-H_C = -0.909 - 2.944k$

$\Rightarrow H_C = 0.909 + 2.944k$

$H_A + H_D = 0$

$-0.746 + 3.69k + 0.909 + 2.944k = 0$

$+0.163 = -6.634k$

$\boxed{k = -0.024}$

$M_{AB} = m'_{AB} + m''_{AB} \times k$

$= 0.991 + 8.243 \times (-0.024)$

$= 1.185\ kN\text{-}m$

$M_{BA} = 1.996 - 6.55 \times (-0.024)$

$= 2.153\ kN\text{-}m$

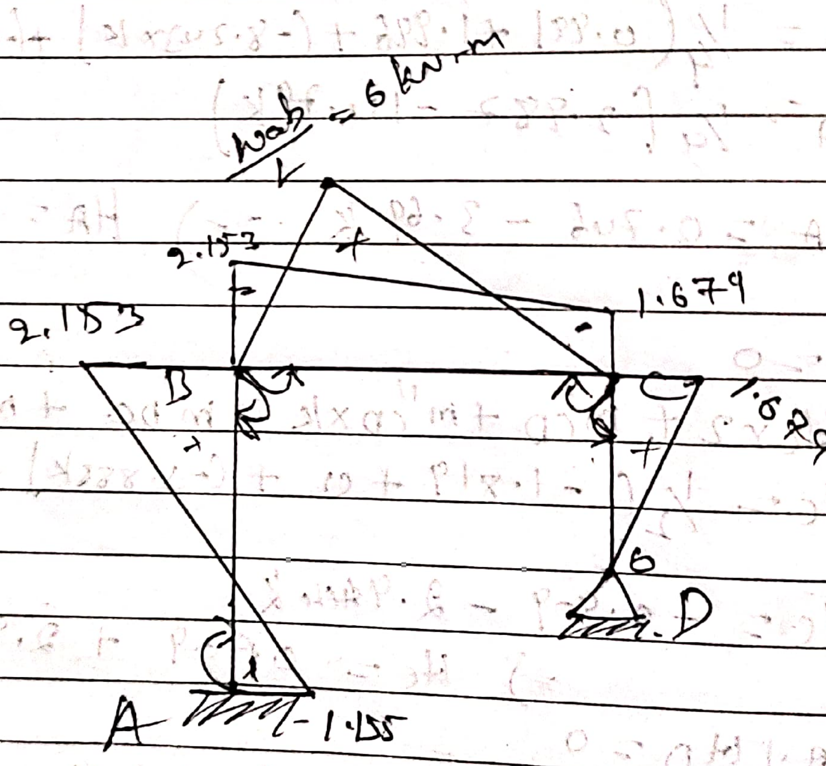
$$M_{BC} = -1.996 + 6.55 \times (-0.024)$$
$$= -2.153 \text{ kN-m}$$

$$M_{CB} = 1.821 + 5.888 \times (-0.024)$$
$$= 1.679 \text{ kN-m}$$

$$M_{CD} = -1.819 - 5.888 \times (-0.024)$$
$$= -1.677 \text{ kN-m}$$

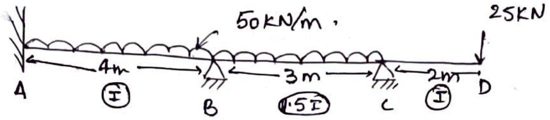
$$M_{DC} = 0$$

BMD



Module - 3

Q5. Analyse the continuous beam loaded shown in Fig Q5 by Kani's rotation method. Draw BMD.



Step 1 - Fixed end moments -

$M_{FAB} = -\frac{wl^2}{12} = -\frac{50\times4^2}{12} = -66.67$

$M_{FBA} = \frac{wl^2}{12} = +\frac{50\times4^2}{12} = 66.67$

$M_{FBC} = -\frac{wl^2}{12} = -\frac{50\times3^2}{12} = -37.5$

$M_{FCB} = \frac{wl^2}{12} = \frac{50\times3^2}{12} = 37.5$

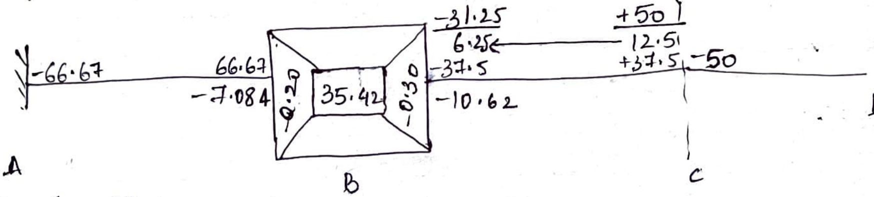
Overhanging -

$M_{CD} = -25\times2 = -50$ KN-m.

Step 2 - Stiffness Factor, distribution Factor

Joint	Member	K	ΣK	$DF = -\frac{1}{2}\frac{K}{\Sigma K}$
B	BA	$\frac{I}{l} = \frac{I}{4} = 0.25I$	0.62I	-0.201
	BC	$\frac{3}{4}\frac{I}{l} = \frac{3\times1.5\,I}{4\times3} = 0.37$		-0.30

Step 3 -



Step 4 - Final moments

$M_{AB} = -66.67 + 2(0) + (-7.084) = -73.75$ KN-m

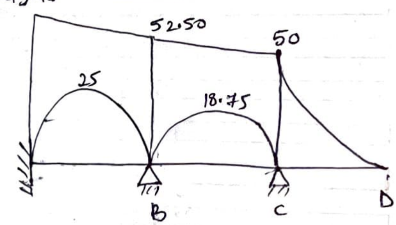
$M_{BA} = 66.67 + 2(-7.084) + 0 = 52.50$ KN-m

$M_{BC} = -31.25 + 2(-10.62) + 0 = -52.49$ KN-m

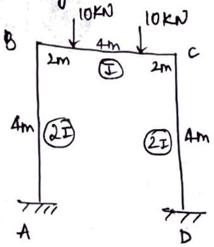
$M_{CB} = +50$ KN-m

$M_{CD} = -50$ KN-m

BMD



Q 6. Analyse the frame shown in fig by Kani's method. Take advantage of symmetry.



Step 1 - Fixed end moments -

$M_{F_{AB}} = 0$ $\quad M_{F_{CD}} = 0$

$M_{F_{BA}} = 0.$ $\quad M_{F_{DC}} = 0$

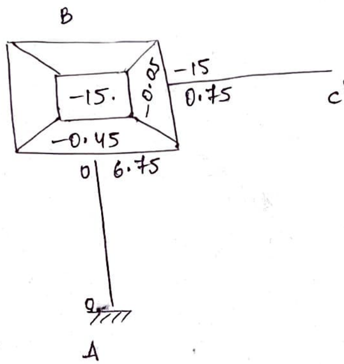
$M_{F_{BC}} = -\frac{wab^2}{l^2} = -\frac{10 \times 2 \times 6^2}{8^2} - \frac{10 \times 6 \times 2^2}{8^2} = -15 \text{ KN-m}$

$M_{F_{CB}} = +\frac{wa^2b}{l^2} = +\frac{10 \times 2^2 \times 6}{8^2} + \frac{10 \times 6^2 \times 2}{8^2} = +15 \text{ KN-m}$

Step 2 - Distribution Factor -

Joint	Member	K	ΣK	$DF = -\frac{1}{2}\frac{K}{\Sigma K}$
B	BA	$\frac{I}{l} = \frac{2I}{4} = 0.5I$	0.56	$-0.45.$
	BC	$\frac{K}{2} = \frac{I}{2l} = \frac{I}{2 \times 8} = 0.06$		$-0.05.$

Step 3 -



Step 4 - Final moments.

$M_{AB} = 0 + 2(0) + 6.75 = 6.75 \text{ KN-m}$

$M_{BA} = 0 + 2(6.75) + 0 = 13.5 \text{ KN-m}$

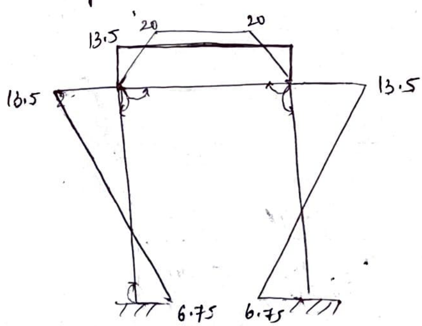
$M_{BC} = -15 + 2(0.75) + 0 = -13.5 \text{ KN-m}$

$M_{CB} = +13.5 \text{ KN-m}$

$M_{CD} = -13.5 \text{ KN-m}$

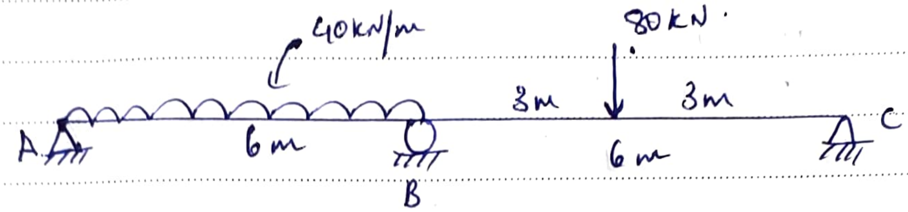
$M_{DC} = -6.75 \text{ KN-m}.$

Step 5 - BMD



A S - MOD-5

9) Analyse the continuous beam by stiffness matrix method shown in fig. Draw BMD. EI is constant.



Step ①: Fixed end moments

$M_{FAB} = -wl^2/12 = -120\,kNm$

$M_{FBA} = wl^2/12 = 120\,kNm$

$M_{FBC} = -Wl/8 = -60\,kNm$

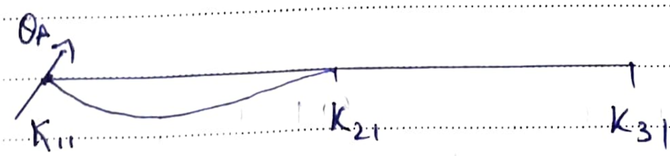
$M_{FCB} = Wl/8 = 60\,kNm$

Step ②: Determine $[\Delta]$, $[P]$, $[P_L]$.

$[\Delta] = \begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \end{bmatrix}$ $\quad P = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad P_L = \begin{bmatrix} M_{FAB} \\ M_{FBA}+M_{FBC} \\ M_{FCB} \end{bmatrix} = \begin{bmatrix} -120 \\ 60 \\ 60 \end{bmatrix}$

Step ③:

Apply unit rotation at A coordinate - ①.

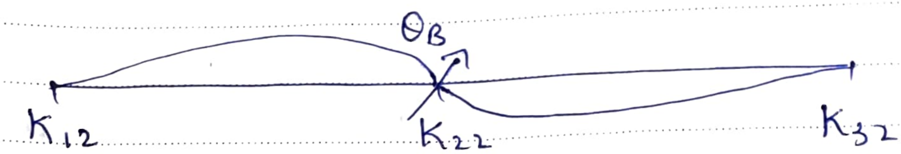


$K_{11} = 4EI/l = 0.667\,EI$

$K_{21} = 2EI/l = 0.334\,EI$

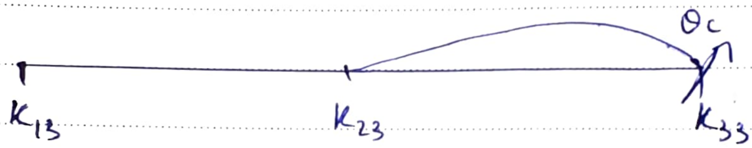
$K_{31} = 0$.

Apply unit rotation at B coordinate (2)



$K_{12} = 2EI/l = 0.334EI.$
$K_{22} = 4EI/l + 4EI/l = 1.334EI.$
$K_{32} = 2EI/l = 0.334EI$

Apply unit rotation at C, coordinate (3)



$K_{13} = 0$
$K_{23} = 2EI/l = 0.334EI$
$K_{33} = 4EI/l = 0.667EI.$

$$[K] = \frac{1}{EI}\begin{bmatrix} 0.667 & 0.334 & 0 \\ 0.334 & 1.334 & 0.334 \\ 0 & 0.334 & 0.667 \end{bmatrix}$$

$$[\Delta] = [K]^{-1}[P - P_L].$$

$$=$$

$$\begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 0.667 & 0.334 & 0 \\ 0.334 & 1.334 & 0.334 \\ 0 & 0.334 & 0.667 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 0+120 \\ 0-60 \\ 0-60 \end{bmatrix}.$$

$\theta_A = \left(\frac{225.02}{EI}\right)$ $\theta_B = \left(\frac{-90.09}{EI}\right)$ $\theta_C = \left(\frac{-44.84}{EI}\right)$

Step (4): Final moments. (SDE)

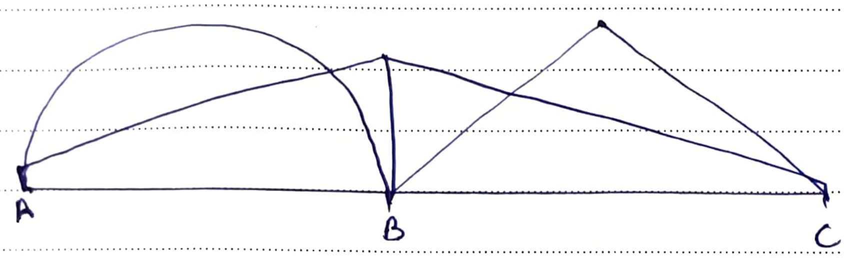
$M_{AB} = -0.01667$ kNm

$M_{BA} = 134.94$ kNm

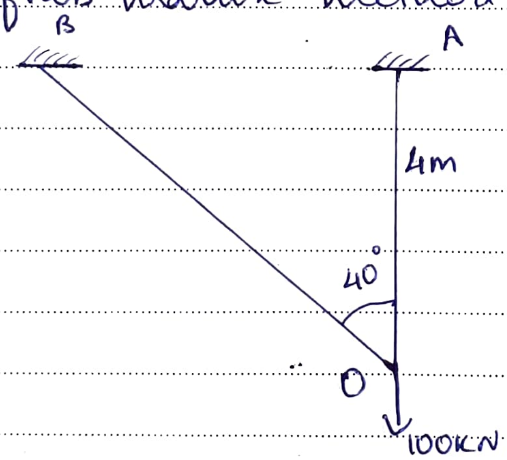
$M_{BC} = -135.00$ kNm.

$M_{CB} = 0.076$ kNm

Step (5): BMD.



(10) Find the forces in the members of a joint 'O' shown in fig. by stiffness matrix method.



A = 200mm² for OA & OB

E = 200kN/mm².

Step (1): Determine KI.

In this problem, A & B are fixed & hence can't move. O is free to move vertically.

$\therefore KI = 1$