

**Seventh Semester B.E. Degree Examination, Feb./Mar. 2022**  
**Power System Analysis - II**

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Explain the following terms in network topology with an example. (06 Marks)  
 i) Tree ii) Basic loops iii) Basic cut-sets.  
 b. Consider an oriented graph of the power system network shown below Fig Q1(b). Choose branches 1, 3 and 5 as twigs. Build a bus incidence matrix  $A$  and basic cut-set matrix  $B$  for the oriented graph. Select node 2 as reference.

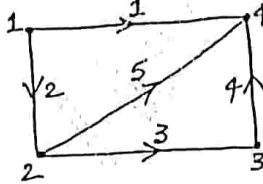


Fig Q1(b)

(08 Marks)

- c. A power system consists of four buses. The generators are connected at buses 1 and 3. The transmission lines are connected between buses 1-2, 1-4, 2-3 and 3-4 which have reactances of  $j0.25$ ,  $j0.5$ ,  $j0.4$  and  $j0.1$  respectively. Develop a bus admittance matrix by direct inspection method. Choose bus 1 as reference. (06 Marks)

OR

- 2 a. Build bus incidence matrix  $A$  and then bus admittance matrix  $Y_{bus}$  using singular transformation method for the power system network shown below in Fig Q2(a). Choose bus 1 as reference. The linedata of the power system are given in Table Q2(a) below.

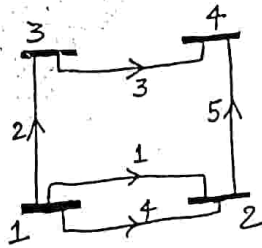


Fig Q2(a)

line No	Bus code (p-q)	Z(pu)	Mutual temperature $Z_m$ (pu)
1	1-2	0.6	0.2 (line 2)
2	1-3	0.5	-
3	3-4	0.5	-
4	1-2	0.4	0.1 (line 1)
5	2-4	0.2	-

Table Q2(a)

(08 Marks)

- b. Define primitive network and explain its two forms with neat representation circuit. Also derive their respective performance equations. (06 Marks)  
 c. Consider an oriented graph of the power system shown below in Fig Q2(c). Choose branches 1, 3 and 5 as twigs to form a tree. Build a basic loop incidence matrix  $C$  for the given oriented graph. Select node 2 as reference.

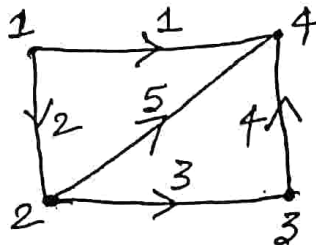


Fig Q2(c)

1 of 3

(06 Marks)

2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42+8 = 50, will be treated as malpractice.

**Module-2**

- 3 a. State the need of load flow study. Derive the static load flow equations or power flow equating to conduct load flow study in usual notations. (06 Marks)
- b. For a 4 bus power system network shown below in Fig Q3(b), the generators are connected at all four buses, while loads are at buses 2 and 3. The real and reactive powers are listed below in table 3(b). Assuming a flat voltage start compute the unknown variables in all the buses other than the slack at the end of first GS iteration. Take acceleration factor as 1.4

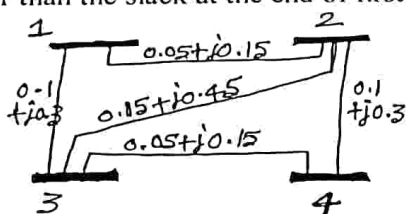


Fig Q3(b)

Bus No.	$P_i$ (pu)	$Q_i$ (pu)	$V_i$ (pu)
1	-	-	$1.04 \angle 0^\circ$
2	0.5	-0.2	-
3	-1	0.5	-
4	0.3	-0.1	-

Table Q3(b)

(14 Marks)

OR

- 4 a. Explain the algorithm for Gauss – Seidel method to obtain load flow solution of a power system network with i) Absence of PV buses ii) Presence of PV buses. (10 Marks)
- b. For the power system network shown below in Fig Q4(b), the line impedance are marked in pu. The bus data of the power system are shown below Table Q4(b). Compute the voltage in all buses other than slack at the end of first iteration using Gauss – Seidel method. Take  $0 < Q_2 < 0.35$ pu.

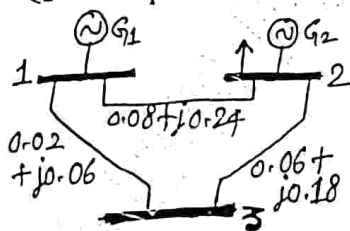


Fig Q4(b)

Bus No.	Voltage (pu)	Generation		Load	
		$P_G$	$Q_G$	$P_D$	$Q_D$
1	$0.05 \angle 0^\circ$	-	-	-	-
2	1.03	0.2	-	0.5	0.2
3	-	0	0	0.6	0.25

Table Q4(b)

(10 Marks)

**Module-3**

- 5 a. Derive the general expression for Jacobian elements in polar form with usual notations in NR method to obtain load flow solution. (10 Marks)
- b. Explain the algorithm of Fast Decoupled Load Flow method with a neat flow chart for the load flow solution of a power system network. (10 Marks)

OR

- 6 a. In a two bus power system network shown below in Fig Q6(a), the bus – 1 is a slack bus with  $V_1 = 1 \angle 0^\circ$  pu and bus 2 is a load bus with  $P_2 = 100$ MW,  $Q_2 = 50$ MVAR. The line impedance is  $(0.12 + j0.16)$ pu on a base of 100MVA. Using NR method of load flow solution, compute the voltage at bus 2 at the end of first iteration.



Fig Q6(a)

(10 Marks)

- b. Compare Gauss – Seidal, Newton Raphson and Fast decoupled load flow method of load flow solution with respect to various parameters. (10 Marks)

Module-4

- 7 a. A constant load of 300MW is supplied by two 200MW generators 1 and 2 for which the respective incremental fuel costs are,  $\frac{dC_1}{dP_{G1}} = 0.1P_{G1} + 20$  and  $\frac{dC_2}{dP_{G2}} = 0.12P_{G2} + 15$ , where  $P_G$ 's in MW and costs  $C_1$  and  $C_2$  are in Rs/hr. Determine : i) the most economical division of load between the generators and ii) the saving in Rs./day there by obtained compared to equal load sharing between generators. (10 Marks)
- b. Explain various constraints involved in unit commitment solution. (10 Marks)

OR

- 8 a. Two units are connected at two buses through a transmission line. If 100MW is transmitted from unit 1 at bus 1 to the load at bus 2, a line loss of 10MW is incurred. The incremental cost curve of the two units are,  
 $IC_1 = 16 + 0.02 P_1$  Rs./MWhr and  
 $IC_2 = 20 + 0.04 P_2$  Rs./MWhr  
 If the system incremental cost is Rs.26/MWhrs no load fuel costs are Rs. 250 and Rs. 350 per hour for units 1 and 2 respectively, then determine the following :  
 i) Power generations from both units and the power received by the load if the losses are included and also coordinated  
 ii) Power generating from both units for the power received by the load as calculated above, if the losses are included but not coordinated  
 iii) Net saving in fuel cost by coordinating the losses. (12 Marks)
- b. Explain the Dynamic program algorithm with the recursive relation and also explain forward DP approach with a neat flow chart. (08 Marks)

Module-5

- 9 a. Explain the algorithm for short circuit studies to be carried out in large power systems. (08 Marks)
- b. A 20MVA, 50Hz generator delivers 18MW over a double circuit line to an infinite bus. The generator has kinetic energy of 2.52MJ/MVA at rated speed. The generator has a transient reactance of 0.35pu. Each transmission line has a reactance of 0.2pu on a 20MVA base. The generator excitation voltage  $|E'| = 1.1$ pu and infinite bus voltage  $V = 1 \angle 0^\circ$  pu. A three phase short circuit occurs at the midpoint of one of the lines. Plot the swing curve with the fault cleared by simultaneous opening of breakers at both ends of the line at 2.5 cycles after the occurrence of fault. Take a step size of time as 0.05sec. Also, calculate the critical clearing angle. Use point by point method. (12 Marks)

OR

- 10 a. For a three bus power system network show below in Fig Q10(a), the pu impedances are shown therein. Build bus impedance matrix  $Z_{bus}$  using step by step building algorithm. Add the elements in the order specified. (10 Marks)

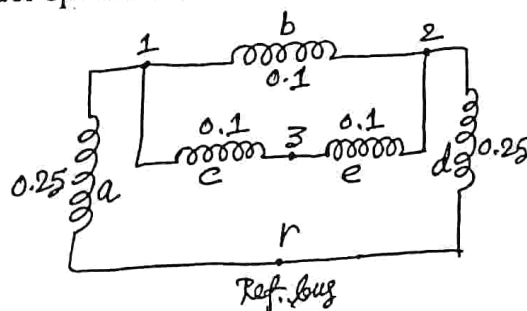


Fig Q10(a)

- b. Build an algorithm for numerical solution of swing equation by Runge - Kutta method. (10 Marks)

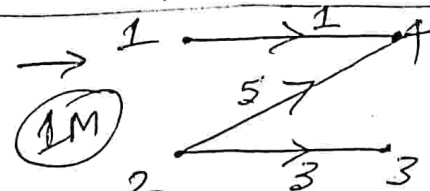
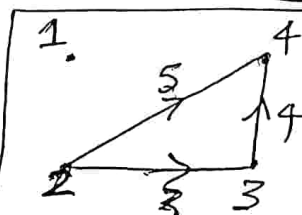
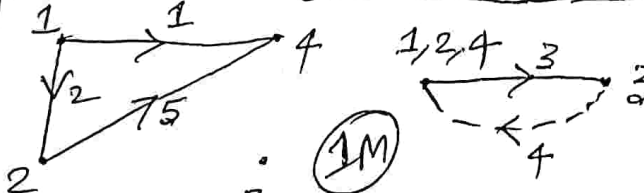
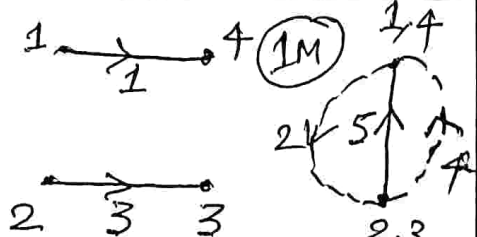
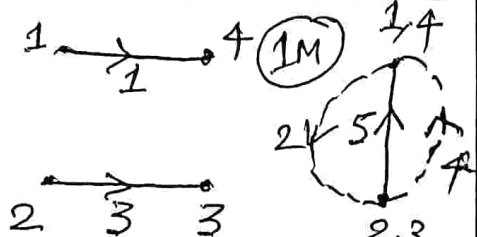
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Scheme & Solutions

Signature of Scrutinizer

Subject Title : POWER SYSTEM ANALYSIS - 2 Subject Code : 18EE71

Question Number	Solution	Marks Allocated
1(a)	<p>Tree - A connected sub-graph containing all the nodes of the graph but having no closed loops. Explanation with an example - (2M)</p> <p>Basic loops - A loop formed by adding one link to a tree. Explanation with an example (2M)</p> <p>Basic cut-sets - Minimal set of branches of the graph in which all the branches are links except for that one twig, removal of them makes the graph into two parts and reduces the rank of the graph by one. Explanation with an example -&gt; (2M)</p>	6M
1(b)	<p>Tree <math>\rightarrow</math>  (1M)</p> <p>Basic cut-sets:- cut-set-1: {1, 2} cut-set-2: {3, 4} cut-set-3: {5, 4, 2}</p> <p>Basic incidence matrix <math>A = \begin{bmatrix} 1 &amp; 0 &amp; -1 \\ 1 &amp; 0 &amp; 0 \\ 0 &amp; -1 &amp; 0 \\ 0 &amp; 1 &amp; -1 \\ 0 &amp; 0 &amp; -1 \end{bmatrix}</math> (2M)</p> <p>Basic cut-set 1 <math>\rightarrow</math>  (1M)</p>	8M
	<p> (1M)</p> <p>Basic cut-set 2 <math>\rightarrow</math> </p>	
	<p>Basic cut-set matrix <math>B = \begin{bmatrix} 1 &amp; 1 &amp; 0 &amp; 0 &amp; 0 \\ 0 &amp; 0 &amp; 1 &amp; -1 &amp; 0 \\ 0 &amp; -1 &amp; 0 &amp; 1 &amp; 1 \end{bmatrix}</math> (2M)</p> <p>Basic cut-set 3 <math>\rightarrow</math> </p> <p>Basic cut-set matrix <math>B = \begin{bmatrix} 1 &amp; 3 &amp; 5 &amp; 2 &amp; 4 \\ 1 &amp; 0 &amp; 0 &amp; 1 &amp; 0 \\ 0 &amp; 1 &amp; 0 &amp; 0 &amp; -1 \\ 0 &amp; 0 &amp; 1 &amp; -1 &amp; 1 \end{bmatrix} = [U; B_e]</math></p>	

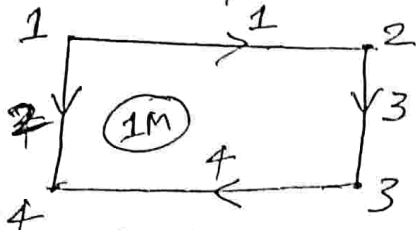
Question Number

Solution

Marks Allocated

1)(c)

SLD of the p/s network / oriented graph:



$Z_{12} = j0.25, Z_{14} = j0.5,$   
 $Z_{23} = j0.4, Z_{34} = j0.1.$

$Y_{bus} = \begin{bmatrix} Y_{22} & Y_{23} & Y_{24} \\ Y_{32} & Y_{33} & Y_{34} \\ Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$

Diagonal elements are,

$Y_{22} = \frac{1}{Z_{23}} + \frac{1}{Z_{12}} = -6.5j$

$Y_{33} = \frac{1}{Z_{34}} + \frac{1}{Z_{23}} = -12.5j$

$Y_{44} = \frac{1}{Z_{14}} + \frac{1}{Z_{34}} = -12j$  (2M)

off-diagonal elements are,  $Y_{23} = Y_{32} = -\frac{1}{Z_{23}} = 2.5j$

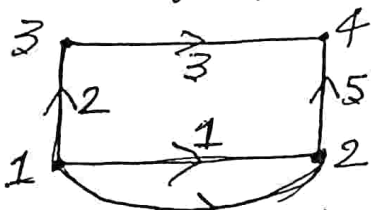
$Y_{24} = Y_{42} = \frac{-1}{Z_{24}} = 0$       $Y_{34} = Y_{43} = \frac{-1}{Z_{34}} = 10j$  (2M)

$\therefore Y_{bus} = \begin{bmatrix} -6.5j & 2.5j & 0 \\ 2.5j & -12.5j & 10j \\ 0 & 10j & -12j \end{bmatrix}$  (1M)

6M

2)(a)

oriented graph:



Bus incidence matrix  $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$  (2M)

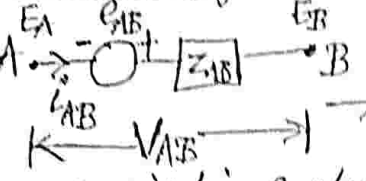
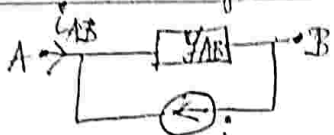
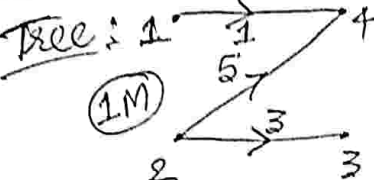
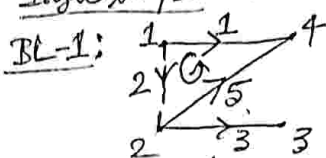
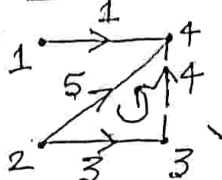
Primitive impedance matrix  $Z = \begin{bmatrix} 0.6 & 0.2 & 0 & 0.1 & 0 \\ 0.2 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}$  (1M)

Primitive Admittance matrix  $Y = \frac{1}{Z} = \begin{bmatrix} 2.02 & -0.81 & 0 & -0.51 & 0 \\ -0.81 & 2.32 & 0 & 0.20 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -0.51 & 0.20 & 0 & 2.63 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$  (2M)

$\therefore Y_{bus} = A^T Y A$

$Y_{bus} = \begin{bmatrix} 8.6364 & -0.6061 & -5 \\ -0.6061 & 4.3232 & -2 \\ -5 & -2 & 7 \end{bmatrix}$  (3M)

8M

Question Number	Solution	Marks Allocated
2(b)	<p>Primitive network - set of unconnected elements. <math>\rightarrow</math> (1M)</p> <p>Impedance form:</p>  <p>performance equation of n/w,  <math>V = E = Z \cdot I</math> derivation. <math>\rightarrow</math> (1M)</p> <p>Description &amp; applying KVL,  <math>V_{AB} + E_{AB} = Z_{AB} I_{AB}</math>. <math>\rightarrow</math> (1M)</p> <p>Admittance form:</p>  <p>KCC at node A, <math>I_{AB} + I_{AB} = Y_{AB} V_{AB}</math>.          performance equation of the n/w,  <math>I + J = YV</math> <math>\rightarrow</math> (1M)</p> <p>Description <math>\rightarrow</math> (1M)</p>	6M
2(c)	<p>Tree: 1 <math>\rightarrow</math> 4 <math>\rightarrow</math> 3 <math>\rightarrow</math> 2 <math>\rightarrow</math> 3</p>  <p>(1M)</p> <p>Basic loops: 2.</p> <p>BL-1: 1 <math>\rightarrow</math> 4 <math>\rightarrow</math> 3 <math>\rightarrow</math> 2 <math>\rightarrow</math> 1</p>  <p><math>\{2, 1, 5\}</math> <math>\rightarrow</math> (2M)</p> <p>BL-2: 1 <math>\rightarrow</math> 4 <math>\rightarrow</math> 3 <math>\rightarrow</math> 2 <math>\rightarrow</math> 1</p>  <p><math>\{4, 3, 5\}</math> <math>\rightarrow</math> (2M)</p> <p>loop incidence matrix,</p> <p>BL1 <math>\begin{bmatrix} 1 &amp; 2 &amp; 3 &amp; 4 &amp; 5 \\ -1 &amp; 1 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p> <p>BL2 <math>\begin{bmatrix} 0 &amp; 0 &amp; 1 &amp; 1 &amp; -1 \end{bmatrix}</math></p> <p><math>C = \begin{bmatrix} 1 &amp; 3 &amp; 5 &amp; 2 &amp; 4 \\ -1 &amp; 0 &amp; 1 &amp; 1 &amp; 0 \\ 0 &amp; 1 &amp; -1 &amp; 0 &amp; 1 \end{bmatrix}</math> <math>\rightarrow</math> (1M)</p>	6M
3(a)	<p>Need for LFS: (a) Information obtained from LFS is very much necessary for continuous monitoring of current status of the power system.</p> <p>(b) Also, for analyzing effectiveness of alternative plans for future expansion of the network. <math>\rightarrow</math> (2M)</p> <p>derivation of, <math>P_i =  V_i  \sum_{k=1}^n  V_k   Y_{ik}  \cos(\theta_{ik} + \delta_k - \delta_i)</math> and <math>\rightarrow</math> (2M)</p> <p><math>Q_i = - V_i  \sum_{k=1}^n  V_k   Y_{ik}  \sin(\theta_{ik} + \delta_k - \delta_i)</math>. <math>\rightarrow</math> (2M)</p>	6M
3(b)	<p><math>Y_{bus} = \begin{bmatrix} 3-j9 &amp; -2+j6 &amp; -1+j3 &amp; 0 \\ -2+j6 &amp; 3.6667-j11 &amp; -0.6667+j2 &amp; -1+j3 \\ -1+j3 &amp; -0.6667+j2 &amp; 3.6667-j11 &amp; -2+j6 \\ 0 &amp; -1+j3 &amp; -2+j6 &amp; 3-j9 \end{bmatrix}</math> <math>\rightarrow</math> (3M)</p>	

Question Number

Solution

Marks Allocated

$$V_2^1 = 1.0191 + j0.0464 \text{ pu} = 1.0202 \angle 2.605^\circ \text{ pu} \rightarrow (2M)$$

$$(V_2^1)_{acc} = V_2^1 + \alpha(V_2^1 - V_2^0) = (1.0458 + j0.1114) \text{ pu} \rightarrow (1M)$$

$$V_3^1 = 1.0329 - j0.0752 \text{ pu} = 1.0356 \angle -4.16^\circ \text{ pu} \rightarrow (3M)$$

$$(V_3^1)_{acc} = V_3^1 + \alpha(V_3^1 - V_3^0) = (1.079 - j0.1805) \text{ pu} = 1.094 \angle -9.5^\circ$$

$$V_4^1 = 1.0679 - j0.0499 \text{ pu} = 1.0691 \angle -2.67^\circ \text{ pu} \rightarrow (1M)$$

$$(V_4^1)_{acc} = 1.163 - j0.1198 \text{ pu} = 1.1692 \angle -5.88^\circ \text{ pu} \rightarrow (1M)$$

14M

4)(a) Algorithm for GS method with absence of PV buses:-

Step 1: Identification of buses and their specified variables.

Step 2: Formation of  $Y_{bus}$

Step 3: Assume flat voltage start.

Iterative computations of bus voltages. (5M)

$$V_i^{r+1} = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{r+1} - \sum_{k=i+1}^n Y_{ik} V_k^r \right]$$

Continue iterative process until,  $|\Delta V_i^{r+1}| = |V_i^{r+1} - V_i^r| \leq \epsilon$

Step 4: computation of slack bus power ( $P_1, Q_1$ ) using

Step 5: computation of line flows. ( $S_{ik}$  &  $S_{ki}$ ) (5M)

Algorithm for GS method with presence of PV buses:

Step 1 & 2: - Same as above.

Step 3: - Assume flat voltage start. For PQ buses  $V_i = 1 \angle 0^\circ$  (10M)

Compute reactive power iteratively for PV buses, For PV buses,  $\delta_i = 0$

$$Q_i^{r+1} = -\text{Imag} \left\{ (V_i^r)^* \sum_{k=1}^{i-1} Y_{ik} V_k^{r+1} + (V_i^r)^* \sum_{k=i+1}^n Y_{ik} V_k^r \right\}$$

Iterative computation of voltages

$$V_i^{r+1} = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i^{r+1}}{(V_i^r)^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{r+1} - \sum_{k=i+1}^n Y_{ik} V_k^r \right]$$

Obtain the revised value of  $\delta_i$

$$\delta_i^{r+1} = \angle V_i^{r+1} \quad \therefore V_i^{r+1} = |V_i| \angle \delta_i^{r+1} \quad (5M)$$

Continue iterative process until  $|\Delta V_i^{r+1}| = |V_i^{r+1} - V_i^r| \leq \epsilon$

Steps 4 & 5: - Same as above.

(b)

$$Y_{bus} = \begin{bmatrix} 6.25 - j18.75 & -1.25 + j3.75 & -5 + j15 \\ -1.25 + j3.75 & 2.9167 - j8.75 & -1.6667 + j5 \\ -5 + j15 & -1.6667 + j5 & 6.6667 - j20 \end{bmatrix} \rightarrow (2M)$$

$$P_2 = -0.3 \text{ pu}$$

$$P_3 = -0.6 \text{ pu}$$

$$Q_3 = -0.25 \text{ pu}$$

Question Number

Solution

Marks Allocated

Flat voltage start:  $V_2^0 = 1.03 \angle 0^\circ \text{ pu}$ ,  $V_3^0 = 1 \angle 0^\circ \text{ pu}$   $\rightarrow$  (1M)

For  $r=0$ ,  $i=2$ ,  $Q_2^1 = -\text{Imag}\{V_2^0\}^* \sum_{k=1}^3 Y_{2k} V_k^1 - (V_2^0)^* \sum_{k=2}^3 Y_{2k} V_k^0$   $\rightarrow$  (2M)

$$Q_2^1 = 0.07725 \text{ pu} \rightarrow$$

since,  $Q_{2\text{min}} \leq Q_2^1 \leq Q_{2\text{max}}$ ,  $Q_2^1 = 0.07725 \text{ pu}$   $\rightarrow$  (1M)

$$\therefore V_2^1 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2^1}{(V_2^0)^*} - \sum_{k=1}^3 Y_{2k} V_k^1 - \sum_{k=3}^3 Y_{2k} V_k^0 \right] \rightarrow$$

$$V_2^1 = (1.0192 - j0.0325) \text{ pu} = 1.0197 \angle -1.83^\circ \text{ pu}$$

$$V_3^1 = \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{(V_3^0)^*} - \sum_{k=1}^2 Y_{3k} V_k^1 - \sum_{k=4}^3 Y_{3k} V_k^0 \right] \rightarrow$$

$$V_3^1 = (1.0221 - j0.0314) \text{ pu} = 1.0226 \angle -1.76^\circ \text{ pu}$$

5) (a) SLFE are,  $P_i = |V_i| \sum_{k=1}^n |V_k| |Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i)$   
 $Q_i = -|V_i| \sum_{k=1}^n |V_k| |Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i)$

Expanded form of SLFE are,

$$P_i = |V_i| \sum_{k=1}^n |V_k| |Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) + |V_i|^2 |Y_{ii}| \cos(\theta_{ii}) + |V_i| |V_m| |Y_{im}| \cos(\theta_{im} + \delta_m - \delta_i) \rightarrow$$

$$Q_i = -|V_i| \sum_{k=1}^n |V_k| |Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i) = -|V_i|^2 |Y_{ii}| \sin \theta_{ii} - |V_i| |V_m| |Y_{im}| \sin(\theta_{im} + \delta_m - \delta_i) \rightarrow$$

Case-1 ( $i \neq m$ ):

$$H_{im} = \frac{\partial P_i}{\partial \delta_m} = -|V_i| |V_m| |Y_{im}| \sin(\theta_{im} + \delta_m - \delta_i)$$

$$N_{im} = \frac{\partial P_i}{\partial |V_m|} \times |V_m| = |V_i| |V_m| |Y_{im}| \cos(\theta_{im} + \delta_m - \delta_i) \rightarrow$$

$$J_{im} = \frac{\partial Q_i}{\partial \delta_m} = -|V_i| |V_m| |Y_{im}| \cos(\theta_{im} + \delta_m - \delta_i) = -N_{im}$$

$$L_{im} = \frac{\partial Q_i}{\partial |V_m|} \times |V_m| = -|V_i| |V_m| |Y_{im}| \sin(\theta_{im} + \delta_m - \delta_i) = H_{im}$$

Case-2 ( $i=m$ ):

$$H_{ii} = \frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2 |Y_{ii}| \sin \theta_{ii} = -Q_i - B_{ii} |V_i|^2 \rightarrow$$

$$N_{ii} = \frac{\partial P_i}{\partial |V_i|} \times |V_i| = P_i + |V_i|^2 |Y_{ii}| \cos \theta_{ii} = P_i + G_{ii} |V_i|^2$$

$$J_{ii} = \frac{\partial Q_i}{\partial \delta_i} = P_i - |V_i|^2 |Y_{ii}| \cos \theta_{ii} = P_i - G_{ii} |V_i|^2$$

$$L_{ii} = \frac{\partial Q_i}{\partial |V_i|} \times |V_i| = Q_i - |V_i|^2 |Y_{ii}| \sin \theta_{ii} = Q_i - B_{ii} |V_i|^2$$

5) (b) FDLF Algorithm:-

- $\rightarrow$  Neglect shunt reactances & t/f off-nominal taps from [B'].
- $\rightarrow$  Neglect phase angle shifting effects from [B''].
- $\rightarrow$  Divide each vector-matrix equations by  $|V_i|$  & set

10M



Question Number	Solution	Marks Allocated
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$|V_m| = 1$  pu in them.  
 → Ignore series resistance in lines while calculating elements of  $[B']$ .  
 Hence, vector-matrix equations are, (5M)  

$$\begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta V \end{bmatrix} = [B'] \begin{bmatrix} \Delta \delta \\ \Delta \theta \\ \Delta V \end{bmatrix} \quad \& \quad \begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta V \end{bmatrix} = [B''] \begin{bmatrix} \Delta \delta \\ \Delta \theta \\ \Delta V \end{bmatrix}$$
  
 → solve alternatively above equations involving most recent voltage values.  
 → One iteration implies one solution for  $[\Delta \delta]$  to update  $[\delta]$  and then one solution for  $[\Delta V]$  to update  $[V]$  to be called 1- $\delta$  and 1-V iteration. 10M  
 → convergence tests for real & reactive power mismatches are applied as,  $\max[\Delta P] \leq \epsilon_p$ ;  $\max[\Delta Q] \leq \epsilon_q$ .  
 where,  $\epsilon_p$  &  $\epsilon_q$  are respective tolerances.  
 Flow chart for FDLF algorithm → (5M)

6) (a)  $Y_{bus} = \begin{bmatrix} 3-j4 & -3+j4 \\ -3+j4 & 3-j4 \end{bmatrix} = \begin{bmatrix} 5 \angle -53.13^\circ & 5 \angle 126.87^\circ \\ 5 \angle 126.87^\circ & 5 \angle -53.13^\circ \end{bmatrix}$  (2M)  
 $P_2 = 1$  pu  
 $Q_2 = 0.5$  pu

Flat voltage start  $V_2^0 = 1 \angle 0^\circ$  pu.

$\therefore P_2^0 = |V_2^0| \sum_{k=1}^2 |V_k^0| |Y_{2k}^0| \cos(\theta_{2k} + \delta_k^0 - \delta_2^0) = 0$  pu  $\Rightarrow \Delta P_2^0 = 1$  pu

$Q_2^0 = -|V_2^0| \sum_{k=1}^2 |V_k^0| |Y_{2k}^0| \sin(\theta_{2k} + \delta_k^0 - \delta_2^0) = 0$  pu  $\Rightarrow \Delta Q_2^0 = 0.5$  pu. (2M)

vector-matrix equations, 
$$\begin{bmatrix} \Delta P_2^0 \\ \Delta Q_2^0 \end{bmatrix} = \begin{bmatrix} H_{22}^0 & N_{22}^0 \\ J_{22}^0 & L_{22}^0 \end{bmatrix} \begin{bmatrix} \Delta \delta_2^0 \\ \Delta V_2^0 \end{bmatrix}$$

$H_{22}^0 = -Q_2^0 - (|V_2^0|)^2 |Y_{22}^0| \sin \theta_{22} = 4$  pu,  $N_{22}^0 = P_2^0 + (|V_2^0|)^2 |Y_{22}^0| \cos \theta_{22} = 3$  pu (4M)

$J_{22}^0 = P_2^0 - (|V_2^0|)^2 |Y_{22}^0| \cos \theta_{22} = -3$  pu,  $L_{22}^0 = Q_2^0 - (|V_2^0|)^2 |Y_{22}^0| \sin \theta_{22} = 4$  pu.

$\therefore \begin{bmatrix} \Delta \delta_2^0 \\ \Delta V_2^0 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} \Rightarrow \Delta \delta_2^0 = 0.1 \text{ \& } \Delta V_2^0 = 0.2$   
 $\therefore V_2^1 = 1.2 \angle 0.1^\circ$  pu (2M)

6) (b) Parameters	GS	NR	FDLF
Coordinates	rectangular	Polar	Polar
Arithmetic operations	Less	More	Less than NR
Time Consumed	less time	7 times of GS	Less time of NR & GS
Convergence	linear	Quadratic	Geometric

Question Number

Solution

Marks Allocated

Parameters	GS	NR	FDLF
No. of iterations	more & increases with no. of buses	very less & is constant	only 2 to 5
slack bus selection	Affects convergence adversely	minimal affects	moderately sensitive
Accuracy	less accurate	more accurate	moderate
Memory	less	large	only 60% of NR
Usage/application	Small size systems	Large systems optimal LFS	optimization studies multiple LFS, contingency evaluation for security assessment enhancement
Programming logic	Easy	very difficult	moderate
Reliability	Reliable only for small systems	Reliable even for large systems	more reliable than NR method

10M

7(a)

$\frac{dC_1}{dP_{g1}} = \frac{dC_2}{dP_{g2}} = \lambda \Rightarrow 0.1P_{g1} + 20 = 0.12P_{g2} + 15$   
 $0.1P_{g1} - 0.12P_{g2} = -5 \quad \text{--- (1)}$   
 Satisfying load demand;  $P_{g1} + P_{g2} = P_D = 300 \quad \text{--- (2)}$   
 solving (1)/(2);  $P_{g1} = 140.91 \text{ MW}$  &  $P_{g2} = 159.09 \text{ MW}$   
 Fuel costs;  $C_1 = 0.05P_{g1}^2 + 20P_{g1} = \text{Rs. } 3810.98/\text{hr.}$   
 $C_2 = 0.06P_{g2}^2 + 15P_{g2} = \text{Rs. } 3904.93/\text{hr.}$   
 $\therefore (C_T)_{opt.} = C_1 + C_2 = \text{Rs. } 7715.91/\text{hr.}$   
 For equal load sharing  $P_{g1} = P_{g2} = P_D/2 = 150 \text{ MW}$   
 Fuel costs;  $C_1 = 0.05P_{g1}^2 + 20P_{g1} = \text{Rs. } 4125/\text{hr.}$   
 $C_2 = 0.06P_{g2}^2 + 15P_{g2} = \text{Rs. } 3600/\text{hr.}$   
 $(C_T)_{equal} = C_1 + C_2 = \text{Rs. } 7725/\text{hr.}$   
 Net saving =  $(C_T)_{equal} - (C_T)_{opt.} = \text{Rs. } 9.09/\text{hr.} = \text{Rs. } 9.09 \times 24/\text{day}$   
 Net saving =  $\text{Rs. } 218.16/\text{day.}$

10M

7(b)

constraints; (1) spinning Reserve, (2) Thermal Unit constraints  
 (3) start-up costs of thermal units (4) Network constraints  
 (5) Emission constraints, (6) Generation capacity limits  
 (7) Fuel constraints, (8) security constraints (9) Hydel plant constraints  
 Explanation on  $\rightarrow$  spinning Reserve  $\rightarrow$   
 Thermal unit constraints  $\rightarrow$   
 start-up costs of thermal units  $\rightarrow$   
 All other constraints  $\rightarrow$

10M

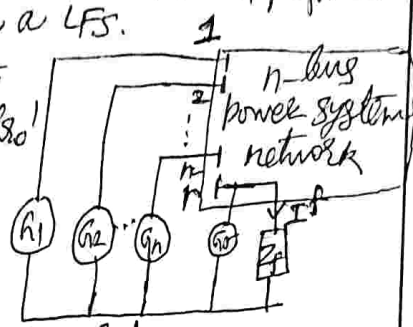
Question Number	Solution	Marks Allocated
8/a)	<p>Since, load is at bus-2, <math>(P_L)_{bus 2} = 0</math>  <math>\Rightarrow B_{22} = B_{12} = B_{21} = 0</math>  <math>\therefore P_L = B_{11} P_{g1}^2</math></p> <p>Given, if <math>P_{g1} = 100 \text{ MW}</math>, <math>P_L = 10 \text{ MW}</math>. <math>\Rightarrow B_{11} = 0.001 \text{ MW}^{-1}</math>  <math>\therefore P_L = 0.001 P_{g1}^2 \rightarrow (2M)</math></p> <p>For unit-1; <math>ITL_1 = 0.002 P_{g1} \Rightarrow L_1 = \frac{1}{1 - ITL_1} = \frac{1}{1 - 0.002 P_{g1}}</math>                  For unit-2; <math>ITL_2 = 0 \Rightarrow L_2 = \frac{1}{1 - ITL_2} = 1 \rightarrow (1M)</math></p> <p>① The exact coordination eqn.,                  For unit-1; <math>IC_1 L_1 = \lambda \Rightarrow (16 + 0.02 P_{g1})^* = 26</math>                  solving, <math>P_{g1} = 138.89 \text{ MW} \rightarrow (2M)</math></p> <p>For unit-2; <math>IC_2 L_2 = \lambda \Rightarrow (20 + 0.04 P_{g2}) = 26</math>                  solving, <math>P_{g2} = 150 \text{ MW} \rightarrow (1M)</math></p> <p><math>\therefore P_L = 0.001 P_{g1}^2 = 19.29 \text{ MW}</math>. <math>\Rightarrow P_{g1} + P_{g2} = P_L + P_D</math>  <math>\therefore P_D = P_{g1} + P_{g2} - P_L = 269.6 \text{ MW}</math></p> <p>Fuel costs; <math>C_1 = \int IC_1 dP_{g1} = 0.01 P_{g1}^2 + 16 P_{g1} + 250 = 2665.1237 \text{ Rs/hr}</math>.  <math>C_2 = \int IC_2 dP_{g2} = 0.02 P_{g2}^2 + 20 P_{g2} + 350 = 3800 \text{ Rs/hr}</math>.  <math>\therefore (C_T)_{\text{coord}} = C_1 + C_2 = 6465.1237 \text{ Rs/hr} \rightarrow (2M)</math></p> <p>② <math>P_D = 269.6 \text{ MW}</math>, losses are included; <math>P_{g1} + P_{g2} = P_L + P_D</math>.                  losses are not coordinated; <math>IC_i = \lambda</math>  <math>\Rightarrow IC_1 = IC_2 \Rightarrow 0.02 P_{g1} - 0.04 P_{g2} = 4 \rightarrow (1)</math>                  losses are included, <math>P_{g1} + P_{g2} - P_L - P_D = 0 \Rightarrow P_{g1} + P_{g2} - 0.001 P_{g1}^2 - P_D = 0</math>  <math>\Rightarrow P_{g1} + P_{g2} - 0.001 P_{g1}^2 - 269.6 = 0 \rightarrow (2) \rightarrow (2M)</math>                  solving ① &amp; ②, <math>P_{g1} = 310.7948 \text{ MW}</math> &amp; <math>P_{g2} = 55.3974 \text{ MW}</math></p> <p>For unit-1; <math>C_1 = 0.01 P_{g1}^2 + 16 P_{g1} + 250 = 6188.6509 \text{ Rs/hr}</math>.                  For unit-2; <math>C_2 = 0.02 P_{g2}^2 + 20 P_{g2} + 350 = 1519.3254 \text{ Rs/hr}</math>.  <math>(C_T)_{\text{uncoord}} = C_1 + C_2 = 7707.9763 \text{ Rs/hr}</math>.  <math>(C_T)_{\text{uncoord}} - (C_T)_{\text{coord}} = 1242.8526 \text{ Rs/hr}</math>.</p> <p>③ Net saving in fuel cost = <math>(C_T)_{\text{uncoord}} - (C_T)_{\text{coord}} = 1242.8526 \text{ Rs/hr}</math>.                  Net saving = <math>1,08,87,389 \text{ Rs/annum} \rightarrow (2M)</math></p>	<p>Marks Allocated</p> <p>12M</p>
8/b)	<p><math>f_N(y)</math> - cost of generating power of 'y' MW with <math>N^{\text{th}}</math> unit alone.  <math>F_N(x)</math> - cost of generating power of 'x' MW with all 'N' units.                  Recursive relation; <math>F_N(x) = \min_y [f_N(y) + F_{N-1}(x-y)]</math> operating. <math>(4M)</math></p> <p>Recursive formula for Forward DP approach.  <math>F_{\text{cost}}(k, c) = \min_{L \leq y \leq U} [F_{\text{cost}}(k, c) + S_{\text{cost}}(k-1, L: k: c) + F_{\text{cost}}(k-1, y)]</math></p> <p>Flow chart for Forward DP approach <math>\rightarrow (4M)</math></p>	<p>8M</p>

Solution

Marks Allocated

Step 1: obtain prefault voltages at all buses and prefault current in all lines through a LFS.

let  $V_{bus}^0 = \begin{bmatrix} V_1^0 \\ V_2^0 \\ \vdots \\ V_n^0 \end{bmatrix}$  let bus r be at 3 $\phi$  sc fault thro' a fault imp.  $Z_f$ .



$\therefore V_{bus}^f = V_{bus}^0 + \Delta V$  (2M)

Step-2: To compute  $\Delta V$ , draw a passive <sup>and</sup> thevenin's network.

Step 3: Excite the passive thevenin's network with prefault voltage reversely connected at the faulted bus.

Let  $I^f$  be the current injected and consider bus 'i'.

$\therefore$  vector of changes in bus voltages  $\Delta V = Z_{bus} I^f$ .

where,  $\Delta V = [\Delta V_1, \Delta V_2, \dots, \Delta V_i, \Delta V_r, \dots, \Delta V_n]^T$  (2M)

$I^f = [0, 0, \dots, 0, -I^f, \dots, 0]^T$

$\Rightarrow \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_i \\ \Delta V_r \\ \vdots \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} -Z_{1r} I^f \\ -Z_{2r} I^f \\ \vdots \\ -Z_{ir} I^f \\ -Z_{rr} I^f \\ \vdots \\ -Z_{nr} I^f \end{bmatrix}$

At bus r,  $\Delta V_r = Z_{rr} (-I^f)$   
 voltage at bus r after fault,  
 $V_r^f = V_r^0 + \Delta V_r = V_r^0 - Z_{rr} I^f$   
 At bus r, in general,  $V_r^f = Z_f I^f$   
 $\Rightarrow V_r^0 - Z_{rr} I^f = Z_f I^f$  (2M)

At bus 'i',  $\Delta V_i = -Z_{ir} I^f \therefore I^f = V_r^0 / (Z_{rr} + Z_f)$  (2M)

$V_i^f = V_i^0 + \Delta V_i = V_i^0 - Z_{ir} V_r^0 / (Z_{rr} + Z_f)$

Post-fault currents in all lines,  $I_{ik}^f = Y_{ik} (V_i^f - V_k^f)$

9) b) let base MVA = 20. inertia  $M = \frac{H}{180} = 2.8 \times 10^{-4} \text{ s}^2/\text{elec. deg.}$   
 constant  
 Fault clearing time = 2.5 cycles = 2.5/50 sec. = 0.05 sec. (1M)

Prefault condition

$X_{bf} = 0.35 + j0.2$   
 $X_{bf} = 0.45 \text{ pu}$   
 $(P_{e,bf}) = \frac{E|V| \sin \delta}{X_{bf}}$   
 $(P_e) = 2.44 \text{ pu}$   
 $(P_e)_0 = P_m = 0.9 \text{ pu}$   
 $\Rightarrow 2.44 \sin \delta_0 = 0.9$   
 $\delta_0 = 21.64^\circ$  (2M)

During fault:-

converting star to delta,  
 $X_{df} = 1.25 \text{ pu}$   
 $(P_{e,df}) = \frac{E|V| \sin \delta}{X_{df}}$   
 $(P_e)_{df} = 0.88 \sin \delta$   
 $(P_e)_{max} = 0.88$  (2M)

Post fault:-

$X_{af} = j0.55 \text{ pu}$   
 $(P_{e,af}) = \frac{E|V| \sin \delta}{X_{af}}$   
 $(P_e)_{af} = 2 \sin \delta$   
 $(P_e)_{max} = 2 \text{ pu}$  (2M)

let  $\Delta t = 0.05 \text{ sec.}$   
 At  $t=0$ ,  $(P_a)_0 = \frac{(P_a)_0^- + (P_a)_0^+}{2}$   
 $(P_a)_0^- = 0 \text{ pu}, (P_a)_0^+ = 0.9 - 0.88 \sin 21.64^\circ$   
 $(P_a)_0 = 0.288 \text{ pu} = 0.576 \text{ pu}$

Question Number

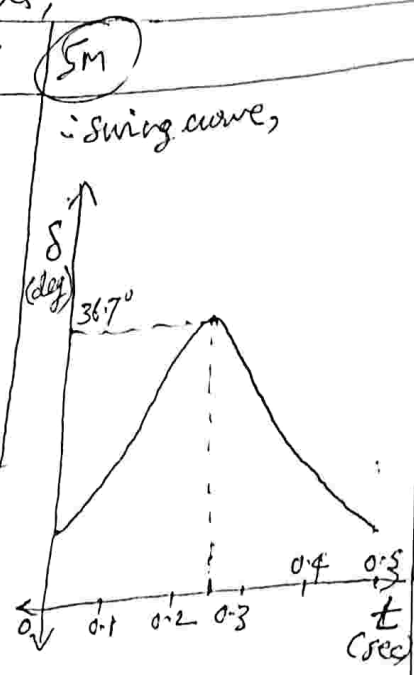
Solution

Marks allocated

$$\begin{aligned} \therefore \Delta \delta_1 &= \Delta \delta_0 + \frac{\Delta t^2}{M} (P_a)_0 = 0 + \frac{0.05^2}{2.8 \times 10^{-4}} \times 0.288 = 2.57^\circ \\ \therefore \delta_1 &= \delta_0 + \Delta \delta_1 = 21.64 + 2.57 = 24.21^\circ \\ \text{At } t=0.05, (P_a)_{0.05} &= \frac{(P_a)_{0.05}^- + (P_a)_{0.05}^+}{2} \\ \therefore \Delta \delta_2 &= \Delta \delta_1 + \frac{\Delta t^2}{M} (P_a)_1 = 2.57 + \frac{0.05^2}{2.8 \times 10^{-4}} \times 0.31 = 5.33 \\ \therefore \delta_2 &= \delta_1 + \Delta \delta_2 = 24.21 + 5.33 = 29.54^\circ \end{aligned}$$

complete calculation for swing curve is,

t (sec)	P <sub>max</sub> (pu)	δ (deg)	P <sub>e</sub> (pu)	P <sub>a</sub> =0.9P <sub>e</sub> (pu)	Δδ (deg)
0-	2.44	21.64	0.9	0	—
0+	0.88	21.64	0.324	0.576	—
0	—	21.64	—	0.288	2.57
0.05-	0.88	24.21	0.36	0.54	—
0.05+	2	24.21	0.82	0.08	—
0.05	2	24.21	—	0.31	2.767
0.1	2	29.54	0.986	-0.086	-0.767
0.15	2	34.1	1.12	-0.22	-1.96
0.2	2	36.7	1.19	-0.29	-2.58
0.25	2	36.72	1.19	-0.29	-2.58
0.3	2	34.36	1.12	-0.22	-1.96
0.35	2	29.84	0.989	-0.089	-0.79
0.4	2	24.33	0.82	0.08	0.71
0.45	2	17.73	0.675	0.225	2
0.5	2	17.13	—	—	—



12M

Critical clearing angle  $\delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) - P_{mf} \times \cos \delta_0 + P_{mf} \times \cos \delta_{max}}{P_{mf} - P_{mdf}}$

$\delta_{cr} = 118.62^\circ$

10(a) Step 1:-  $Z_{bus} = [0.25]$  type-1 (1M)

Step 2:- Type 2  $Z_{bus} = \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.35 \end{bmatrix}$  (2M)

Step 3:- Type 2  $Z_{bus} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix}$  (2M)

Step 4:- Type 3  $Z_{bus} = \begin{bmatrix} 0.1458 & 0.1042 & 0.1458 \\ 0.1042 & 0.1458 & 0.1042 \\ 0.1458 & 0.1042 & 0.2458 \end{bmatrix}$  (2M)

Step 5:- Type-4.  $Z_{bus} = \begin{bmatrix} 0.1397 & 0.1103 & 0.125 \\ 0.1103 & 0.1397 & 0.125 \\ 0.125 & 0.125 & 0.175 \end{bmatrix}$  (3M)

10M

10(b) Algorithm for numerical solution of swing equation by Runge-Kutta method.

10M