CBCS SCHEME

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20SCN/SCE/SSE/SCS/SIT/SIS/SFC/SAM/SAD/SDS11

First Semester M.Tech. Degree Examination, Feb./Mar. 2022
Mathematical Foundations of Computer Science

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define subspace. **(03 Marks)**

b. Prove that if w_1 and w_2 are subspaces of vector space V(F) then $w_1 + w_2$ is a subspace of V(F). **(07 Marks)**

c. Show that the set $S = \{t^2+1,\ t-1,\ 2t+2\}$ is a basis for the vector space P_2. **(10 Marks)**

OR

2 a. Define linear span. **(03 Marks)**

b. Let $S = \{(1,-3,2),(2,4,1),(1,1,1)\}$ be a subset of $V_3(R)$. Show that the vectors (3, –7, 6) is in L[S]. **(07 Marks)**

c. Let $S = \{V_1, V_2, V_3, V_4\}$ be a basis for R^4, where $V_1 = (1, 1, 0, 0)$, $V_2 = (2, 0, 1, 0)$, $V_3 = (0, 1, 2, -1)$ and $V_4 = (0, 1, -1, 0)$. If $V = (1, 2, -6, 2)$, compute $[V]_S$. **(10 Marks)**

Module-2

3 a. Define an inner product space. **(03 Marks)**

b. For any vectors α, β in an inner product space V and any scalar C, prove that

(i) $\|c\alpha\| = c\|\alpha\|$ (ii) $\|\alpha\| > 0$ for $\alpha \neq 0$

(iii) $\|(\alpha|\beta)\| \le \|\alpha\| + \|\beta\|$ (iv) $\|\alpha+\beta\| \le \|\alpha\| + \|\beta\|$ **(07 Marks)**

c. Construct an orthogonal basis for w given

$$x_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix},\quad x_2 = \begin{bmatrix}3\\1\\1\\3\end{bmatrix},\quad x_3 = \begin{bmatrix}5\\0\\2\\3\end{bmatrix}$$ **(10 Marks)**

OR

4 a. Define orthogonal projection. **(03 Marks)**

b. Prove that an orthogonal set of non-zero vectors is linearly independent and hence forms the basis for the subspace spanned by the set. **(07 Marks)**

c. Find the least square solution of the system Ax = b for $A = \begin{bmatrix}4 & 0\\0 & 2\\1 & 1\end{bmatrix}$, $b = \begin{bmatrix}2\\0\\11\end{bmatrix}$ and also determine the least square error in the solution of Ax = b. **(10 Marks)**

Module-3

5 a. Diagonalize the matrix $A = \begin{bmatrix}4 & 1\\2 & 3\end{bmatrix}$, find A^6. **(10 Marks)**

b. Convert the quadratic form $Q(x) = x_1^2 - 8x_1x_2 - 5x_2^2$ into quadratic form with no cross product. **(10 Marks)**

OR

6 a. Find the maximum and minimum values of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$ subject to the constraint $X^TX = 1$. (10 Marks)

b. Find the singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$. (10 Marks)

Module-4

7 a. Find the correlation coefficient and the line of regression of y on x for the

x	1	2	3	4	5
y	2	5	3	8	7

(10 Marks)

b. Fit a straight line for the following data:

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(10 Marks)

OR

8 a. Show that $\tan\theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$ where θ is acute angle. Explain the significance of the formula when $r = 0$ and $r = \pm 1$. (10 Marks)

b. Fit a parabola $y = a + bx + cx^2$ for the following data:

x	1	2	3	4
y	1.7	1.8	2.3	3.2

(10 Marks)

Module-5

9 a. A random variable X has the following pmf for various values of X.

X	0	1	2	3	4	5	6	7	8
F(X)	K	3K	5K	7K	9K	11K	13K	15K	17K

Solve: (i) Value of K (ii) $P(X < 3)$, $P(X \geq 3)$, $P(0 < X <= 5)$
(iii) Find cumulative distribution function
(iv) What is the smallest value of X for which $P(X \leq x) > 0.5$? (10 Marks)

b. A certain injection administered to each of 12 patients resulted in the following increases of B.P: 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be concluded that the injection will be, in general, accompanied by an increase in BP? (10 Marks)

OR

10 a. A random variable X has the following pdf:

$$P(X) = \begin{cases} Ke^{-x} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Solve: (i) Value of constant K (ii) Mean (iii) Variance (iv) F(0.5) (10 Marks)

b. The following data show defective articles produced by 4 machines:

Machine	A	B	C	D
Production time	1	1	2	3
No. of defective	12	30	63	98

Do the figures indicate a significant difference in the performance of the machines? [Use $\chi^2_{0.05}(\gamma = 3) = 7.815$] (10 Marks)

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