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Internal Assesment Test – V

Sub:	Information Theory and Coding		Sec	*ALL*			Code:	18EC54
Date:	07 / 02 / 2022	Duration:	90 mins	Max Marks:	50	Sem: V	Branch:	ECE

	Answer Any FIVE FULL Questions	Marks	OBE CO	OBE RBT
1	Define the terms Self information content, average information content, average rate of information and hence derive the expression for average information content in long independent sequences.	10	CO1	L1
2	Mention different properties of entropy and prove that entropy is additive.	10	CO1	L2
3	The state diagram of a Markov source is shown in the fig. 3. Show that $G_1 \geq G_2 \geq H$.	10	CO1	L3

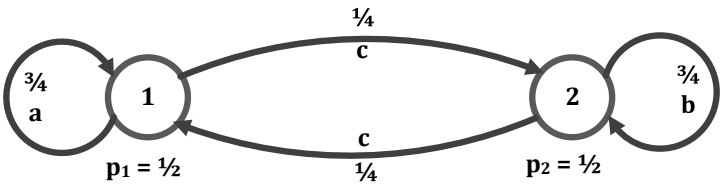

Fig. 3

PTO

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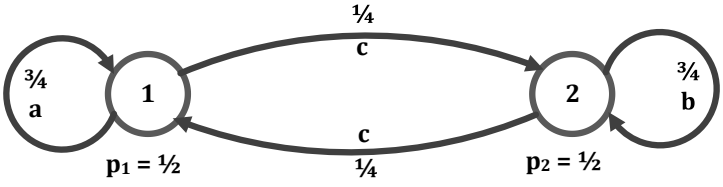

Fig. 3

PTO

4	Consider a discrete memory less source $S = (X, Y, Z)$ Given its second order extension source symbols and probabilities as given in **Table 1**, compute the code words, efficiency and redundancy using Shannon-fano algorithm.	10	CO2	L2
5	Consider a discrete memory less source $S = (X, Y, Z)$ Given its second order extension source symbols and probabilities as given in **Table 1**, compute the code words, efficiency and redundancy using Huffman encoding algorithm.	10	CO2	L3
6	Explain the Shannon's encoding algorithm, clearly indicating the steps involved with the help of a simple example.	10	CO2	L1

Table 1: Data for question 4 and question 5

Symbol	Probability	Symbol	Probability
XX	0.25	YZ	0.06
XY	0.15	ZX	0.10
XZ	0.10	ZY	0.06
YX	0.15	ZZ	0.04
YY	0.09		

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Table 2: Data for question 4 and question 5

Symbol	Probability	Symbol	Probability
XX	0.25	YZ	0.06
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XZ	0.10	ZY	0.06
YX	0.15	ZZ	0.04
YY	0.09		

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18EC54 - ITC - IA5

Scheme & solutions

(1) Self information Content — def $I(w) = \log_2\left(\frac{1}{P_i}\right)$ → 2M

Avg information Content — def

$$H(S) = \sum_{i=1}^{M} P_i \log\left(\frac{1}{P_i}\right) \text{ b/sym} \longleftrightarrow 2M$$

Avg rate of information — def

$$R = r_s \times H(S) \longrightarrow 2M$$

derivation

$S_1 \rightarrow P_1 N \quad I(s_1) = \log\left(\frac{1}{P_1}\right)$

$S_2 \rightarrow P_2 N \quad I(s_2) = \log\left(\frac{1}{P_2}\right)$

$S_3 \rightarrow P_3 N \quad I(s_3) = \log\left(\frac{1}{P_3}\right)$

$S_i \rightarrow P_i N \quad I(s_i) = \log\left(\frac{1}{P_i}\right)$ } 4M

Total Information Content

$$= P_1 N \log\left(\frac{1}{P_1}\right) + P_2 N \log\left(\frac{1}{P_2}\right) + P_3 N \log\left(\frac{1}{P_3}\right) + \cdots + P_i N \log\left(\frac{1}{P_i}\right)$$

$$\text{Avg} = \frac{\text{Total}}{N}$$

$$= P_1 \log\left(\frac{1}{P_1}\right) + P_2 \log\left(\frac{1}{P_2}\right) + \cdots + P_i \log\frac{1}{P_i}$$

$$\therefore \boxed{H(S) = \sum_{i=1}^{M} P_i \log\frac{1}{P_i}}$$

(2) properties of entropy

1. Entropy is Continuous function
2. Entropy is symmetric function

$$H(p, (1-p)) = H((1-p), p)$$

3. Entropy is measure of uncertainity

} 5M

4. if only one symbol is edited from the info source it's entropy
5. $H'(s) \geq H(s)$ 6. $H(s) = \log_2 M$

Additivity property proof ⟶ 5M.

$p_1\ p_2\ p_3\ p_4 : \frac{1}{2}\ \frac{1}{5}\ \frac{1}{2}\ \frac{1}{3}$

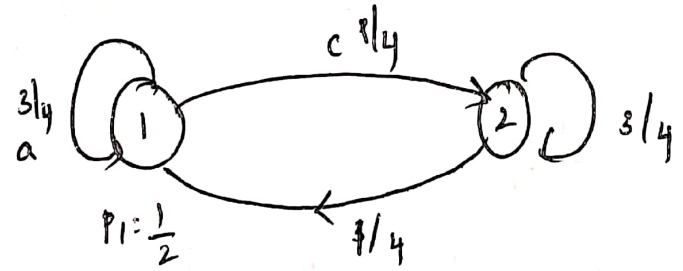
$\frac{1}{2}\ \frac{1}{3}\ \frac{1}{5}$

$H(s) = 1.992$

$H'(s) = 2.954$

$H'(s) \geq H(s)$

(3)

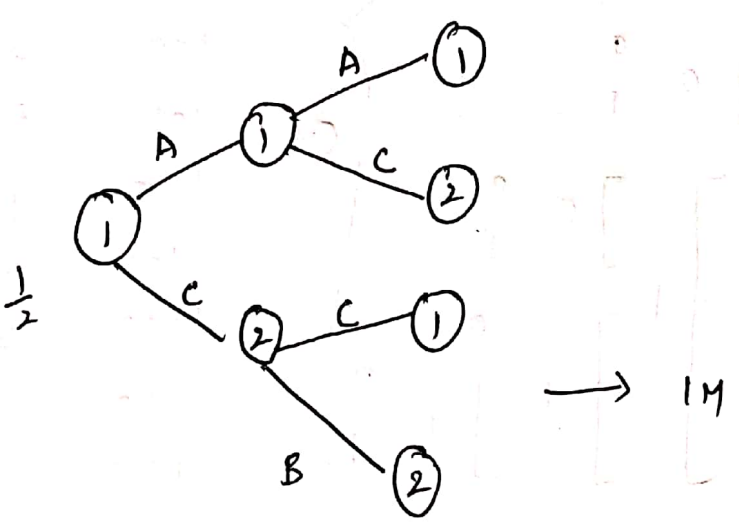


$H_1 = \frac{3}{4}\log\frac{4}{3} + \frac{1}{4}\log 4$

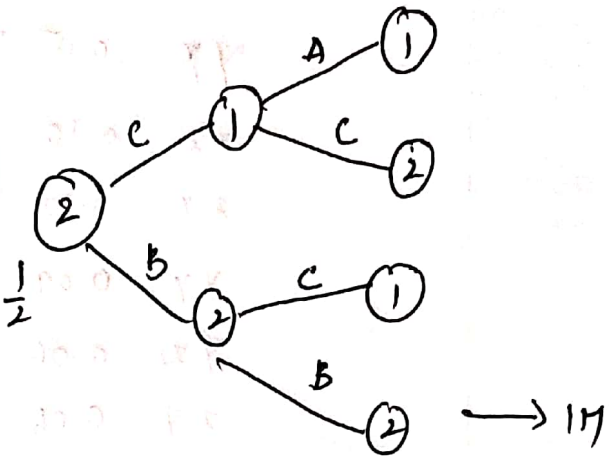
$= 0.8113$ bits/sym ⟶ 1M

$H_2 = \frac{1}{4}\log 4 + \frac{3}{4}\log 4 = 0.8113$ bits/sym ⟶ 1M

$H = P_1H_1 + P_2H_2 = 0.8113$ bits/sym $\longrightarrow$ 1M



$\frac{1}{2}$ $\longrightarrow$ 1M



$\frac{1}{2}$ $\longrightarrow$ 1M

Messages of length 1

$P(A) = 3/8$ $P(B) = 3/8$ $P(C) = \frac{1}{4}$ $\longrightarrow$ 2M

$G_1 = 1.5614$ b/sym

Messages of length 2

$P(AA) = 9/32$ $P(AC) = 3/32$ $P(CC) = 2/32$ $P(CB) = 3/32$

$P(CA) = 3/32$ $P(BC) = 3/32$ $P(BB) = 9/32$ } 3M

$G_2 = 1.2801$ b/sy

$\therefore$ $G_1 \geq G_2 \geq H$

4.

		Codeword	n_i
xx	0.25	00	2
xy	0.15	01	2
yx	0.15	100	3
x2	0.10	101	3
2x	0.10	1100	4
yy	0.09	1101	4
y2	0.06	1110	4
2y	0.06	11110	5
22	0.04	11111	5

5M

2M

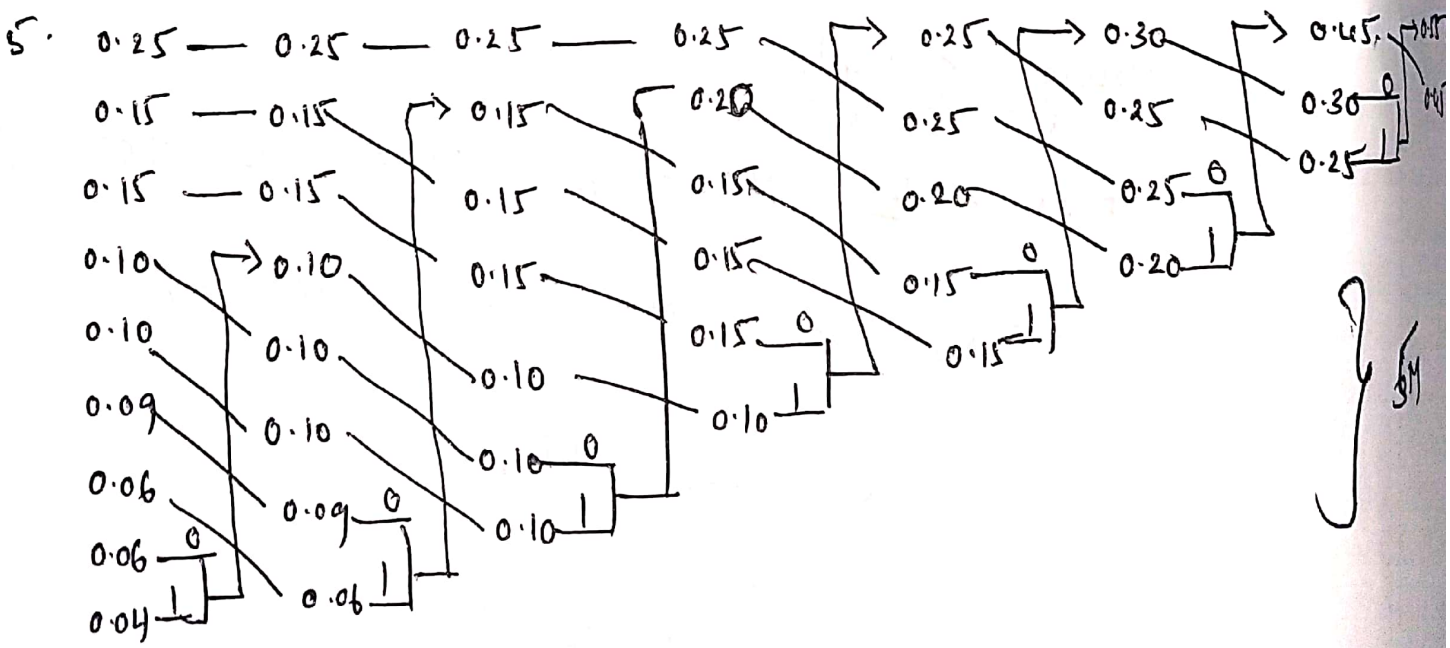
$H(s) = 2.9706$ bits/sym

$HN^1 = 3.05$

efficiency $= 97.4$ bits/sym

3M.

5.



3M

XX	01	2
XY	001	3
YX	010	3
XZ	110	3
ZX	111	3
YY	0000	4
YZ	0001	4
ZY	0110	4
ZZ	0111	4

2M

$H(s) = 2.97$

$H\hat{N} = 8.04$

$e = 97.726\%$

3M

(b)

1. Arrange the probabilities of the given symbols in descending order.
2. Calculate n_i

$$\log\left(\frac{1}{P_i}\right) \leq n_i < 1 + \log\left(\frac{1}{P_i}\right)$$

3. $F_1 = 0$

$$F_i = \sum_{k=1}^{i-1} P_k$$

4. Find the Binary equivalent of F_i
5. Find the Code word.
6. Find $H(s)$. 7. Find $H\hat{N}$. 8. Find efficiency

Any Example ⟶ 4M.

6M