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CMR INSTITUTE OF TECHNOLOGY, BENGALURU.
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Internal Assessment Test 1 – July 2022

| Sub: | Engineering Physics Theory | | | | Sub Code: | 21PHY22 | Branch: | ECE/EEE/AIML/AIDS |
|---|---|---|---|---|---|---|---|---|
| Date: | 09/07/2022 | Duration: | 90 min's | Max Marks: 50 | Sem/Sec: | II Sem / I,J,K,L,M,N & O | | OBE |

**Answer any FIVE FULL Questions**

**Given: c = 3 × $10^8$ m/s; h = 6.625 × $10^{-34}$Js; k = 1.38 × $10^{-23}$ J/K; $m_e$ = 9.1 × $10^{-31}$kg; e = 1.6 × $10^{-19}$C**

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| 1 (a) | Derive time independent Schrodinger wave equation for a particle moving in one dimension | [07] | CO1 | L3 |
| (b) | State any three applications of shock waves. | [03] | CO3 | L1 |
| 2 (a) | With the help of a neat diagram, explain the construction and working of Reddy shock tube. | [07] | CO3 | L3 |
| (b) | An electron has a speed of 6.5 x $10^5$ m/s accurate to 0.18%. With what accuracy can the position of electron be located? | [03] | CO1 | L3 |
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| 4 (a) | State Heisenberg's uncertainty principle and show that electrons cannot exist in the nucleus. | [07] | CO1 | L3 |
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**PTO**


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| 5 (a) | What are damped oscillations. Discuss the theory of damped oscillations. Represent overdamping, critical damping and under damping by graph. | [7] | CO3 | L3 |
| (b) | A free particle is executing S.H.M in straight line with a period of 20 seconds. At the equilibrium point, the velocity is found to be 0.9m/s. Find the displacement at the end of 4 seconds, and also the amplitude of oscillation. | [3] | CO2 | L3 |
| 6 (a) | Find the eigen function and energy eigen values for a particle in a one-dimensional potential well of infinite height. | [7] | CO3 | L3 |
| (b) | In a shock tube experiment, the shock wave propagates between the two sensors separated by 10cm in a duration of $100x10^{-6}$ s. Find the Mach number assuming velocity of sound as 340m/s. | [3] | CO3 | L4 |
| 7 (a) | Applying Hooke's law derive the expression for the effective spring constants of Series and Parallel combination of springs. | [6] | CO2 | L3 |
| (b) | A spectral line of wavelength 6500Å has a width of $10^{-5}$Å. Evaluate the minimum time spent by the electron in the upper energy state between the excitation and de-excitation processes. | [4] | CO1 | L4 |
| 8 (a) | Define SHM and mention any two examples. Derive the differential equation using Hooke's law. | [6] | CO2 | L3 |
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## IAT-1 2021-22 EVEN SEM

## SCHEME

### 1.a) <u>Time independent Schrödinger equation</u>

A matter wave can be represented in complex form as

$$\Psi = A\sin kx(\cos wt + i\sin wt) \text{ (1 mark)}$$

$$\Psi = A\sin kx e^{iwt}$$

Differentiating wrt x

$$\frac{d\Psi}{dx} = kA\cos kx e^{iwt}$$

$$\frac{d^2\Psi}{dx^2} = -k^2 A\sin kx e^{iwt} = -k^2\Psi \quad \ldots\ldots\ldots\ldots (1) \text{ (1 mark)}$$

From Debroglie's relation

$$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\Pi p}{h}$$

$$k^2 = 4\Pi^2 \frac{p^2}{h^2} \quad \ldots\ldots\ldots\ldots\ldots (2) \text{ (2 marks)}$$

Total energy of a particle

E = Kinetic energy + Potential Energy

$$E = \frac{p^2}{2m} + V$$

$$E = \frac{1}{2} m v^2 + V$$

$$p^2 = (E - V)2m$$

Substituting in (2)

$$k^2 = \frac{4\Pi^2 (E - V)2m}{h^2}$$

$\therefore$ From (1) (3 marks)

$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 m(E - V)\Psi}{h^2} = 0}$$

### 1B (1 mark for each application)

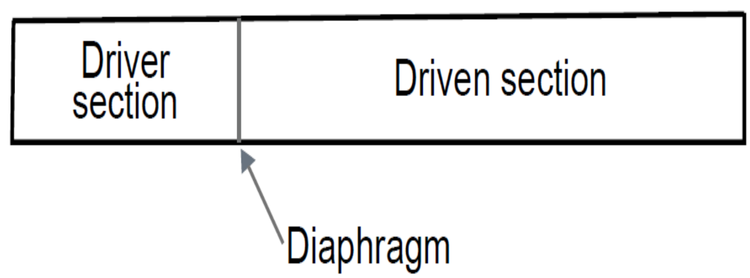
- Aerodynamics – hypersonic shock tunnels, scramjet engines.
- High temperature chemical kinetics – ignition delay
- Rejuvenating depleted bore wells
- Material studies – effect of sudden impact pressure, blast protection materials
- Investigation of traumatic brain injuries – Nerve activation
- Needle-less drug delivery
- Wood preservation – Sandlewood oil extraction

### 2A

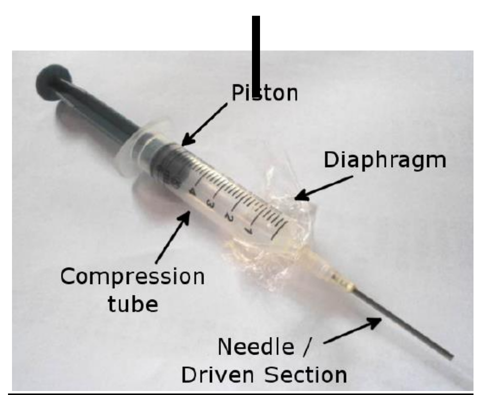
**Reddy shock tube: (Construction 3 marks)**

A shock tube is a device used to study the changes in pressure & temperature which occur due to the propagation of a shock wave. A shock wave may be generated by an explosion caused by the buildup of high pressure which causes diaphragm to burst.

It is a hand driven open ended shock tube. It was conceived with a medical syringe. A plastic sheet placed between the plastic syringe part and the needle part constitutes the diaphragm.




- A high pressure (driver) and a low pressure (driven) side separated by a diaphragm.
- When diaphragm ruptures, a shock wave is formed that propagates along the driven section.
- Shock strength is decided by driver to driven pressure ratio, and type of gases used.

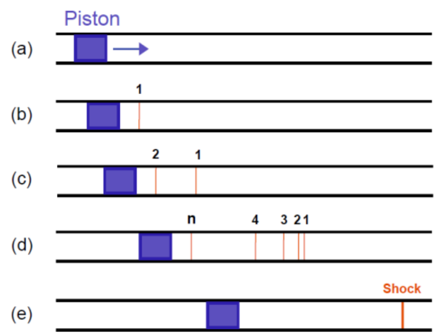


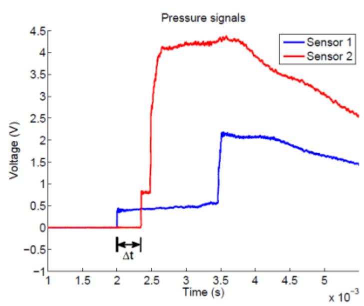

<u>Working:</u> **(Working 4 marks)**

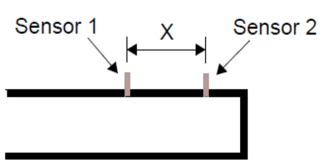
- The piston is initially at rest and accelerated to final velocity V in a short time t.
- The piston compresses the air in the compression tube. At high pressure, the diaphragm ruptures and the shock wave is set up. For a shock wave to form, $V_{piston} > V_{sound}$.

**Formation of shock wave:**

As the piston gains speed, compression waves are set up. Such compression waves increase in number. As the piston travels a distance, all the compression waves coalesce and a single shock wave is formed. This wave ruptures the diaphragm.




$$U_s = \frac{X}{\Delta t}$$

Mach number $M = \frac{V_{Shock}}{V_{Sound}}$

**2B** (Formula-1mark+ Substitution-1mark +Answer-1mark)

$$\Delta v = \frac{0.18}{100} \times 6.5 \times 10^5 = 1770 m/s$$

$$\Delta x = \frac{h}{4\pi m \Delta v} = 3.27 \times 10^{-8} m$$

**3A** ( 2 marks)

1. a black body is imagined to be consisting of large number of electrical oscillators.
2. an oscillator emits or absorbs energy in discrete units. It can emit or absorb energy by making a transition from one quantum state to another in the form of discrete energy packets known as photons whose energy is an integral multiple of hν where h is the planks constant and ν is the frequency.
3. the key point in Planck's theory is the radical assumption of quantized energy states. This development marked the birth of quantum theory.

Based on these ideas, Planck was able to derive an expression that agreed remarkably well with the experimental curves. It is given by

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]} - 1}\right] d\lambda$$

(1 mark)

Where h is Planck's constant, c is velocity of light, T is absolute temperature, λ is the wavelength and k is Boltzmann constant

**Deduction of Weins law: ( 2 marks)**

It is applicable at smaller wavelengths.

For smaller wavelengths $e^{\frac{h\gamma}{kT}} >> 1$

$$\therefore e^{\frac{h\gamma}{kT}} >> 1 = e^{\frac{h\gamma}{kT}}$$

So Planck's radiation law becomes

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]}}\right]$$

**Deduction of Rayleigh Jeans Law: (2 marks)**

It is applicable at longer wavelengths.

For longer wavelengths $\frac{h\gamma}{kT} << 1$

$$\therefore e^{\frac{h\gamma}{kT}} = 1 + \frac{h\gamma}{kT} + \left(\frac{h\gamma}{kT}\right)^2 \frac{1}{2|} + ............ = 1 + \frac{h\gamma}{kT}$$

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{1 + \frac{h\gamma}{kT} - 1} d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

## 3B

**Mach number** is the ratio of velocity of fluid causing the shock wave to the velocity of sound in the medium. It represents the compressibility nature of the medium. (1 mark)
**Supersonic waves:** Shock waves whose Mach number is greater than 1(1 mark)
**Ultrasonic wave :** Sound waves of frequency greater than 20,000Hz. (1 mark)

**4A**

**HEISENBERG'S UNCERTAINTY PRINCIPLE:**

The position and momentum of a particle cannot be determined accurately and simultaneously. The product of uncertainty in the measurement of position $(\Delta x)$ and momentum is always greater than or equal to $\frac{h}{2\Pi}$. **(2 marks)**

$$\boxed{(\Delta x) . (\Delta p) \geq \frac{h}{4\Pi}}$$

**TO SHOW THAT ELECTRON DOES NOT EXIST INSIDE THE NUCLEUS:**

We know that the diameter of the nucleus is of the order of $10^{-14}$m.If the electron is to exist inside the nucleus, then the uncertainty in its position $\Delta x$ cannot exceed the size of the nucleus

$$\Delta x = 5x10^{-15} m$$

Now the uncertainty in momentum is

$$\Delta x = 5x10^{-15} m$$

$$\Delta P = \frac{h}{4\pi x \Delta x} = 0.1x10^{-19} kg.m/s$$ **(2 marks)**

Then the momentum of the electron can atleast be equal to the uncertainty in momentum.

$$P \approx \Delta P = 0.1x10^{-19} kg.m/s$$

Now the energy of the electron with this momentum supposed to be present in the nucleus is given by (for small velocities -non-relativistic-case)

$$E = \sqrt{p^2c^2 + m_o^2c^4} = 548.8x10^{-13} J = 343MeV$$

**(2 marks)**

The beta decay experiments have shown that the kinetic energy of the beta particles (electrons) is only a fraction of this energy. This indicates that electrons do not exist within the nucleus. They are produced at the instant of decay of nucleus ( $n \rightarrow p + e + \bar{\nu}$ / $p \rightarrow n + \overset{+}{e} + \nu$ ). **(1 mark)**

**4B** (Formula-1mark+ Substitution-1mark +Answer-1mark)

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$E = \frac{h^2}{\lambda^2 .2.m} = 7.48x10^{-18} J = 46.44eV$$

**5A:** In damped oscillations, the oscillator looses energy due to frictional forces causing the decrease in amplitude. **(1 mark)**

Let us assume that in addition to the elastic force F =- kx, there is a force that is opposed to the velocity, F = b v where b is damping coefficient
For the oscillating mass in a medium with resistive coefficient b, the equation of motion is given by

$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

This is a homogeneous, linear differential equation of second order.

The auxiliary equation is $D^2 + \frac{b}{m}D + \frac{k}{m} = 0$

The roots are $D_1 = -\frac{b}{2m} + \frac{1}{2m}\sqrt{b^2 - 4mk}$ and $D_2 = -\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2 - 4mk}$

The solution can be derived as $x(t) = Ce^{-\left(\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2-4mk}\right)t} + De^{-\left(\frac{b}{2m} + \frac{1}{2m}\sqrt{b^2-4mk}\right)t}$ …….(1)

Note: This can be expressed as x(t) = $Ae^{-\frac{b}{2m}t}\cos(\omega t - \phi)$ where $\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

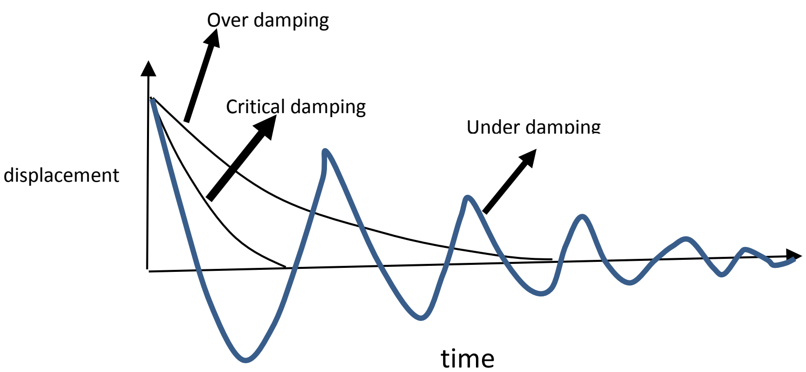
$A = \sqrt{C^2 + D^2}$ $\phi = \tan^{-1}(D/C)$ **(4 marks)**

Here, the term $Ae^{-\frac{b}{2m}t}$ represents the decreasing amplitude and (ωt-ϕ) represents phase**(2 marks)**

Case 1: $b^2 > 4mk$ OVER DAMPING

Case 2: $b^2 < 4mk$ UNDER DAMPING

Case 3: $b^2 = 4mk$ CRITICAL DAMPING




**5B(Formula-1mark+ Substitution-1mark +Answer-1mark)**

$v_{max} = \omega_o A$

$\omega_0 = \frac{2\pi}{T}$

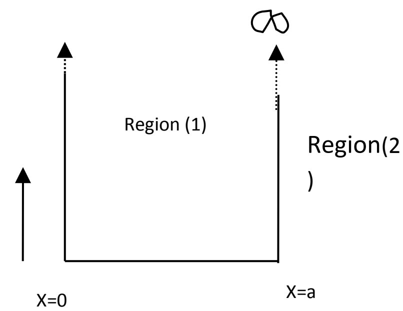
$A = \frac{0.9}{\frac{2\pi}{T}} = 2.86m$

$x = A\sin\omega_o t = 2.86\sin\left(\frac{2\pi}{T}x4\right)$ =2.72m

## 6A

**Particle in an infinite potential well problem:**

Consider a particle of mass m moving along X-axis in the region from X= 0 to X = a in a one dimensional potential well as shown in the diagram. The potential energy is assumed to be zero inside the region and infinite outside the region.




**(1 mark)**

Applying, Schrodingers equation for region (1) as particle is supposed to be present in region (1)

$$\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 mE\psi}{h^2} = 0 \because V = 0 \text{ for region (1)}$$

But $k^2 = \frac{8\Pi^2 mE}{h^2}$

$$\therefore \frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

Auxiliary equation is $(D^2 + k^2)x = 0$

Roots are D = +ik and D = -ik

The general solution is **(2 marks)**

$$
\begin{aligned}
x &= Ae^{ikx} + Be^{-ikx} \\
&= A(\cos kx + i\sin kx) + B(\cos kx - i\sin kx) \\
&= (A+B)\cos kx + i(A-B)\sin kx \\
&= C\cos kx + D\sin kx
\end{aligned}
$$

The boundary conditions are

1. At x=0, $\Psi = 0 \therefore C = 0$

2. At x=a, $\Psi = 0$

D sin ka = 0 $\Rightarrow$ ka = n$\Pi$ ………(2)

where n = 1, 2 3…

$$\therefore \Psi = D\sin\left(n\frac{\Pi}{a}\right)x$$

From (1) and (2) E = $\frac{n^2h^2}{8ma^2}$ **(2 marks)**

**To evaluate the constant D:**

Normalisation: For one dimension

$$\int_0^a \Psi^2 dx = 1$$

$$\int_0^a D^2 \sin^2\left(\frac{n\Pi}{a}\right)x dx = 1$$

But $\cos 2\theta = 1 - 2\sin^2\theta$

$$\int_0^a D^2 \frac{1}{2}\left(1 - \cos 2\left(\frac{n\Pi}{a}\right)x\right)dx = 1$$

$$\int_0^a \frac{D^2}{2}dx - \int_0^a \frac{1}{2}\cos 2\left(\frac{n\Pi}{a}\right)x)dx = 1$$

$$\frac{D^2 a}{2} - \quad [\sin 2\left(\frac{n\Pi}{a}\right)\frac{x}{2}]_0^a = 1$$

$$D^2\frac{a}{2} - 0 = 1$$

$$D = \sqrt{\frac{2}{a}}$$

$$\therefore \Psi_n = \sqrt{\frac{2}{a}}\sin\left(n\frac{\Pi}{a}\right)x$$ **(2 marks)**

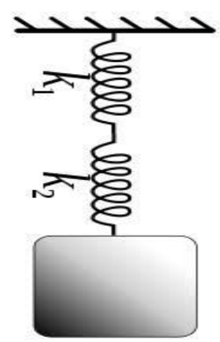
**6B** **(Formula-1mark+ Substitution-1mark +Answer-1mark)**

$$V_{shock} = \frac{dx}{dt} = \frac{0.1}{100x10^{-6}} = 1000 m/s$$

$$M = \frac{V_{Shock}}{V_{sound}} = \frac{1000}{340} = 2.94$$

**7A**

**Expression for Spring Constant for Series Combination (3marks)**




Consider a load suspended through two springs with spring constants $k_1$ and $k_2$ in series combination. Both the springs experience same stretching force. Let $\Delta x_1$ and $\Delta x_2$ be their elongation.

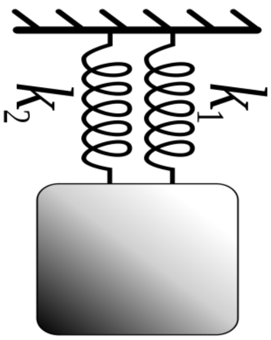
Total elongation is given by

$$\Delta X = \Delta X_1 + \Delta X_2 = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{F}{k_{eqv}} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{1}{k_{eqv}} = \frac{1}{k_1} + \frac{1}{k_2}$$

**Expression for Spring Constant forParallel Combination(3marks)**




Consider a load suspended through two springs with spring constants $k_1$ and $k_2$ in parallel combination. The two individual springs both elongate by x but experience the load non uniformly.

Total load across the two springs is given by

$$F = F_1 + F_2$$

$$k_{eqv}.\Delta X = k_1.\Delta X + k_2.\Delta X$$

$$k_{eqv} = k_1 + k_2$$

**7B** (Formula-1mark+ Substitution-1mark +Answer-2marks)

$$\Delta E.\Delta t = \frac{h}{4\pi}$$

$$\Delta E = \frac{hc}{\lambda^2}\Delta\lambda = 4.7x10^{-28}\,J$$

$$\Delta t = \frac{h}{4\pi.\Delta E}$$

$=1.12x10^{-7}$ s

## 8A

**SIMPLE HARMONIC MOTION**

It is the periodic oscillations of an object caused when the restoring force on the object is proportional to the displacement. The restoring force is directed opposite to displacement.

Ex: 1. Oscillation of mass connected to spring

2. Oscilations of prongs of Tuning fork

3. Simple pendulum

Restoring force α – displacement

F = -k x **(3marks)**

Here k is the proportionality constant known as spring constant. It represents the amount of restoring force produced per unit elongation and is a relative measure of stiffness of the material.

$$F_{\text{Re}\,storing} = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$Let\ \omega_o{}^2 = \frac{k}{m}$$

$$\frac{d^2x}{dt^2} + \omega_o{}^2 x = 0$$

Here $\omega_o$ is angular velocity = 2.π.f

f is the natural frequency $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

The Solution is of the form $x(t) = A\cos\omega_o t + B\sin\omega_o t$.

This can also be expressed as $x(t) = C\cos(\omega_o t-\theta)$ where

$C = \sqrt{A^2 + B^2}$ $\tan\theta = B/A$ **(3marks)**

**8B** (Formula-1mark+ Substitution-1mark +Answer-2marks)

n=3

$$\Pr obability = \int_{L/3}^{2L/3} \psi^2 dx = \int \left(\frac{2}{L}\right) \sin^2 \left({}^{n\pi}\!/\!_{L}\right) \; dx$$

$$= \frac{2}{2L}\int dx - \frac{2}{2L}\int \cos\left(\frac{2n\pi}{L}\right) x.dx$$

$$= \frac{1}{L}\left(\frac{L}{3}\right) - \frac{1}{L}\left(\frac{\sin\left(\frac{2n\pi}{L}x\right)_{L/3}^{2L/3}}{\frac{2n\pi}{L}}\right)$$

$$= 0.33$$