USN

INTERNAL ASSESSMENT TEST – IV

Sub:	DIGITAL SIGNAL PROCESSING							Code:	18EC52
Date:	04/ 02 / 2022	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE

Answer any 5 full questions

		Marks	CO	RBT
1	Derive an expression for DFT and IDFT of a finite length sequence.	[10]	CO1	L2
2	Compute the 6-point DFT of the sequence $x[n] = [1,3,5,7]$. Plot the magnitude spectrum and the phase spectrum.	[10]	CO1	L2
3(a)	Compute the 4-point DFT of $x[n] = [1,2,3,4]$ using matrix method. Plot the magnitude spectrum and the phase spectrum.	[06]	CO1	L2
3(b)	Compute the IDFT of $X[k] = [9, -3 + j1.7321, -3 - j1.7321]$ using matrix method.	[04]	CO1	L2

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4	The first 5 samples of 8-point DFT of a real 8-point sequence are as follows. $X[k] = [36, -4 + 9.6569j,\ -4 + 4j,\ -4 + 1.6569j,\ -4]$. Determine the remaining samples of $X[k]$. Evaluate the following without explicitly determining $x[n]$. i) $x[0]$ ii) $x[4]$ iii) $\sum_{n=0}^{7} x[n]$ iv) $\sum_{n=0}^{7} \lvert x[n]\rvert^2$	[10]	CO1	L3
5	Derive the relationship between DFT and Z-transform of a finite length sequence $x[n], 0 \le n \le N-1$. Compute the Z-transform of the sequence $x[n] = [0.5, 0, 0.5, 0]$. Using Z-transform compute the DFT of $x[n]$.	[10]	CO1	L2
6(a)	Compute the DFT of the sequence $x[n] = 0.5^n, 0 \le n \le 3$ by evaluating the DFT of $x[n] = a^n, 0 \le n \le N-1$ and $0 < a < 1$.	[05]	CO1	L2
6(b)	Prove the periodicity and linearity properties of DFT.	[05]	CO1	L2

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6(b)	Prove the periodicity and linearity properties of DFT.	[05]	CO1	L2

Solutions

1 $$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$X\left(\frac{2\pi}{N}k\right) = \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$0 \le k \le N-1$

$$= \sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$x_p(n) = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$\therefore a_k = \frac{1}{N} X(k)$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn}$$

$0 \le n \le N-1$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$0 \le k \le N-1$

2

k	$X(k)$	$\lvert X(k)\rvert$	$\angle X(k)$
0	16	16	0
1	$-7-6.9282j$	9.8489	-2.3613
2	$4+1.7321j$	4.3589	0.4086
3	-4	4	3.1416
4	$4-1.7321j$	4.3589	-0.4086
5	$-7+6.9282j$	9.8489	2.3613

3a

k	$X(k)$	$\lvert X(k) \rvert$	$\angle X(k)$
0	10	10	0
1	$-2+2j$	2.8284	2.3562
2	-2	2	3.1416
3	$-2+2j$	2.8284	-2.3562

3b $x(n) = (1, 3, 5)$

4 $$x(0) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)$$ $\qquad x(n) = (1,2,3,4,5,6,7,8)$

$$= 1$$

$$x(4) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)(-1)^k$$

$$= 5$$

$$\sum_{n=0}^{7} x(n) = X(0)$$
$$= 36$$

$$\sum_{n=0}^{7} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{7} |X(k)|^2$$

$$= 204$$

5 $$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n}$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$X(k) = X(z)\Big|_{z = e^{j\frac{2\pi}{N}k}}$$

$x(n) = (0.5, 0, 0.5, 0)$

$X(z) = 0.5 + 0.5 z^{-2}$

$X(k) = 0.5 + 0.5 e^{-j\frac{2\pi}{4}k(\cdot 2)}$

$= 0.5 + 0.5 e^{-j\pi k}$

$= (1, 0, 1, 0)$

6a $\quad x(n) = a^n$

$X(k) = \sum_{n=0}^{N-1} \left(a e^{-j\frac{2\pi}{N}k}\right)^n$

$= \dfrac{1 - a^N}{1 - a e^{-j\frac{2\pi}{N}k}}$

$x(n) = 0.5^n, \quad 0 \le n \le 3$

$X(k) = \dfrac{1 - 0.5^4}{1 - 0.5 e^{-j\frac{2\pi}{4}k}}$

$= \dfrac{0.9375}{1 - 0.5 e^{-j\frac{\pi}{2}k}}$

6b $\quad X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$

$X(k+N) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}(k+N)n}$

$= X(k)$

$a x_1(n) + b x_2(n) \xleftrightarrow{DFT} a X_1(k) + b X_2(k)$