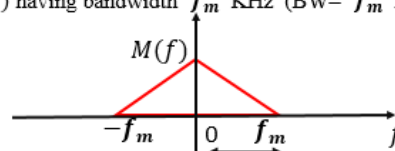
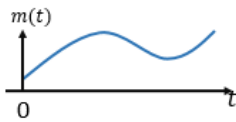


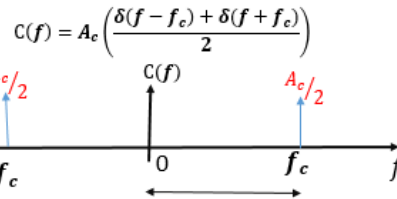
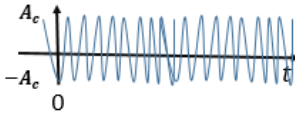
Amplitude Modulation description in time domain and frequency domain

Consider an arbitrary message signal $m(t)$, with Fourier transform $M(f)$ having bandwidth ' f_m ' KHz (BW= ' f_m ' KHz)



Consider a carrier signal $c(t) = A_c \cos 2\pi f_c t$, with Fourier transform $C(f)$ having bandwidth ' f_c ' KHz (BW= ' f_c ' MHz)

$$c(t) = A_c \cos(2\pi f_c t)$$

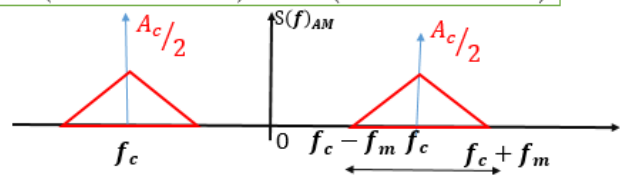
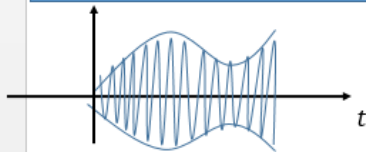


The AM wave in time domain is given by

$$s(t)_{AM} = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t$$

$$s(t)_{AM} = A_c (1 + k_a m(t)) \cos 2\pi f_c t$$

$$S(f)_{AM} = A_c \left(\frac{\delta(f - f_c) + \delta(f + f_c)}{2} \right) + A_c k_a \left(\frac{M(f - f_c) + M(f + f_c)}{2} \right)$$



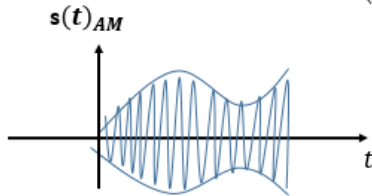
The AM wave in time domain is given by

$$s(t)_{AM} = A_c (1 + k_a m(t)) \cos 2\pi f_c t$$

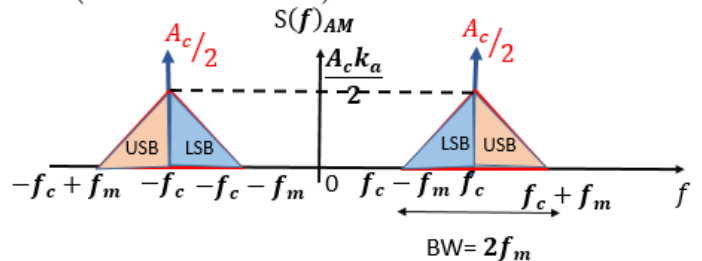
$$s(t)_{AM} = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t \dots \dots \dots (1)$$

Taking Fourier transform of equation(1), both the sides

$$S(f)_{AM} = A_c \left(\frac{\delta(f - f_c) + \delta(f + f_c)}{2} \right) + A_c k_a \left(\frac{M(f - f_c) + M(f + f_c)}{2} \right) \dots \dots \dots (2)$$



F.T. \longleftrightarrow



Bandwidth requirement for AM

$$BW = 2f_m = 2(\text{Message signal BW})$$

Power requirement for AM

$$P_t = P_c + P_{USB} + P_{LSB}$$

2)

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Envelope detector (AM detection)

$s(t)_{AM}$
 $A_c(1 + k_a m(t)) \cos 2\pi f_c t$

AM wave $s(t)$

Fig (a)

$A_c(1 + k_a m(t))$

Demodulation is used to recover the original modulating wave from the incoming modulated wave

The demodulation of an AM wave can be accomplished using various devices; here, we describe a simple, yet highly effective device known as the *envelope detector*

The AM wave has to be narrow-band, which requires that the carrier frequency be large compared to the message bandwidth. Moreover, the percentage modulation must be less than 100 percent.

An envelope detector of the series type is shown in Figure (a) which consists of a diode and a resistor-capacitor (RC) filter.

WORKING

On a positive half-cycle of the input signal, the diode is forward-biased and the capacitor C charges up rapidly to the peak value of the input signal.

When the input signal falls below this value, the diode becomes reverse-biased and the capacitor C discharges slowly through the load resistor R_L

The discharging process continues until the next positive half-cycle. When the input signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated

We assume that the AM wave applied to the envelope detector is supplied by a voltage source of internal impedance R_s

Selection of Time constant

The charging time constant R_s must be short compared with the carrier period $\frac{1}{f_c}$, while discharging time constant must be compared with message signal bandwidth $\frac{1}{f_m}$, so ideal time constant for envelope detector is

$$\frac{1}{f_c} \ll R_L C \ll \frac{1}{f_m}$$

3) VSB Generation and Demodulation

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Message signal $m(t)$

Product modulator

$A_c \cos(2\pi f_c t)$

$u(t)$

Band-pass filter $H(f)$

Modulated signal $s(t)$

(a)

This signal is further bandlimited using a bandpass filter for VSB having impulse response $h(t)$, then the VSB modulated signal is given by

$$s(t) = u(t) * h(t)$$

$$\downarrow FT$$

$$S(f) = U(f)H(f)$$

$$S(f) = \frac{A_c}{2} \{M(f - f_c) + M(f + f_c)\} H(f)$$
2

Part (a) represents the modulation process used for the generation of VSB

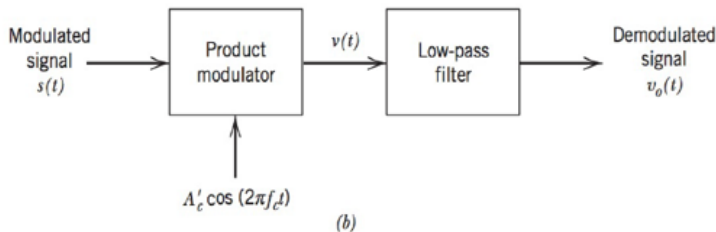
In the product modulator message signal is multiplied with carrier signal to produce a DSB-SC signal indicated as $u(t)$

$$u(t) = m(t)A_c \cos 2\pi f_c t$$

$$\downarrow FT$$

$$U(f) = \frac{A_c}{2} \{M(f - f_c) + M(f + f_c)\}$$
1

$$S(f) = \frac{A_c}{2} \{M(f - f_c) + M(f + f_c)\}H(f)$$



Part(b) represents the demodulation process used for the recovery of message signal from VSB modulated wave. The transmitted VSB signal $s(t)$ is received and function as one of the input for product modulator in part (b), this signal is further multiplied by the carrier signal locally produced in the receiver.

The signal $v(t)$ is given by

$$v(t) = s(t)A'_c \cos 2\pi f_c t$$

$\downarrow FT$

$$V(f) = \frac{A'_c}{2} \{S(f - f_c) + S(f + f_c)\} \quad (3)$$

$$\text{If, } S(f) = \frac{A_c}{2} \{M(f - f_c) + M(f + f_c)\}H(f)$$

$$\text{then, } S(f - f_c) = \frac{A_c}{2} \{M(f - f_c - f_c) + M(f - f_c + f_c)\}H(f - f_c)$$

$$\text{then, } S(f - f_c) = \frac{A_c}{2} \{M(f - 2f_c) + M(f)\}H(f - f_c) \quad (4)$$

$$\text{and, } S(f + f_c) = \frac{A_c}{2} \{M(f + f_c - f_c) + M(f + f_c + f_c)\}H(f + f_c)$$

$$\text{and, } S(f + f_c) = \frac{A_c}{2} \{M(f) + M(f + 2f_c)\}H(f + f_c) \quad (5)$$

$$V(f) = \frac{A'_c}{2} \{S(f - f_c) + S(f + f_c)\}$$

$$V(f) = \frac{A'_c A_c}{2} \left\{ \frac{1}{2} \{M(f - 2f_c) + M(f)\}H(f - f_c) + \frac{1}{2} \{M(f) + M(f + 2f_c)\}H(f + f_c) \right\}$$

Output of LPF

$$V(f) = \frac{A'_c A_c}{2} \{M(f)\}H(f - f_c) + \{M(f)\}H(f + f_c) \quad (6)$$

$$V_0(f) = \frac{A'_c A_c}{2} \{M(f)\} \{H(f - f_c) + H(f + f_c)\}$$

$\downarrow IFT$

$$v(t) = \frac{A'_c A_c}{4} m(t) \quad \because H(f - f_c) + H(f + f_c) = 1 \quad (6)$$

4)

Problem 1

A modulating signal $m(t) = 10 \cos(2\pi \times 10^3 t)$ is amplitude modulated with a carrier signal $c(t) = 50 \cos(2\pi \times 10^5 t)$. Find the modulation index, the carrier power, and the power required for transmitting AM wave.

Given, the equation of modulating signal as

$$m(t) = 10 \cos(2\pi \times 10^3 t)$$

We know the standard equation of modulating signal as

$$m(t) = A_m \cos(2\pi f_m t)$$

Amplitude of modulating signal as $A_m = 10 \text{ volts}$

and Frequency of modulating signal as

$$f_m = 10^3 \text{ Hz} = 1 \text{ KHz}$$

Given, the equation of carrier signal is

$$c(t) = 50 \cos(2\pi \times 10^5 t)$$

By comparing these two equations, we will get

Amplitude of carrier signal as $A_c = 50 \text{ volts}$

and Frequency of carrier signal as $f_c = 10^5 \text{ Hz} = 100 \text{ KHz}$

We know the formula for modulation index as

$$\mu = \frac{A_m}{A_c}$$

Substitute, A_m and A_c values in the above formula.

$$\mu = \frac{10}{50} = 0.2$$

The formula for Carrier power

$$P_c = \frac{A_c^2}{2R}$$

Assume $R = 1 \Omega$ and substitute A_c value in the above formula.

$$P_c = \frac{(50)^2}{2(1)} = 1250 \text{ W}$$

Substitute P_c and μ values in the above formula.

$$P_t = 1250 \left(1 + \frac{(0.2)^2}{2} \right) = 1275 \text{ W}$$

2.3) NOISE IN AM RECEIVERS

- An AM signal is given by

$$s(t) = A_c(1 + k_a m(t)) \cos 2\pi f_c t \quad 1$$

where,

$m(t)$ = message signal and let us assume that the message signal power is 'P' watts

$c(t) = A_c \cos 2\pi f_c t$ = carrier signal and

the average power of the carrier signal is $\frac{A_c^2}{2}$

$$s(t) = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t \quad 2$$

So average power in $s(t) = \frac{A_c^2}{2} + \frac{A_c^2 k_a^2 P}{2} = \frac{A_c^2}{2} (1 + k_a^2 P)$

- The combination $s(t) + w(t)$ is applied to a bandpass filter, the BPF is actually a narrow- BPF such that $f_c \gg B_T$,

- After passing from BPF, wideband noise $w(t)$ gets converted into narrowband noise $n(t)$

- The filtered signal $x(t)$ available for demodulation is defined by

$$x(t) = s(t) + n(t)$$

$$x(t) = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t + n(t) \quad 3$$

From equation (3)

$$x(t) = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t + n(t)$$

$$x(t) = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

$$x(t) = A_c (1 + k_a m(t) + n_I(t)) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \quad 5$$

After passing from envelope detector, the envelope of output $x(t)$ is given by

$y(t)$ = Envelope of $x(t)$

$$y(t) = \left\{ (A_c (1 + k_a m(t) + n_I(t)))^2 + (n_Q(t))^2 \right\}^{1/2} \quad 6$$

However, when the average carrier power is large compared with the average noise power, so that the receiver is operating satisfactorily, then the signal term $A_c (1 + k_a m(t))$ will be large compared with the noise terms $n_I(t)$ and $n_Q(t)$, at least most of the time. Then we may approximate the output $y(t)$ as

$$y(t) \approx A_c (1 + k_a m(t) + n_I(t))$$

After passing from LPF

$$y(t) \approx A_c k_a m(t) + n_I(t) \quad 7$$

Demodulated Signal

Noise

The average power of the demodulated signal is = $\frac{A_c^2 k_a^2 P}{2}$ 8

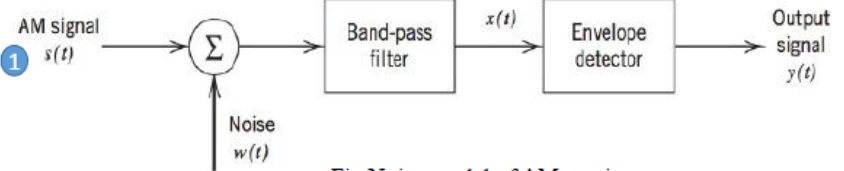


Fig Noisy model of AM receiver.

- The power of the noise $n(t)$ is given by $N_o W$, where W is the bandwidth of message signal

- we define the channel signal-to-noise ratio,

$$(SNR)_c = \frac{\text{average power of the modulated signal}}{\text{the average power of noise in the message bandwidth}}$$

$$(SNR)_c = \frac{\frac{A_c^2}{2} (1 + k_a^2 P)}{N_o W} = \frac{A_c^2 (1 + k_a^2 P)}{2 N_o W} \quad 4$$

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- The power of the noise $n_I(t)$ is given by $N_o W$, where W is the bandwidth of message signal

- The output signal-to-noise ratio, average power of the demodulated message signal
- $$(SNR)_o = \frac{\text{average power of the demodulated message signal}}{\text{the average power of the noise}}$$

$$(SNR)_o = \frac{A_c^2 k_a^2 P}{2 N_o W} \quad 9$$

Finally we need to find out 'Figure of Merit' of AM as

$$FOM = \frac{(SNR)_o}{(SNR)_c}$$

$$FOM = \frac{\frac{A_c^2 k_a^2 P}{2 N_o W}}{\frac{A_c^2 (1 + k_a^2 P)}{2 N_o W}}$$

$$FOM = \frac{k_a^2 P}{1 + k_a^2 P} \quad 10$$

FOM of AM receiver is given by

$$FOM = \frac{k_a^2 P}{1 + k_a^2 P}$$

Suppose, if the message signal is a singletone waveform then, $m(t) = A_m \cos 2\pi f_m t$,

Then average message signal power is

$$P = \frac{A_m^2}{2} \quad (11)$$

So,

$$FOM = \frac{k_a^2 \left(\frac{A_m^2}{2}\right)}{1 + k_a^2 \left(\frac{A_m^2}{2}\right)} = \frac{\mu^2}{2 + \mu^2} \quad (\because \mu = k_a A_m) \quad (12)$$

Equation (12) is, however, valid only if the following two conditions are satisfied:

1. The average noise power is small compared to the average carrier power at the envelope detector input.
 2. The amplitude sensitivity k_a is adjusted for a percentage modulation less than or equal to 100 percent
- figure of merit of an AM receiver using envelope detection is always less than unity. In other words, *the noise performance of an AM receiver is always inferior to that of a DSB-SC receiver*. This is due to the wastage of transmitter power, which results from transmitting the carrier as a component of the AM wave.

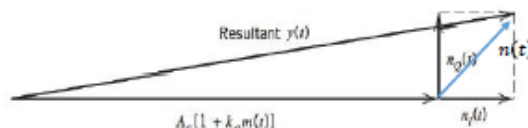


Fig. Phasor diagram for AM wave plus narrowband noise for the case of high carrier-to-noise ratio.

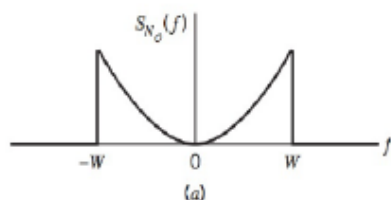
$$n(t) = n_i(t) + jn_q(t)$$

$$r(t) = \sqrt{n_i^2 + n_q^2}$$

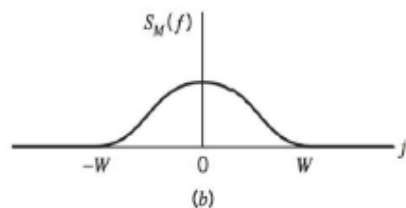
6) (iii)Pre emphasis and (iv) Deemphasis

➤ The power spectral density of output noise is given by

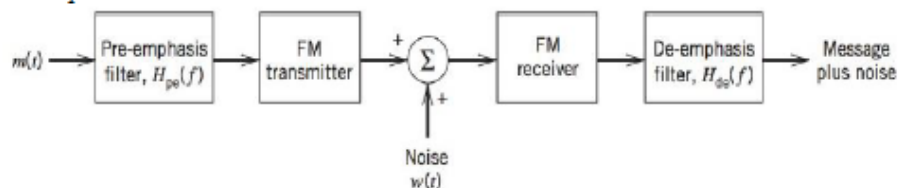
$$S_{No}(f) = \begin{cases} \frac{f^2}{A_c} N_0 : |f| \leq B_T = W & (1) \\ 0; \text{otherwise} \end{cases}$$



➤ The power spectral density of a typical message source; audio and video signals typically have spectra of this form, shown in figure (b),



- Near the cut-off frequency noise becomes more dominant compared to message signal, obviously SNR will go down.
- A more satisfactory approach to the efficient utilization of the allowed frequency band is based on the use of *pre-emphasis* in the transmitter and *de-emphasis* in the receiver



- In this method, we artificially emphasize the high-frequency components of the message signal prior to modulation in the transmitter
- Then, at the discriminator output in the receiver, we perform the inverse operation by de-emphasizing the high-frequency components, so as to restore the original signal-power distribution of the message
- In order to produce an undistorted version of the original message at the receiver output, the pre-emphasis filter in the transmitter and the de-emphasis filter in the receiver must ideally have transfer functions that are the inverse of each other.

$$H_{de}(f) = \frac{1}{H_{pe}(f)} \quad (2)$$

➤ Simple pre-emphasis filter that emphasizes high frequencies and is commonly used in practice is defined by the transfer function

$$H_{pe}(f) = 1 + \frac{jf}{f_0} \quad (3)$$

Hence,

$$H_{de}(f) = \frac{1}{1 + \frac{jf}{f_0}} \quad (4)$$

Average output noise power

$$\begin{aligned} \text{with de-emphasis} &= |H_{de}(f)|^2 \int_{-W}^W S_{N_0}(f) df \\ &= \frac{N_0}{A_c^2} \int_{-W}^W |H_{de}(f)|^2 f^2 df \quad (5) \end{aligned}$$

The improvement in output signal-to-noise ratio produced by the use of pre-emphasis in the transmitter and de-emphasis in the receiver is defined by

$$I = \frac{\text{average output noise power without pre-emphasis and de-emphasis}}{\text{average output noise power with pre-emphasis and de-emphasis}}$$

$$I = \frac{\frac{2N_0W^3}{3A_c^2}}{\frac{N_0}{A_c^2} \int_{-W}^W |H_{de}(f)|^2 f^2 df} = \frac{2W^3}{3 \int_{-W}^W |H_{de}(f)|^2 f^2 df}$$

$$I = \frac{2W^3}{3 \int_{-W}^W |H_{de}(f)|^2 f^2 df}$$

$$I = \frac{2W^3}{3 \int_{-W}^W \left| \frac{1}{1 + \frac{jf}{f_0}} \right|^2 f^2 df}$$

$$I = \frac{(W/f_0)^3}{3 \left[(W/f_0) + \tan^{-1}(W/f_0) \right]} \quad (7)$$

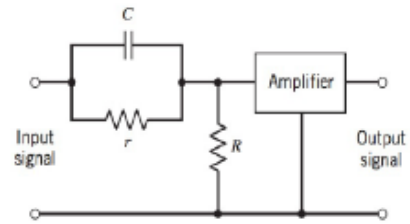


Fig.(c) Pre-emphasis filter

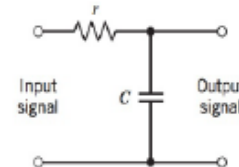
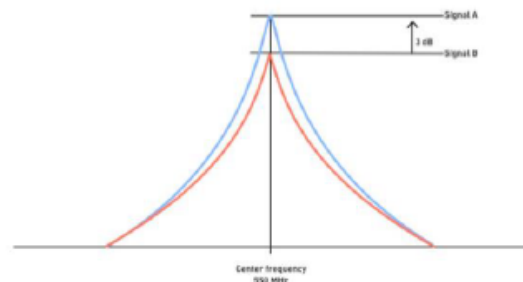


Fig.(d) de-emphasis filter

i) Capture effect

2.6) CAPTURE EFFECT

- In a radio receiver, the capture effect, or FM capture effect, is a phenomenon associated with FM reception in which only the stronger of two signals at, or near, the same frequency or channel will be demodulated.
- The capture effect is defined as the complete suppression of the weaker signal at the receiver's limiter (if present) where the weaker signal is not amplified, but attenuated.
- When both signals are nearly equal in strength, or are fading independently, the receiver may switch from one to the other and exhibit picket fencing.
- The capture effect can occur at the signal limiter, or in the demodulation stage.
- Some types of radio receiver circuits have a stronger capture effect than others. The measurement of how well a receiver can reject a second signal on the same frequency is called the capture ratio for a specific receiver.
- It is measured as the lowest ratio of the power of two signals that will result in the suppression of the smaller signal.



- The capture effect occurs with very low ratios between a signal of interest and a competing FM signal. This ratio depends on the receiver type and quality, but a separation of 3-4 dB is needed between two signals for the receiver to "lock on" to one instead of the other.

(ii) Threshold effect

2.7) FM THRESHOLD EFFECT

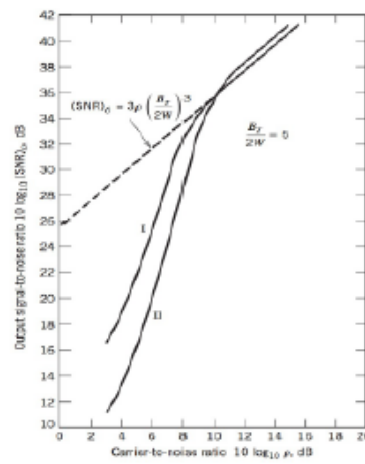
- An important aspect of analog FM systems is FM threshold effect. In FM systems where the signal level is well above noise received carrier-to-noise ratio then below FOM expression is valid

$$FOM = \frac{3k_f^2 P}{W^2} \quad (1)$$

- The expression however does not apply when the carrier-to-noise ratio decreases below a certain point. Below this critical point the signal-to-noise ratio decreases significantly, this is known as the FM threshold effect
- Below the FM threshold point the noise signal (whose amplitude and phase are randomly varying), may instantaneously have an amplitude greater than that of the wanted signal.
- When this happens the noise will produce a sudden change in the phase of the FM demodulator output. In an audio system this sudden phase change makes a "click".
- To characterize threshold performance, let the carrier-to-noise ratio be defined by

$$\rho = \frac{A_c^2}{N_0 B_T} \Rightarrow \rho = \frac{A_c^2}{2N_0 B_T} \quad (2)$$

- As ρ is decreased, the average number of clicks per unit time increases. When this number becomes appreciably large, the threshold is said to occur.



- In most practical cases of interest if the carrier-to-noise ratio ρ is equal to or greater than 20 or, equivalently, 13 dB. Thus, using Eq. (2) we find that the loss of message at the discriminator output is negligible if

$$\frac{A_c^2}{2N_0 B_T} \geq 20$$

or, equivalently, if the average transmitted power $\frac{A_c^2}{2}$ satisfies the condition

$$\frac{A_c^2}{2} \geq 20N_0 B_T$$