CMR INSTITUTE OF TECHNOLOGY

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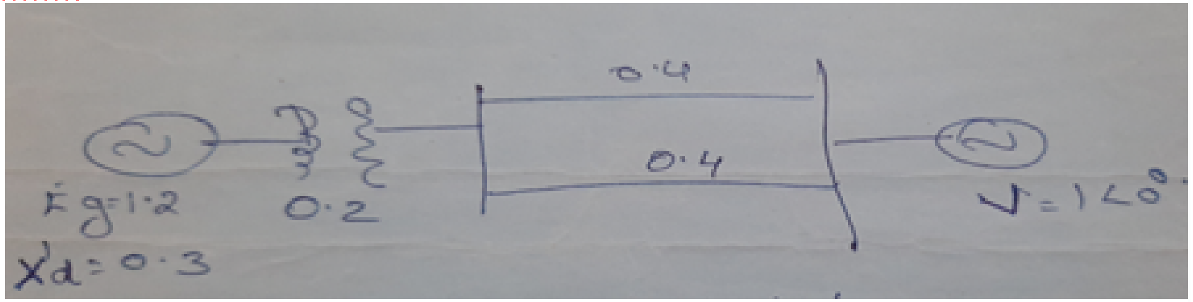
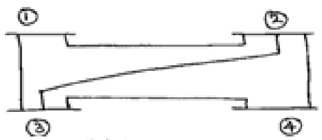
Internal Assesment Test - IV

Sub:	Power System Analysis II							Code:	18EE71/17EE71
Date:	01/02/2022	Duration:	90 mins	Max Marks:	50	Sem:	7	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE	
			CO	RBT
1	A 50 Hz synchronous generator having an inertia constant H=5.2 MJ/MVA and x_d' =0.3 pu is connected to an infinite bus through a double circuit line as shown in fig.1 .The reactance of the connecting HT transformer is 0.2 pu and reactance of each line is 0.4 pu. $\lvert E_g \rvert$ =1.2 pu and $\lvert V \rvert$=1.0 pu and Pe=0.8 pu. Plot the swing curve if a 3 phase fault occurs at the middle of one of the transmission lines by Runge Kutta method. 	[10]	CO6	L4
2	Derive the algorithm for the formation of bus impedance matrix Z_{bus} for a single phase system when a branch element is added to the partial network	[10]	CO5	L3
3	Obtain the Gauss –Seidal load flow solution at the end of first iteration for the power system shown in fig .Assume flat start for bus voltages V_3 and V_4. Given :0.2<=Q2<=1.0	[10]	CO2	L3



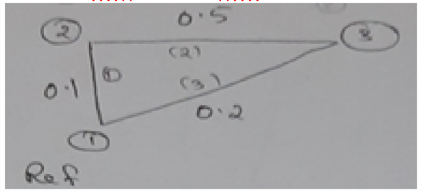
Line Dat

SB	EB	R(p.u)	X(p.u)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

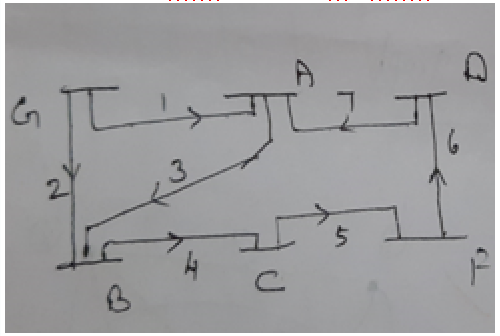
Bus Data

Bus No.	P_i	Q_i	V_i
1	-	-	$1.04\angle 0°$
2	0.5	-	1.04
3	-1.0	0.5	-
4	-0.3	-0.1	-

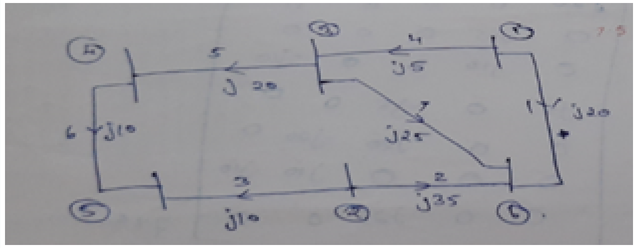
4 Frame Z_{bus} using Z_{bus} building algorithm



5 For the power system shown in fig, assume G as reference and AB and DF as links , prove $A_bK^t=U$ and $B_l=A_lK^t$ [10] CO1 L4



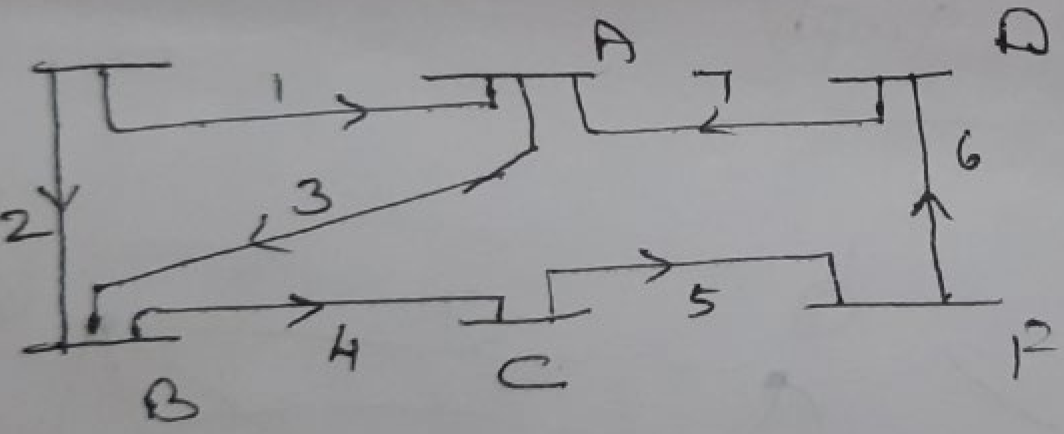
6 For the system given below, determine bus admittance matrix by singular transformation method .select bus 6 as reference and a tree with elements 6 and 7 as links [10] CO1 L4



Line No	1	2	3	4	5	6	7
Bus code p-q	1-6	2-6	2-5	1-3	3-4	4-5	3-6
Admittance in pu	j20	j35	j10	j5	j20	j10	j25

Question 1

For the sample power system shown in fig obtain A, B and C matrices. Assume G as reference bus and AB and DF as links.

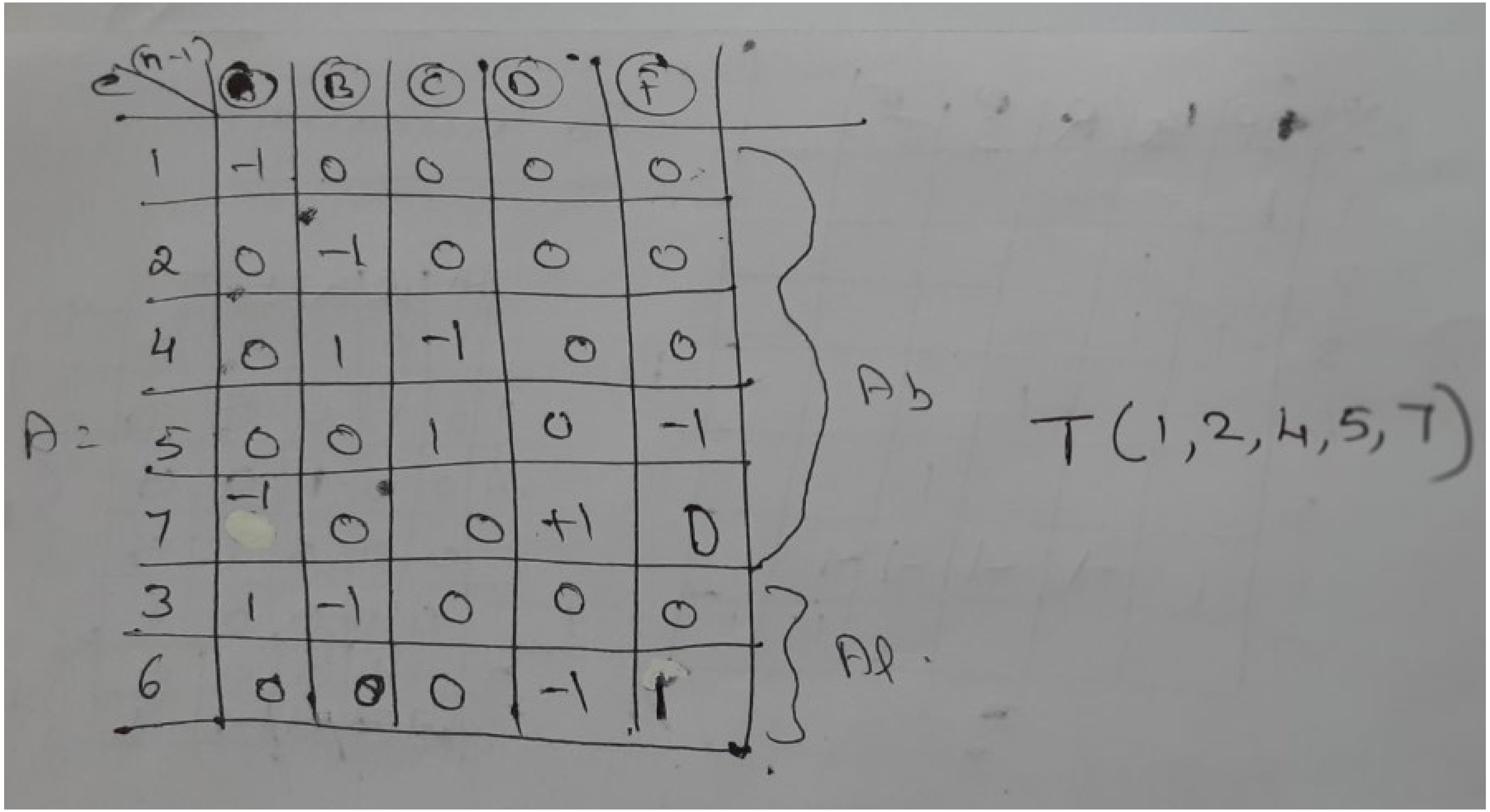


Answer

A =

	A	B	C	D	E
1	-1	0	0	0	0
2	0	-1	0	0	0
3	1	-1	0	0	0
4	0	1	-1	0	0
5	0	0	1	0	-1
6	0	0	0	-1	1
7	-1	0	0	1	0

Rearranging the branches on top & links in the bottom



$A =$

e \ (n-1)	[illegible]	B	C	D	F
1	-1	0	0	0	0
2	0	-1	0	0	0
4	0	1	-1	0	0
5	0	0	1	0	-1
7	-1	0	0	+1	0
3	1	-1	0	0	0
6	0	0	0	-1	1

A_b (rows 1, 2, 4, 5, 7)

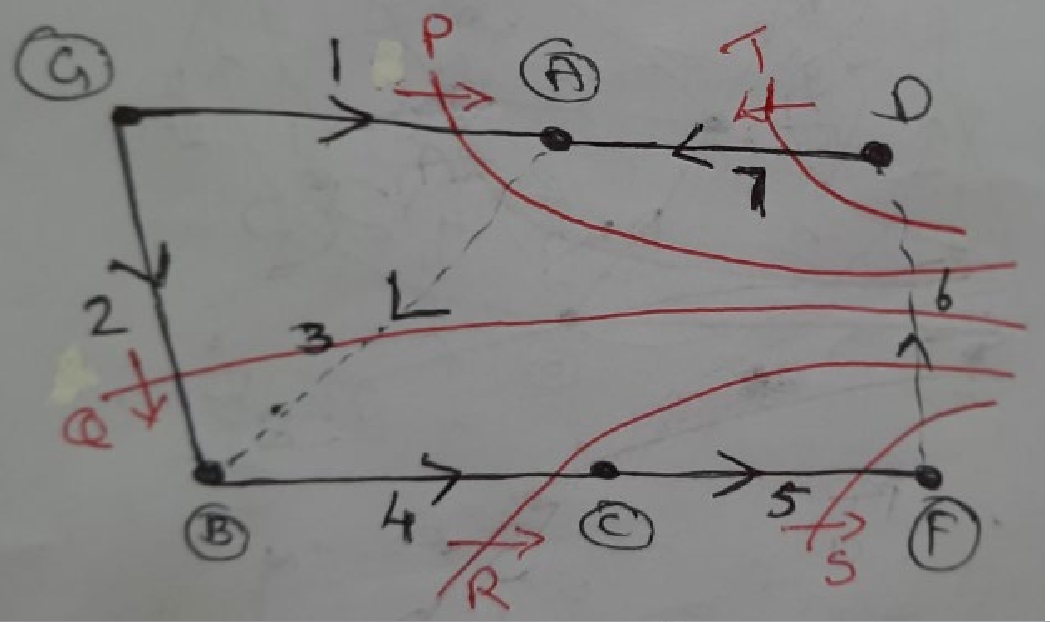
A_l (rows 3, 6)

$T(1, 2, 4, 5, 7)$

$$K = \begin{array}{c|c|c|c|c|c|}
 & A & B & C & D & F \\ \hline
1 & -1 & 0 & 0 & -1 & 0 \\ \hline
2 & 0 & -1 & -1 & 0 & -1 \\ \hline
4 & 0 & 0 & -1 & 0 & -1 \\ \hline
5 & 0 & 0 & 0 & 0 & -1 \\ \hline
7 & 0 & 0 & 0 & 1 & 0 \\ \hline
\end{array}$$

$$A_b \cdot K^t = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \underline{\underline{U}}$$



$$
B = \begin{array}{c|ccccc}
e \backslash b & P & Q & R & S & T \\
\hline
1 & 1 & & & & \\
2 & & 1 & & & \\
3 & -1 & 1 & & & \\
4 & & & 1 & & \\
5 & & & & 1 & \\
6 & 1 & -1 & -1 & -1 & -1 \\
7 & & & & & +1
\end{array}
$$

Re arranging

$$
B = \begin{array}{c|ccccc}
 & P & Q & R & S & T \\
\hline
1 & 1 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 \\
4 & 0 & 0 & 1 & 0 & 0 \\
5 & 0 & 0 & 0 & 1 & 0 \\
7 & 0 & 0 & 0 & 0 & 1 \\
3 & -1 & 1 & 0 & 0 & 0 \\
6 & -1 & -1 & -1 & -1 & -1
\end{array}
\begin{array}{l}
\left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right\} U_b \\
\left.\begin{array}{c} \\ \\ \end{array}\right\} B_l
\end{array}
$$

$B_l = A_l \cdot K^t$

$$B_l = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

$$A_l \cdot K^t = \begin{bmatrix} +1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & -1 & 0 \end{bmatrix}$$

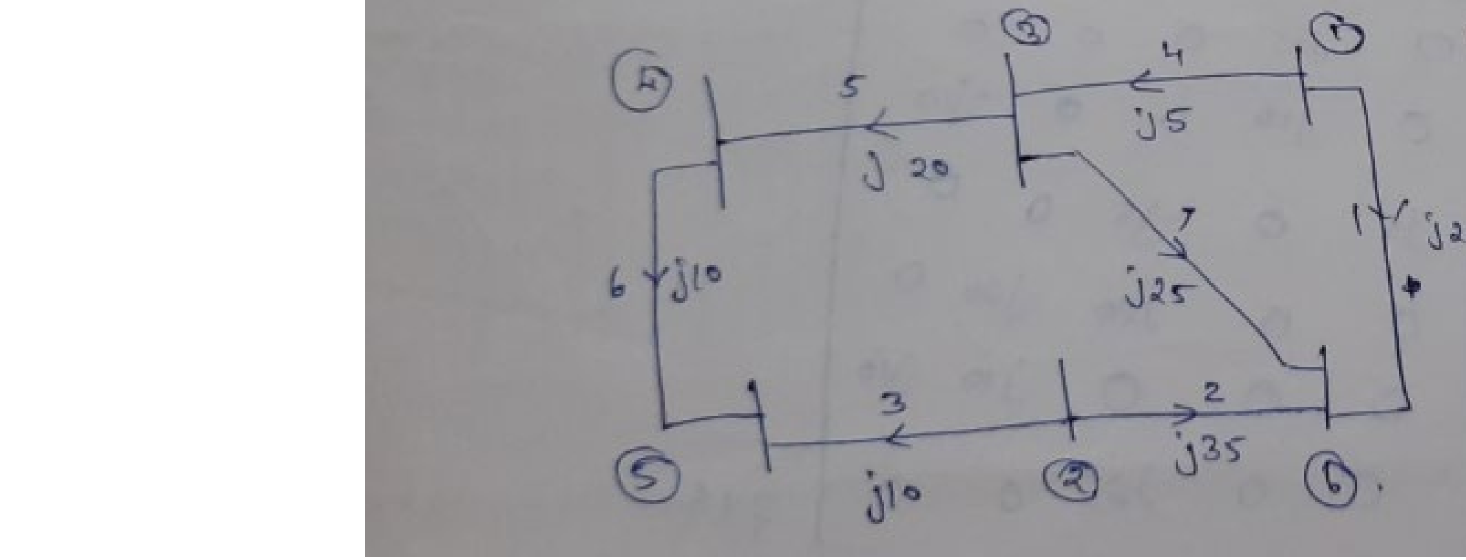
$$= \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

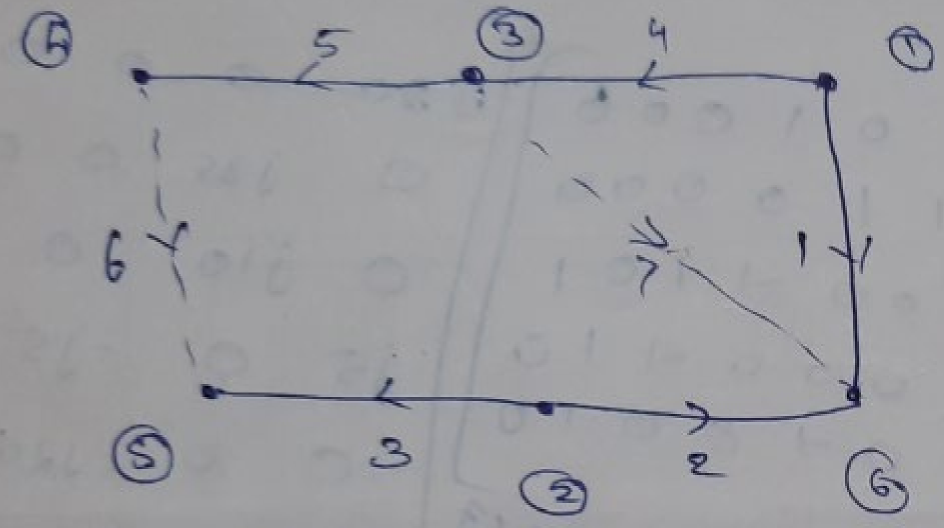
$\therefore B_l = A_l K^t$

Question 2

a) For the system defined by the line data shown in fig, determine the bus admittance matrix by singular transformation. Select bus ⑥ as reference and a tree with elements 6 and 1 as links.

Line No	1	2	3	4	5	6	7
Bus code p-q	1-6	2-6	2-5	1-3	3-4	4-5	3-6
Admittance in pu	j20	-j35	j10	j5	j20	j10	j25





$A =$

	①	②	③	④	⑤
1	1	0	0	0	0
2	0	1	0	0	0
3	0	1	0	0	-1
4	1	0	-1	0	0
5	0	0	1	-1	0
6	0	0	0	1	-1
7	0	0	1	0	0

$[y]$

	1	2	3	4	5	6	7
1	j20	0	0	0	0	0	0
2	0	j35	0	0	0	0	0
3	0	0	j10	0	0	0	0
4	0	0	0	j5	0	0	0
5	0	0	0	0	j20	0	0
6	0	0	0	0	0	j10	0
7	0	0	0	0	0	0	j25

$$\underline{\underline{y \cdot A}} = \underbrace{\begin{bmatrix} j20 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & j35 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & j10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & j5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & j10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & j25 \end{bmatrix}}_{7\times 7} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}}_{7\times 5}$$

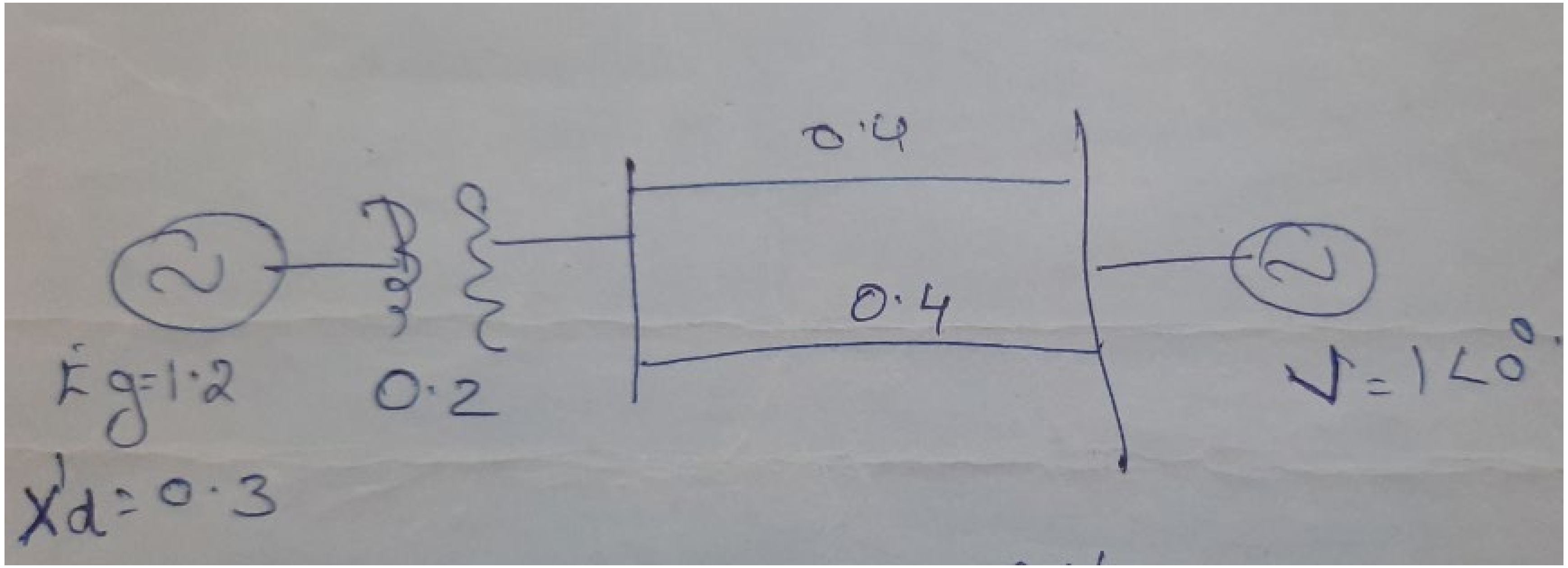
$$= \begin{bmatrix} j20 & 0 & 0 & 0 & 0 \\ 0 & j35 & 0 & 0 & 0 \\ 0 & j40 & 0 & 0 & -j10 \\ j5 & 0 & -j5 & 0 & 0 \\ 0 & 0 & j20 & -j20 & 0 \\ 0 & 0 & 0 & j10 & -j10 \\ 0 & 0 & j25 & 0 & 0 \end{bmatrix}_{7 \times 5}$$

$$\tilde{A}\,y\,A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 \end{bmatrix}_{5\times 7} \begin{bmatrix} j20 & 0 & 0 & 0 & 0 \\ 0 & j35 & 0 & 0 & 0 \\ 0 & j10 & 0 & 0 & -j10 \\ j5 & 0 & -j5 & 0 & 0 \\ 0 & 0 & j20 & -j20 & 0 \\ 0 & 0 & 0 & j10 & -j10 \\ 0 & 0 & j25 & 0 & 0 \end{bmatrix}_{5\times 6}$$

$$= \begin{bmatrix} j25 & 0 & -j5 & 0 & 0 \\ 0 & j45 & 0 & 0 & -j10 \\ -j5 & 0 & j50 & -j20 & 0 \\ 0 & 0 & -j20 & j30 & -j10 \\ 0 & -j10 & 0 & -j10 & j20 \end{bmatrix}$$

Question 3)

Example :A 50 Hz, synchronous generator having inertia constant H = 5.2 MJ/MVA and $x_d' = 0.3$ pu is connected to an infinite bus through a double circuit line as shown in Fig. 9.21. The reactance of the connecting HT transformer is 0.2 pu and reactance of each line is 0.4 pu. $|E_g| = 1.2$ pu and $|V| = 1.0$ pu and $P_e = 0.8$ pu. Obtain the swing curve using Runge Kutta method for a three phase fault occurs at the middle of one of the transmission lines and is cleared by isolating the faulted line.



using Runge Kutta method

The two first order differential equations to be solved to obtain solution for the swing equation are:

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{M} = \frac{P_m - P_{max} \sin \delta}{M}$$

Starting from initial value δ_0, ω_0, t_0 and a step size of Δt the formulae are as follows

$$k_1 = \omega_0 \; \Delta t$$

$$l_1 = \left[\frac{P_m - P_{max} \sin \delta_0}{M}\right] \Delta t$$

$$k_2 = \left(\omega_0 + \frac{l_1}{2}\right) \Delta t$$

$$l_2 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{k_1}{2}\right)}{M}\right] \Delta t$$

$$k_3 = \left(\omega_0 + \frac{l_2}{2}\right) \Delta t$$

$$l_3 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{k_2}{2}\right)}{M}\right] \Delta t$$

$$k_4 = (\omega_0 + l_3) \; \Delta t$$

$$l_4 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + k_3\right)}{M}\right] \Delta t$$

$$\delta_1 = \delta_0 + \frac{1}{6} \; [k_1 + 2k_2 + 2k_3 + k_4]$$

$$\omega_1 = \omega_0 + \frac{1}{6} \; [l_1 + 2l_2 + 2l_3 + l_4]$$

$\delta_0 = 27.8$, $\omega_0 = 0$.

$K_1 = 0 \times .05 = 0$, $l_1 = \left[\frac{0.8 - 0.63 \sin \delta_0}{.000544}\right] .05 = 46.52$.

$K_2 = \left(0 + \frac{46.52}{2}\right) .05 = \underline{\underline{1.163}}$ $\quad l_2 = \left[\frac{0.8 - 0.63 \sin\left(27.8 + \frac{0}{2}\right)}{.000544}\right] .05$

$= \underline{\underline{46.52}}$

$K_3 = \left(0 + \frac{46.52}{2}\right) .05 = 1.163$ $\quad l_3 = \left[\frac{0.8 - 0.63 \sin\left(27.8 + \frac{1.163}{2}\right)}{.000544}\right] .05$

$= 46$

$K_4 = (0 + 46) .05 = 2.3$ $\quad l_4 = \left[\frac{0.8 - 0.63 \sin(27.8 + 1.163)}{.000544}\right] .05 = \underline{\underline{45.54}}$

$\delta_1 = 27.8 + \frac{1}{6}\left[0 + 2 \times 1.163 + 2 \times 1.163 + 2.3\right] = \underline{\underline{28.9}}$

$\omega_1 = 0 + \frac{1}{6}\left[46.5 + 2 \times 46.52 + 2 \times 46 + 45.54\right] = \underline{\underline{46.18}}$

$$K_1 = 46.18 \times .05 = 2.309 \qquad l_1 = \left[\frac{0.8 - 0.63 \sin 28.9}{.000544}\right].05 = \underline{\underline{45.54}}$$

$$K_2 = \left(46.18 + \frac{45.54}{2}\right).05 = \underline{\underline{3.4475}} \qquad l_2 = \left[\frac{0.8 - 0.63 \sin\left(28.9 + \frac{2.309}{2}\right)}{.000544}\right].05 = \underline{\underline{44.53}}$$

$$K_3 = \cancel{2.309}\left(46.18 + \frac{44.53}{2}\right).05 = \underline{\underline{3.422}} \qquad l_3 = \left(\frac{0.8 - 0.63 \sin\left(28.9 + \frac{3.4475}{2}\right)}{.000544}\right).05 = \underline{\underline{44.64}}$$

$$K_4 = (46.18 + 44.04).05 = \underline{\underline{4.5112}} \qquad l_4 = \left(\frac{0.8 - 0.63 \sin(28.9 + 3.422)}{.000544}\right).05 = 42.56$$

$$\delta_2 = 28.9 + \frac{1}{6}\left[2.309 + 2 \times 3.4475 + 2 \times 3.422 + 4.5112\right] = \underline{\underline{32.326}}$$

$$\omega_2 = 46.18 + \frac{1}{6}\left[45.54 + 2 \times 44.53 + 2 \times 44.04 + 42.56\right] = \underline{\underline{90.386}}$$

Question 4

For the sample system of Fig. 6.5 the generators are connected at all the four buses, while loads are at buses 2 and 3. Values of real and reactive powers are listed in Table 6.3. All buses other than the slack are *PQ* type.

Assuming a flat voltage start, find the voltages and bus angles at the three buses at the end of the first GS iteration.

Solution

Table 6.3 Input data

Bus	P_i, pu	Q_i, pu	V_i, pu	*Remarks*
1	–	–	$1.04 \angle 0°$	Slack bus
2	0.5	– 0.2	–	*PQ* bus
3	– 1.0	0.5	–	*PQ* bus
4	0.3	– 0.1	–	*PQ* bus

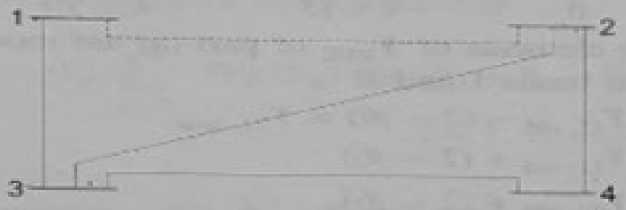


Fig. 6.5 Sample system for Example 6.2

Table 6.1

Line, bus to bus	*R*, pu	*X*, pu
1–2	0.05	0.15
1–3	0.10	0.30
2–3	0.15	0.45
2–4	0.10	0.30
3–4	0.05	0.15

Solution

$$Y_{BUS} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

$$V^1_2 = \frac{1}{Y_{22}}\left\{\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right\}$$

$$= \frac{1}{Y_{22}}\left\{\frac{0.5 + j0.2}{1 - j0} - 1.04(-2 + j6) - (-0.666 + j2) - (-1 + j3)\right\}$$

$$= \frac{4.246 - j11.04}{3.666 - j11} = 1.019 + j0.046 \text{ pu}$$

$$V_3^1 = \frac{1}{Y_{33}} \left\{ \frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31} V_1 - Y_{32} V_2^1 - Y_{34} V_4^0 \right\}$$

$$= \frac{1}{Y_{33}} \left\{ \frac{-1 - j0.5}{1 - j0} - 1.04\,(-1 + j3) \right.$$

$$\left. - (-0.666 + j2)\,(1.019 + j0.046) - (-2 + j6) \right\}$$

$$= \frac{2.81 - j11.627}{3.666 - j11} = 1.028 - j0.087 \text{ pu}$$

$$V_4^1 = \frac{1}{Y_{44}} \left\{ \frac{P_4 - jQ_4}{\left(V_4^0\right)^*} - Y_{41} V_1 - Y_{42} V_2^1 - Y_{43} V_3^1 \right\}$$

$$= \frac{1}{Y_{44}} \left\{ \frac{0.3 + j0.1}{1 - j0} - (-1 + j3)\,(1.019 + j0.046) \right.$$

$$\left. - (-2 + j6)\,(1.028 - j0.087) \right\}$$

$$= \frac{2.991 - j9.253}{3 - j9} = 1.025 - j0.0093 \text{ pu}$$

In Example 6.4, let bus 2 be a PV bus now with $|V_2| = 1.04$ pu. Once again assuming a flat voltage start, find Q_2, δ_2, V_3, V_4 at the end of the first GS iteration.

Given: $0.2 \leq Q_2 \leq 1$.

From Eq. (6.5), we get (*Note* $\delta_2^0 = 0$, i.e. $V_2^0 = 1.04 + j0$)

$$Q_2^1 = -\,\mathrm{Im}\,\{(V_2^0)^* Y_{21} V_1 + (V_2^0)^*[Y_{22} V_2^0 + Y_{23} V_3^0 + Y_{24} V_4^0]\}$$

$$= -\,\mathrm{Im}\,\{1.04\,(-2 + j6)\,1.04 + 1.04\,[(3.666 - j11)\,1.04 + (-0.666 + j2) + (-1 + j3)]\}$$

$$= -\,\mathrm{Im}\,\{-0.0693 - j0.2079\} = 0.2079 \text{ pu}$$

$$\therefore\ Q_2^1 = 0.2079 \text{ pu}$$

From Eq. (6.51)

$$\delta_2^1 = \angle\left\{\frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2^1}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]\right\}$$

$$= \angle\left\{\frac{1}{3.666 - j11}\left[\frac{0.5 - j0.2079}{1.04 - j0} - (-2 + j6)(1.04 + j0)\right.\right.$$

$$\left.\left. - (-0.666 + j2)(1 + j0) - (-1 + j3)(1 + j0)\right]\right\}$$

$$= \angle\left(\frac{4.2267 - j11.439}{3.666 - j11}\right) = \angle(1.0512 + j0.0339)$$

or $\delta_2^1 = 1.84658° = 0.032$ rad

$$\therefore V_2^1 = 1.04\,(\cos\ \delta_2^1 + j\sin\ \delta_2^1)$$

$$= 1.04\,(0.99948 + j0.0322)$$

$$= 1.03946 + j0.03351$$

$$V_3^1 = \frac{1}{Y_{33}}\left\{\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0\right\}$$

$$= \frac{1}{3.666 - j11}\left[\frac{-1 - j0.5}{(1 - j0)} - (-1 + j3)\,1.04\right.$$

$$\left. - (-0.666 + j2)(1.03946 + j0.03351) - (-2 + j6)\right]$$

$$= \frac{2.7992 - j11.6766}{3.666 - j11} = 1.0317 - j0.08937$$

$$V_4^1 = \frac{1}{Y_{44}} \left\{ \frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41} V_1 - Y_{42} V_2^1 - Y_{43} V_3^1 \right\}$$

$$= \frac{1}{3 - j9} \left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j3)(1.0394 + j0.0335) - (-2 + j6)(1.0317 - j0.08937) \right]$$

$$= \frac{2.9671 - j8.9962}{3 - j9} = 0.9985 - j0.0031$$

$$0.25 \le Q_2 \le 1.0 \text{ pu}$$

It is clear, that other data remaining the same, the calculated Q_2 (= 0.2079) is now less than the $Q_{2,\,min}$. Hence Q_2 is set equal to $Q_{2,\,min}$, i.e.

$$Q_2 = 0.25 \text{ pu}$$

Bus 2, therefore, becomes a *PQ* bus from a *PV* bus. Therefore, $|V_2|$ can no longer remain fixed at 1.04 pu. The value of V_2 at the end of the first iteration is calculated as follows. (*Note* $V_2^0 = 1 + j0$ by virtue of a flat start.)

$$V_2^1 = \frac{1}{Y_{22}}\left(\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right)$$

$$= \frac{1}{3.666 - j11}\left[\frac{0.5 - j0.25}{1 - j0}\right.$$

$$\left. - (-2 + j6)1.04 - (-0.666 + j2) - (-1 + j3)\right]$$

$$= \frac{4.246 - j11.49}{3.666 - j11} = 1.0559 + j0.0341$$

$$V_3^1 = \frac{1}{Y_{33}}\left(\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0\right)$$

$$= \frac{1}{3.666 - j11}\left[\frac{-1 - j0.5}{1 - j0} - (-1 + j3)\ 1.04\right.$$

$$\left. - (-0.666 + j2)(1.0559 + j0.0341) - (-2 + j6)\right]$$

$$= \frac{2.8112 - j11.709}{3.666 - j11} = 1.0347 - j0.0893 \text{ pu}$$

$$V_4^1 = \frac{1}{Y_{44}}\left(\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 - Y_{42}V_2^1 - Y_{43}V_3^1\right)$$

$$= \frac{1}{3 - j9}\left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j3)\ (1.0509 + j0.0341)\right.$$

$$\left. - (-2 + j6)\ (1.0347 - j0.0893)\right]$$

$$= \frac{4.0630 - j9.4204}{3 - j9} = 1.0775 + j0.0923 \text{ pu}$$

Question 5

Derive the algorithm for the formation of bus impedance matrix Z_{bus} for a single phase system when a branch element is added to the partial network.

Z bus Algorithm

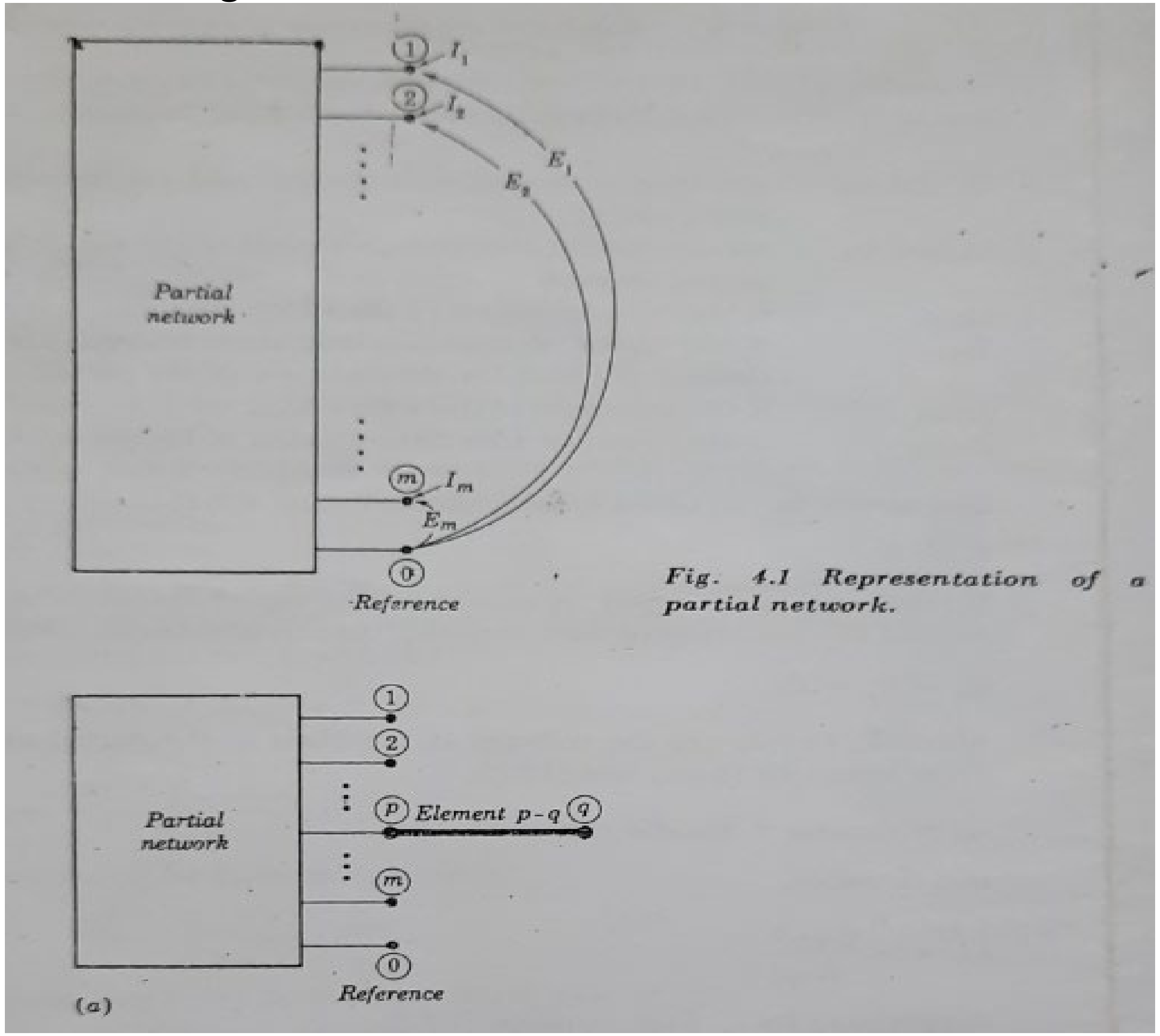
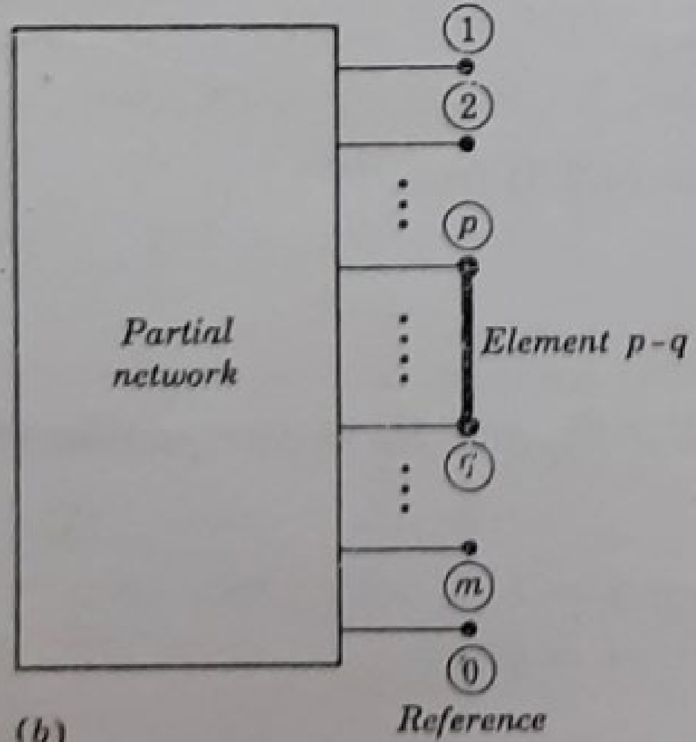


Fig. 4.1 Representation of a partial network.



(b)

Fig. 4.2 **Representations of a partial network with an added element. (a) Addition of a branch; (b) addition of a link.**

Addition of a branch

The performance equation for the partial network with an added branch p–q is

$$
\begin{bmatrix} E_1 \\ E_2 \\ \cdots \\ E_p \\ \cdots \\ E_m \\ E_q \end{bmatrix}
=
\begin{array}{c|ccccccc|}
 & 1 & & & p & & m & q \\
\hline
1 & Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\
 & Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
p & Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
m & Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\
q & Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \\
\hline
\end{array}
\begin{bmatrix} I_1 \\ I_2 \\ \cdots \\ I_p \\ \cdots \\ I_m \\ I_q \end{bmatrix}
\qquad (4.2.1)
$$

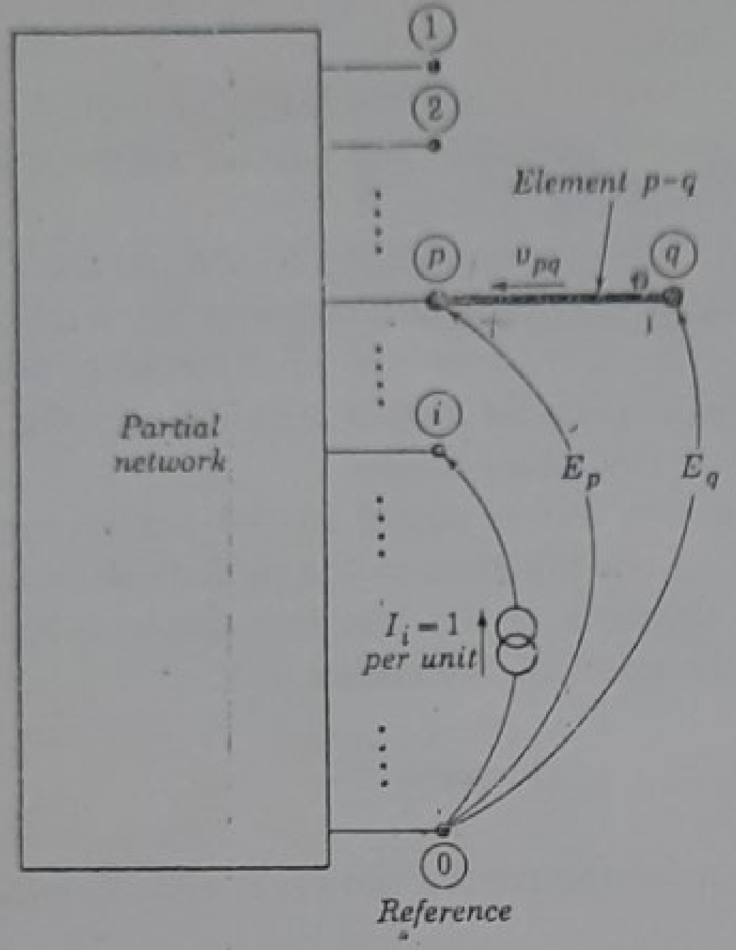


Fig. 4.3 Injected current and bus voltages for calculation of Z_{qi}.

node as shown in Fig. 4.3. Since all other bus currents equal zero, it follows from equation (4.2.1) that

$$\begin{aligned} E_1 &= Z_{1i} I_i \\ E_2 &= Z_{2i} I_i \\ &\cdots\cdots \\ E_p &= Z_{pi} I_i \\ &\cdots\cdots \\ E_m &= Z_{mi} I_i \\ E_q &= Z_{qi} I_i \end{aligned} \qquad (4.2.2)$$

Letting $I_i = 1$ per unit in equations (4.2.2), Z_{qi} can be obtained directly by calculating E_q.

The bus voltages associated with the added element and the voltage across the element are related by

$$E_q = E_p - v_{pq} \qquad (4.2.3)$$

The currents in the elements of the network in Fig. 4.3 are expressed in terms of the primitive admittances and the voltages across the elements by

$$\begin{bmatrix} i_{pq} \\ i_{\rho\sigma} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & y_{pq,\rho\sigma} \\ y_{\rho\sigma,pq} & y_{\rho\sigma,\rho\sigma} \end{bmatrix} \begin{bmatrix} v_{pq} \\ v_{\rho\sigma} \end{bmatrix} \qquad (4.2.4)$$

i_{pq} and v_{pq}	are, respectively, current through and voltage across the added element
$\bar{i}_{\rho\sigma}$ and $\bar{v}_{\rho\sigma}$	are the current and voltage vectors of the elements of the partial network
$y_{pq,pq}$	is the self-admittance of the added element
$\bar{y}_{pq,\rho\sigma}$	is the vector of mutual admittances between the added element p–q and the elements ρ–σ of the partial network
$\bar{y}_{\rho\sigma,pq}$	is the transpose of the vector $\bar{y}_{pq,\rho\sigma}$
$[y_{\rho\sigma,\rho\sigma}]$	is the primitive admittance matrix of the partial network

The current in the added branch, shown in Fig. 4.3, is

$$i_{pq} = 0 \tag{4.2.5}$$

However v_{pq} is not equal to zero since the added branch is mutually coupled to one or more of the elements of the partial network. Moreover,

$$\bar{v}_{\rho\sigma} = \bar{E}_\rho - \bar{E}_\sigma \tag{4.2.6}$$

where $\bar{E}_\rho$ and $\bar{E}_\sigma$ are the voltages at the buses in the partial network. From equations (4.2.4) and (4.2.5),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma} = 0$$

and therefore,

$$v_{pq} = -\frac{\bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma}}{y_{pq,pq}}$$

Substituting for $\bar{v}_{\rho\sigma}$ from equation (4.2.6),

$$v_{pq} = -\frac{\bar{y}_{pq,\rho\sigma} (\bar{E}_\rho - \bar{E}_\sigma)}{y_{pq,pq}} \tag{4.2.7}$$

Substituting for v_{pq} in equation (4.2.3) from (4.2.7),

$$E_q = E_p + \frac{\bar{y}_{pq,\rho\sigma} (\bar{E}_\rho - \bar{E}_\sigma)}{y_{pq,pq}}$$

Finally, substituting for E_q, E_p, $\bar{E}_\rho$, and $\bar{E}_\sigma$ from equation (4.2.2) with $I_i = 1$,

$$Z_{qi} = Z_{pi} + \frac{\bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho i} - \bar{Z}_{\sigma i})}{y_{pq,pq}} \qquad \begin{array}{l} i = 1, 2, \ldots, m \\ i \neq q \end{array} \tag{4.2.8}$$

The element Z_{qq} can be calculated by injecting a current at the qth bus and calculating the voltage at that bus. Since all other bus currents equal zero, it follows from equation (4.2.1) that

$$\begin{aligned} E_1 &= Z_{1q} I_q \\ E_2 &= Z_{2q} I_q \\ &\cdots\cdots \\ E_p &= Z_{pq} I_q \\ &\cdots\cdots \\ E_m &= Z_{mq} I_q \\ E_q &= Z_{qq} I_q \end{aligned} \tag{4.2.9}$$

Letting $I_q = 1$ per unit in equations (4.2.9), Z_{qq} can be obtained directly by calculating E_q.

The voltages at buses p and q are related by equation (4.2.3), and the current through the added element is

$$i_{pq} = -I_q = -1 \tag{4.2.10}$$

The voltages across the elements of the partial network are given by equation (4.2.6) and the currents through these elements by (4.2.4). From equations (4.2.4) and (4.2.10),

$$i_{pq} = y_{pq,pq}v_{pq} + \bar{y}_{pq,\rho\sigma}\bar{v}_{\rho\sigma} = -1$$

and therefore,

$$v_{pq} = -\frac{1 + \bar{y}_{pq,\rho\sigma}\bar{v}_{\rho\sigma}}{y_{pq,pq}}$$

Substituting for $\bar{v}_{\rho\sigma}$ from equation (4.2.6),

$$v_{pq} = -\frac{1 + \bar{y}_{pq,\rho\sigma}(\bar{E}_\rho - \bar{E}_\sigma)}{y_{pq,pq}} \tag{4.2.11}$$

Substituting for v_{pq} in equation (4.2.3) from (4.2.11),

$$E_q = E_p + \frac{1 + \bar{y}_{pq,\rho\sigma}(\bar{E}_\rho - \bar{E}_\sigma)}{y_{pq,pq}}$$

Finally, substituting for E_q, E_p, $\bar{E}_\rho$, and $\bar{E}_\sigma$ from equation (4.2.9) with $I_q = 1$,

$$Z_{qq} = Z_{pq} + \frac{1 + \bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho q} - \bar{Z}_{\sigma q})}{y_{pq,pq}} \tag{4.2.12}$$

If there is no mutual coupling between the added branch and other elements of the partial network, then the elements of $\bar{y}_{pq,\rho\sigma}$ are zero and

$$z_{pq,pq} = \frac{1}{y_{pq,pq}}$$

It follows from equation (4.2.8) that

$$Z_{qi} = Z_{pi} \qquad \begin{matrix} i = 1, 2, \ldots, m \\ i \neq q \end{matrix}$$

and from equation (4.2.12) that

$$Z_{qq} = Z_{pq} + z_{pq,pq}$$

Furthermore, if there is no mutual coupling and p is the reference node,

$$Z_{pi} = 0 \qquad \begin{matrix} i = 1, 2, \ldots, m \\ i \neq q \end{matrix}$$

and

$$Z_{qi} = 0 \qquad \begin{matrix} i = 1, 2, \ldots, m \\ i \neq q \end{matrix}$$

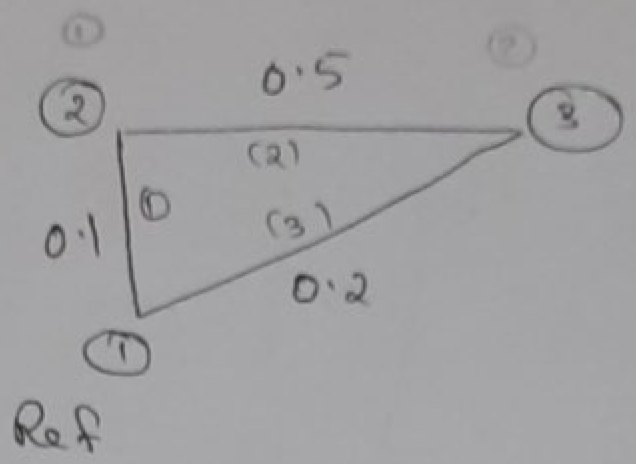
Also

$$Z_{pq} = 0$$

and therefore,

$$Z_{qq} = z_{pq,pq}$$

Question 6 :Form Zbus using Zbus building algorithm



Branch

$Z_{qi} = Z_{pi}$

$Z_{qq} = Z_{pq} + z_{pq\,pq}$

If p is ref.

$Z_{qi} = 0$

$Z_{qq} = z_{pq\,pq}$

Link

$Z_{li} = Z_{pi} - Z_{qi}$

$Z_{ll} = Z_{pl} - Z_{ql} + z_{pq\,pq}$

If p is ref.

$Z_{li} = -Z_{qi}$

$Z_{ll} = -Z_{ql} + z_{pq\,pq}$

step ① - Add branch ① - ②

$$\underset{\text{ref}}{①} - \underset{p}{②}$$

$$Z_{bus} = [j0.1]$$

step ② - Add branch ② - ③

$$\underset{p}{②} - \underset{q}{③}$$

$$Z_{bus} = \begin{array}{cc} ② & ③ \\ \left[\begin{matrix} j0.1_{11} & -_{12} \\ -_{21} & -_{22} \end{matrix}\right] \end{array} \Rightarrow \begin{bmatrix} j0.1 & j0.1 \\ j0.1 & j0.6 \end{bmatrix}$$

$$Z_{qi} = Z_{pi} = j0.1$$

$$Z_{qq} = Z_{pq} + z_{pqpq}$$
$$= j0.1 + j0.5$$
$$= j0.6$$

step③ Add link ① – ③

$\downarrow$ Ref $\quad \downarrow$ q

$$Z_{li} = \quad - Z_{qi}$$

$$Z_{bus} = \begin{array}{ccc} ① & ② & ③ \\ \left[\begin{array}{ccc} j0.1 & j0.1 & - \\ j0.1 & j0.6 & - \\ - & - & - \end{array}\right. & & \left.\right] \end{array} \Rightarrow \begin{bmatrix} j0.1 & j0.1 & -j0.1 \\ j0.1 & j0.6 & -j0.6 \\ -j0.1 & -j0.6 & j0.8 \end{bmatrix}$$

$$Z_{li} = -j0.1$$

$$Z_{l1} = -j0.1$$

$$Z_{l2} = -j0.6$$

$$Z_{ll} = Z_{-ql} + z_{pqR} = j0.6 + j0.2 = \underline{\underline{j0.8}}$$

Eliminate l^{th} node

$$Z_{bus} = \left[\begin{array}{cc|c} j0.1 & j0.1 & -j0.1 \\ j0.1 & j0.6 & -j0.6 \\ \hline -j0.1 & -j0.6 & j0.8 \end{array}\right] \Rightarrow Z_{bus\,new} = \begin{bmatrix} Z_{11\,new} & Z_{12\,new} \\ Z_{21\,new} & Z_{22\,new} \end{bmatrix}$$

$$Z_{11\,new} = Z_{11\,old} - \frac{Z_{l1} \cdot Z_{1l}}{Z_{ll}}$$

$$= j0.1 - \left[\frac{-j0.1 \times -j0.1}{j0.8}\right]$$

$$Z_{12\,new} = Z_{12\,old} - \frac{Z_{1l} \cdot Z_{l2}}{Z_{ll}} = j0.1 - \left[\frac{-j0.1 \times -j0.6}{j0.8}\right]$$

$$Z_{21\,new} = Z_{12\,new}$$

$$Z_{22\,new} = Z_{22\,old} - \frac{Z_{2l} \cdot Z_{l2}}{Z_{ll}} = j0.6 - \left[\frac{-j0.6 \times -j0.6}{j0.8}\right]$$