CMR INSTITUTE OF TECHNOLOGY	USN	CMRIT

Internal Assesment Test-V

Sub:	Electromagnetic Waves							Code:	18EC55
Date:	08/02 /2022	Duration:	90 mins	Max Marks:	50	Sem:	5th	Branch:	ECE(A,B,C,D)

Answer any **FIVE FULL** Questions

		Marks	OBE CO	OBE RBT
1.(a)	Starting from Gauss's law deduce Poisson's and Laplace's equations. Write Laplace's equations in all three co-ordinate systems.	[08]	CO3	L2

Derive Poisson's and Laplace's equation

$$\vec{\nabla}\cdot\vec{D} = \rho_v \quad \ldots (1)$$

$$\vec{D} = \epsilon_0 \vec{E} \quad \ldots (2)$$

$$\vec{E} = -\vec{\nabla} V \quad \ldots (3)$$

$$\vec{\nabla}\cdot\vec{D} = \vec{\nabla}\cdot(\epsilon_0 \vec{E}) = \epsilon_0 (\vec{\nabla}\cdot\vec{E}) = \rho_v$$

or $\epsilon_0 \left(\vec{\nabla}\cdot(-\vec{\nabla} V)\right) = \rho_v$

or $-\epsilon_0 \, \vec{\nabla}\cdot(\vec{\nabla} V) = \rho_v \quad \ldots (4)$

$\cancel{\vec{\nabla}\cdot(\vec{\nabla} V}$ $\vec{\nabla}\cdot(\vec{\nabla} V)$

$$= \left(\hat{a}_x \frac{\partial}{\partial x} + \hat{a}_y \frac{\partial}{\partial y} + \hat{a}_z \frac{\partial}{\partial z}\right) \cdot \left(\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right)$$

$$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \nabla^2 V$$

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<td>∴ eqn. ④ reduces to,
$$\epsilon_0 \nabla^2 V = -\rho_v$$
or $\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon_0}}$ → Poisson's equation.
Now if $\rho_v = 0$ → $\boxed{\nabla^2 V = 0}$ → Laplace's eqn.
→ zero volume charge density, but allowing point charges, line charge and surface density to exist at singular locations as sources of the field.
$\nabla^2 V$ → Laplacian of V.
In rectangular co-ordinate system,
$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$
In cylindrical co-ordinates,
$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$
In spherical co-ordinates,
$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$
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<td>(b)</td>
<td>Given vector V = ($12yx^2 – 6z^2x$). Determine whether the given potential field satisfies Laplace’s equation.</td>
<td>[02]</td>
<td>CO3</td>
<td>L3</td>
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	Given, $V = 12yx^2 - 6z^2x$. $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$. $\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(12yx^2 - 6z^2x) = 24yx - 6z^2$ $\frac{\partial^2 V}{\partial x^2} = 24y$ $\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(12yx^2 - 6z^2x) = 12x^2$. $\frac{\partial^2 V}{\partial y^2} = 0$ $\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(12yx^2 - 6z^2x) = -12zx$ $\frac{\partial^2 V}{\partial z^2} = -12x$. $\therefore \nabla^2 V = 24y - 12x$ Conditionally satisfies Laplace's eqn.			
2.	Find the capacitance between the two concentric spheres of radii r = b, and r = a, such that b>a using Laplace's equation. It is given that, V = 0 at r = b and V = V_0 at r = a.	[10]	CO3	L2

$\int dx = x$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)=0$$

$$\text{or } \frac{d}{dr}\left(r^2\frac{dV}{dr}\right)=0$$

Integrating, $d\left(r^2\frac{dV}{dr}\right)=0$

$$\text{or } r^2\frac{dV}{dr} = c_1$$

$c_1 \rightarrow$ constant of integration.

Note
$V=0$ at $r=b$
$V=V_0$ at $r=a$

$$dV = c_1 \cdot \frac{dr}{r^2}$$

$$\int dV = c_1 \int \frac{dr}{r^2}$$

$$\text{or } \boxed{V = c_1\left[-\frac{1}{r}\right] + c_2}$$

$c_2 \rightarrow$ constant of integration.

$$0 = c_1\left(-\frac{1}{b}\right) + c_2 \quad \cdots (1)$$

$$V_0 = c_1\left(-\frac{1}{a}\right) + c_2 \quad \cdots (2)$$

Subtracting (1) from (2),

$$V_0 = c_1\left(-\frac{1}{a}\right) + \frac{c_1}{b}$$

$$\text{or } V_0 = c_1\left(\frac{1}{b} - \frac{1}{a}\right).$$

$$\therefore c_1 = \frac{V_0}{\left(\frac{1}{b}-\frac{1}{a}\right)}.$$

$$c_2 = \frac{c_1}{b} = \frac{V_0}{\left(\frac{1}{b}-\frac{1}{a}\right)}\cdot\frac{1}{b}$$

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i.e. $$V = \frac{V_0}{\left(\frac{1}{b}-\frac{1}{a}\right)}\left(-\frac{1}{r}\right) + \frac{V_0}{\left(\frac{1}{b}-\frac{1}{a}\right)}\cdot\frac{1}{b}$$

or $$\boxed{V = \frac{V_0}{\left(\frac{1}{b}-\frac{1}{a}\right)}\left[\frac{1}{b}-\frac{1}{r}\right]}$$

$\vec{E}, \vec{D}, \rho_s, Q, C$

$$\vec{E} = -\vec{\nabla}V = -\frac{\partial V}{\partial r}\hat{a}_r$$

$$= -\frac{\partial}{\partial r}\left[\frac{V_0}{\left(\frac{1}{b}-\frac{1}{a}\right)}\cdot\left(-\frac{1}{r}\right)\right]\hat{a}_r$$

$$= \frac{V_0}{\left(\frac{1}{b}-\frac{1}{a}\right)}\cdot\left(-\frac{1}{r^2}\right)\hat{a}_r$$

or $$\vec{E} = \frac{V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)}\cdot\frac{1}{r^2}\hat{a}_r \text{ V/m}$$

$$\vec{D} = \epsilon\vec{E} = \frac{\epsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)}\cdot\frac{1}{r^2}\hat{a}_r \text{ C/m}^2$$

$$\rho_s = |\vec{D}_N| = \frac{\epsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)}\cdot\frac{1}{r^2} \text{ C/m}^2$$

$$Q = \rho_s \times 4\pi r^2 = \frac{\epsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)}\cdot\frac{1}{\cancel{r^2}}\cdot 4\pi\cancel{r^2}$$

$$= \frac{4\pi\epsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)} \text{ C}$$

∴ capacitance,

$$\boxed{C = \frac{Q}{V_0} = \frac{4\pi\epsilon}{\left(\frac{1}{a}-\frac{1}{b}\right)} \text{ Farad}}$$

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<tr><td>3.(a)</td><td>State and prove the uniqueness theorem.</td><td>[08]</td><td>CO3</td><td>L2</td></tr>
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	Uniqueness Theorem: Statement: If a solution to Laplace's equation can be found that satisfies the boundary conditions, then the solution is unique. Let's assume Laplace's equation has two solutions, V_1 and V_2. $\nabla^2 V_1 = 0$ $\nabla^2 V_2 = 0$ $\therefore \nabla^2 (V_1 - V_2) = 0 \quad \cdots (1)$ Value of V_1 on the boundary be $V_{1b} = V_b$ Value of V_2 " " " be $V_{2b} = V_b$. $V_b \rightarrow$ give potential on the boundary				

Vector identity,

$$\vec{\nabla}(V\vec{D}) = V(\vec{\nabla}.\vec{D}) + \vec{D}.(\vec{\nabla}V) \quad -(2)$$

Let the scalar be $(V_1 - V_2)$
and the vector be $\vec{\nabla}(V_1 - V_2)$

From (2),

$$\vec{\nabla}.\left((V_1-V_2)\,\vec{\nabla}(V_1-V_2)\right) = (V_1-V_2)\left(\vec{\nabla}.\,\vec{\nabla}(V_1-V_2)\right) + \vec{\nabla}(V_1-V_2).\,\vec{\nabla}(V_1-V_2)$$

$$\vec{\nabla}.\left((V_1-V_2)\,\vec{\nabla}(V_1-V_2)\right) = (V_1-V_2)\,\cancel{\nabla^2(V_1-V_2)}^{=0} + \left[\vec{\nabla}(V_1-V_2)\right]^2$$

Integrating over the volume enclosed by the boundary surface area,

$$\iiint \underbrace{\vec{\nabla}.\left((V_1-V_2)\,\vec{\nabla}(V_1-V_2)\right)}\,dv = \iiint\left[\vec{\nabla}(V_1-V_2)\right]^2 dv$$

[According to divergence theorem, $\iiint \vec{\nabla}.\vec{A}\,dv = \oiint \vec{A}.\vec{ds}$]

Surface integral over the surface at which the boundaries are specified,

$$\oiint (V_1-V_2)\,\vec{\nabla}(V_1-V_2).\vec{ds} = \oiint (V_{1b}-V_{2b})\,\vec{\nabla}(V_{1b}-V_{2b}).\vec{ds}$$

$$= 0$$

$$\iiint\left[\vec{\nabla}(V_1-V_2)\right]^2 dv = 0$$

	If an integral is zero, - either the integrand is zero or - it is half +ve and half -ve, so that the overall contribution cancels each other. $[\vec{\nabla}(V_1-V_2)]^2 \rightarrow$ can't be -ve. $[\vec{\nabla}(V_1-V_2)]^2 = 0$ or $\nabla(V_1-V_2)=0$ or grad $(V_1-V_2)=0$ $\therefore (V_1-V_2) =$ constant At a point on the boundary, $V_1 - V_2 = V_{1b} - V_{2b} = 0$ $\boxed{V_1 = V_2}$ $\therefore$ Laplace's solution is unique, if that satisfies the same boundary conditions.			
3.(b)	Determine whether or not the following potential field satisfies Laplace's equation $V = \rho\cos\varphi + z$.	[02]	CO3	L3

$$\nabla^2 V = \underbrace{\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right)} + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\frac{\partial V}{\partial \rho} = \frac{\partial}{\partial \rho}(\rho \cos\phi + z) = \cos\phi$$

$$\frac{\partial}{\partial \rho}(\rho \cos\phi) = \cos\phi$$

$$\underline{\text{1st term} = \frac{1}{\rho}\cos\phi}$$

$$\frac{\partial V}{\partial \phi} = \frac{\partial}{\partial \phi}(\rho \cos\phi + z) = -\rho \sin\phi$$

$$\frac{\partial^2 V}{\partial \phi^2} = \frac{\partial}{\partial \phi}(-\rho \sin\phi) = -\rho\cos\phi$$

$$\underline{\text{2nd term}} = \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} = \frac{1}{\rho^2}(-\rho\cos\phi) = \underline{-\frac{\cos\phi}{\rho}}$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(\rho\cos\phi + z) = 1$$

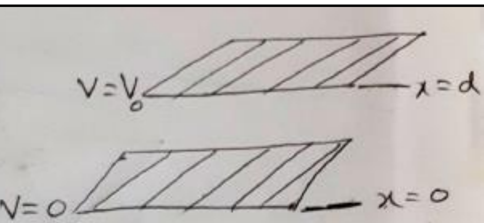
$$\frac{\partial^2 V}{\partial z^2} = 0 = \text{3rd term}$$

$$\therefore \nabla^2 V = \frac{\cos\phi}{\rho} - \frac{\cos\phi}{\rho} + 0 = 0$$

$\therefore$ Laplace's eqn. satisfied.

4.(a)	Consider, the two planes of parallel plate capacitor are separated by a distance ‘d’ and maintained at potential ‘0’ and V_0 respectively. Assuming negligible fringing effect, determine potential at any point between the plates. Also, determine the capacitance of the parallel plate capacitor.	[07]	CO3	L2

Area of the capacitor plate is S.



$\nabla^2 V = 0$

$\frac{d^2V}{dx^2} = 0$

Integrating, $\frac{dV}{dx} = c_1$

$c_1 \to$ constant of integration

$\therefore \int dV = c_1 \int dx$

Integrating, $\boxed{V = c_1 x + c_2}$

$0 = c_1 . 0 + c_2 \Rightarrow c_2 = 0$

$V_0 = c_1 . d \Rightarrow c_1 = \frac{V_0}{d}$.

$\therefore \boxed{V = \frac{V_0}{d} . x}$

$\vec{E} = -\vec{\nabla} V = -\frac{dV}{dx}\hat{a}_x$

$= -\frac{V_0}{d}\hat{a}_x$

Rough

$\vec{\nabla} V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$

$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{a}_x$

$= -\frac{\partial}{\partial x}\left(\frac{V_0}{d}x\right)\hat{a}_x = \frac{-V_0}{d}\hat{a}_x$

$\vec{D} = \epsilon \vec{E} = \frac{-\epsilon V_0}{d}\hat{a}_x$

$\rho_s = |\vec{D}_N| = -\frac{\epsilon V_0}{d}$

Let, area of the capacitor plate be S.

$Q = \rho_s . S = -\frac{\epsilon V_0}{d} S$

$\therefore \boxed{C = \frac{Q}{V_0} = \frac{\epsilon S}{d}}$ Farad

4.(b)	State and explain Biot-Savart's law.	[03]	CO3	L1

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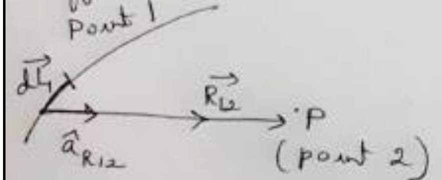
Biot-Savart's law :-
Let's consider a differential current element as a vanishingly small section of a current-carrying filamentary conductor, where a filamentary conductor is the limiting case of a cylindrical conductor of circular cross-section as the radius approaches zero.
We assume a current I flowing in a different vector length of the filament $\vec{dL}$. The Biot-Savart's law then states that, at any point P the mag. of the mag. field intensity produced by the differential element is proportional to the product of the current, the mag. of the differential length and the sine of the angle lying b/w the filament and a line connecting the filament to the point P at which the field is desired. Also, the magnitude of the mag. field intensity is inversely proportional to the square of the distance from the differential element to point P.


$$d\vec{H_2} = \frac{I_1 d\vec{L_1} \times \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$
where $d\vec{H_2}$ → mag. field intensity produced by a differential current element $I_1 d\vec{L_1}$. The direction of $d\vec{H_2}$ is into the page.
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<td>5.</td>
<td>Derive an expression for magnetic field intensity at a point P due to a finite long straight filament carrying a current I.</td>
<td>[10]</td>
<td>CO3</td>
<td>L2</td>
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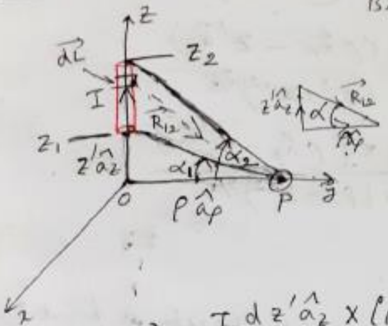
Magnetic Field Intensity due to a finite length straight filament carrying current I :-



Biot-Savart's law,

$$\vec{dH} = \frac{I\vec{dL} \times \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$

Here, $\vec{dL} = dz'\hat{a}_z$

$\vec{R_{12}} = (\rho\hat{a}_\rho - z'\hat{a}_z)$

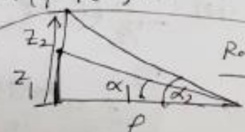
$$\therefore \vec{dH} = \frac{I\,dz'\hat{a}_z \times (\rho\hat{a}_\rho - z'\hat{a}_z)}{4\pi(\rho^2+z'^2)^{3/2}}$$

or $$\vec{dH} = \frac{I\,dz'\rho\;\hat{a}_\phi}{4\pi(\rho^2+z'^2)^{3/2}}.$$

$\therefore$ The total intensity at P due to finite current carrying filament is,

$$\vec{H} = \int \vec{dH} = \int_{z_1}^{z_2} \frac{I\,dz'\rho\;\hat{a}_\phi}{4\pi(\rho^2+z'^2)^{3/2}}$$

Rough



$z_1 = \rho\tan\alpha_1$

$z_2 = \rho\tan\alpha_2$

Let, $z' = \rho\tan\alpha$

$\therefore dz' = \rho\sec^2\alpha\,d\alpha$

as, $z' = z_1$, $\alpha = \alpha_1$

$z' = z_2$, $\alpha = \alpha_2$

	$\therefore \vec{H} = \int_{\alpha_1}^{\alpha_2} \frac{I\rho \; \rho \sec^2\alpha \, d\alpha}{4\pi \left(\rho^2 + \rho^2 \tan^2\alpha\right)^{3/2}} \hat{a}_\phi$ $= \frac{I\rho}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho \sec^2\alpha \, d\alpha}{\rho^3 \sec^3\alpha} \hat{a}_\phi$ $= \frac{I}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha \; \hat{a}_\phi = \frac{I}{4\pi\rho} \left[\sin\alpha\right]_{\alpha_1}^{\alpha_2}$ $= \frac{I}{4\pi\rho} \left(\sin\alpha_2 - \sin\alpha_1\right) \hat{a}_\phi \; A/m$ For infinite conductor, $\alpha_2 = \pi/2$ $\alpha_1 = -\pi/2$ $\therefore \vec{H} = \frac{I}{4\pi\rho}\left(\sin(\pi/2) - \sin(-\pi/2)\right) \hat{a}_\phi$ $= \frac{I}{2\pi\rho} \hat{a}_\phi \quad A/m$			
6.(a)	Discuss the scalar and vector magnetic potential.	[06]	CO3	L2

	** Explain scalar and vector magnetic potential Scalar Magnetic Potential :- Designated as $\boxed{V_m}$ $\boxed{\vec{H} = -\vec{\nabla} V_m}$ → Def.^{n} This def.^{n} should not contradict our previous definitions. $\vec{\nabla} \times \vec{H} = \vec{J}$. But $\vec{\nabla} \times (-\vec{\nabla} V_m) = 0$ as $\vec{H} = (-\vec{\nabla} V_m)$ If $\vec{H}$ to be defined then $\vec{J}$ should be zero throughout the region in which scalar mag. potential (V_m) is defined. ① i.e. $\boxed{\vec{H} = -\vec{\nabla} V_m \quad (\vec{J} = 0)}$ ② V_m satisfies Laplace's eqn. $\vec{\nabla} \cdot \vec{B} = \mu_0 \vec{\nabla} \cdot \vec{H} = 0$ (in free space) $\therefore \mu_0 \vec{\nabla} \cdot (-\vec{\nabla} V_m) = 0$ or $\mu_0 \vec{\nabla} \cdot (\vec{\nabla} V_m) = 0$ or $\mu_0 \nabla^2 V_m = 0$ or $\nabla^2 V_m = 0$. $(\vec{J} = 0)$.				

(3) V_m is not a single valued f^n of position.

For a co-axial cable, $a < \rho < b$

Then, $\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$

We have, $\vec{H} = -\vec{\nabla} V_m$

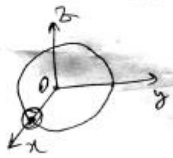
i.e. $\frac{I}{2\pi\rho} = -\frac{1}{\rho}\frac{\partial V_m}{\partial \phi}$

or $\frac{\partial V_m}{\partial \phi} = -\frac{I}{2\pi}$

or $\partial V_m = -\frac{I}{2\pi}\partial\phi$

Integrating; $V_m = -\frac{I}{2\pi}\phi$ [setting constant of integration = 0]

Let, $V_m = 0$ at $\phi = 0$



After one complete rotation,
$\phi = 2\pi$

then, $V_m = -\frac{I}{2\pi}\cdot 2\pi = \underline{\underline{-I}}$

$\therefore V_{m_{a,b}} = -\int_b^a \vec{H}\cdot\vec{dl}$ (specified path)

Vector Magnetic potential

$\vec{\nabla}\cdot\vec{B} = 0$

And divergence of a curl of any vector is zero.

i.e. $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{P}) = 0$

If we choose, $\boxed{\vec{B} = \vec{\nabla}\times\vec{A}.}$

where, $\vec{A} \rightarrow$ magnetic vector potential.

$\therefore \vec{H} = \frac{1}{\mu_0}(\vec{\nabla}\times\vec{A})$. $\left[\because \vec{B} = \vec{\nabla}\times\vec{A}\right.$

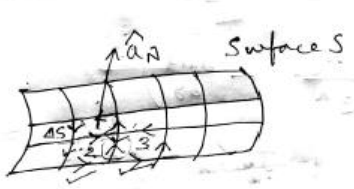
$\left[\mu_0\vec{H} = \vec{\nabla}\times\vec{A}\right.$

$\therefore \vec{\nabla}\times\vec{H} = \vec{J} = \frac{1}{\mu_0}(\vec{\nabla}\times\vec{\nabla}\times\vec{A})$

$$\vec{A} = \oint\frac{\mu_0 I\vec{dl}}{4\pi R}$$

6.(b)	State and prove Stoke's theorem.	[04]	CO3	L2

Stokes theorem



The surface S is broken up into incremental surfaces of area ΔS.

Applying the def^n of curl to one of the incremental areas

$$\frac{\oint \vec{H} \cdot \vec{dl}_{\Delta S}}{\Delta S} = (\vec{\nabla} \times \vec{H})_N \quad \text{--- (1)}$$

$N \rightarrow$ indicating right hand normal to the surface.

$\vec{dl}_{\Delta S} \rightarrow$ closed path of an incremental area ΔS.

$\hat{a}_N \rightarrow$ unit vector in N direction

From (1), $\frac{\oint \vec{H} \cdot \vec{dl}_{\Delta S}}{\Delta S} = (\vec{\nabla} \times \vec{H}) \cdot \hat{a}_N$.

or $\oint \vec{H} \cdot \vec{dl}_{\Delta S} = (\vec{\nabla} \times \vec{H}) \cdot (\hat{a}_N \, \Delta S) = (\vec{\nabla} \times \vec{H}) \cdot \vec{\Delta S}$

or $\oint \vec{H} \cdot \vec{dl}_{\Delta S} = (\vec{\nabla} \times \vec{H}) \cdot \vec{\Delta S}$ --- (2)

We evaluate the circulation for every ΔS.

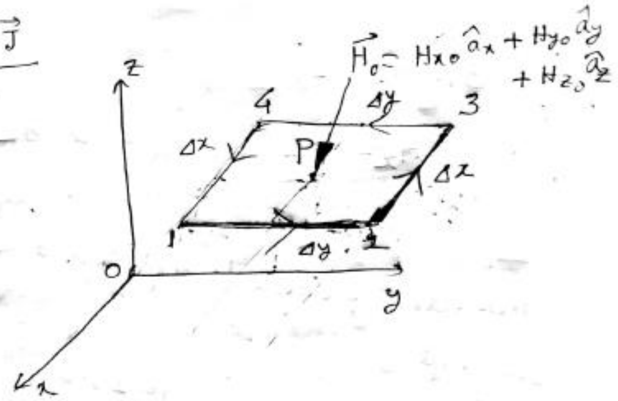
- every interior wall is covered in each direction.
- So some cancellation occurs.
- Only outside ~~boundary~~ no cancellation.

$$\boxed{\oint \vec{H} \cdot \vec{dl} = \iint (\vec{\nabla} \times \vec{H}) \cdot \vec{ds}}$$

$\vec{dl}$ is taken ~~and~~ on the perimeter of S.

7.	Derive point form of Ampere's Law.	[10]	CO3	L2

$\vec{\nabla} \times \vec{H} = \vec{J}$



- Incremental closed path in rectangular co-ordinate system.
- Length of 1-2 side is Δy
  " " 2-3 " is Δx

$$\oint \vec{H} \cdot \vec{dL} = \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1$$

$$\left(\vec{H} \cdot \vec{\Delta L}\right)_{(1-2)} = \left(H_{y0} + \frac{\partial H_y}{\partial x}\left(\frac{\Delta x}{2}\right)\right)(\Delta y)$$

or $$\left(\vec{H} \cdot \vec{\Delta L}\right)_{1-2} = \left(H_{y0} + \frac{\partial H_y}{\partial x}\left(\frac{\Delta x}{2}\right)\right)(\Delta y)$$

$$\left(\vec{H} \cdot \vec{\Delta L}\right)_{2-3} = \left(H_{x0} + \frac{\partial H_x}{\partial y}\left(\frac{\Delta y}{2}\right)\right)(-\Delta x)$$

$$\left(\vec{H} \cdot \Delta L\right)_{3-4} = \left(H_{y0} - \frac{\partial H_y}{\partial x}\left(\frac{\Delta x}{2}\right)\right)(-\Delta y)$$

$$\left(\vec{H} \cdot \vec{\Delta L}\right)_{4-1} = \left(H_{x0} - \frac{\partial H_x}{\partial y}\left(\frac{\Delta y}{2}\right)\right)(\Delta x)$$

$$\oint \vec{H}\cdot\vec{dL} = H_{y_0}\Delta y + \frac{\partial H_y}{\partial x}\left(\frac{\Delta x \Delta y}{2}\right) - H_{x_0}\Delta x - \frac{\partial H_x}{\partial y}\left(\frac{\Delta x \Delta y}{2}\right)$$

$$- H_{y_0}\Delta y + \frac{\partial H_y}{\partial x}\left(\frac{\Delta x \Delta y}{2}\right)$$

$$+ H_{x_0}\Delta x - \frac{\partial H_x}{\partial y}\left(\frac{\Delta x \Delta y}{2}\right)$$

$$= \left(\frac{\partial H_y}{\partial x}\Delta x \Delta y - \frac{\partial H_x}{\partial y}\Delta x \Delta y\right)$$

$$\oint \vec{H}\cdot\vec{dl} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial x}\right)\Delta x \Delta y = I_{enclosed}$$

$$= J_z(\Delta x \Delta y).$$

$$\frac{\oint \vec{H}\cdot\vec{dl}}{\Delta x \Delta y} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = J_z$$

as $$\lim_{\Delta x \Delta y \to 0} \frac{\oint \vec{H}\cdot\vec{dl}}{\Delta x \Delta y} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = J_z \quad \text{--- (1)}$$

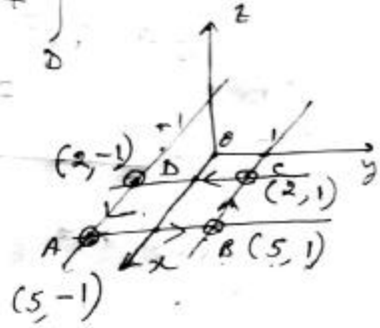
choosing the closed path that are $\perp^r$ to each of the remaining two co-ordinate axes,

$$\lim_{\Delta y \Delta z \to 0} \frac{\oint \vec{H}\cdot\vec{dl}}{\Delta y \Delta z} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) = J_x \quad \text{--- (2)}$$

$$\lim_{\Delta z \Delta x \to 0} \frac{\oint \vec{H}\cdot\vec{dl}}{\Delta z \Delta x} = \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) = J_y \quad \text{--- (3)}$$

	Curl The curl of a vector is a vector any any component of curl is given by the limit of the quotient of the closed line integral of vector about a small path in a plane normal to that component desired and the area enclosed, as the path shrinks to zero. $(\text{curl } \vec{H})_N = \lim_{\Delta S_N \to 0} \frac{\oint \vec{H} \cdot \vec{dl}}{\Delta S_N}$ $\Delta S_N \to$ planar are enclosed by the closed path. Note: $\lim_{\Delta x \Delta y \to 0} \frac{\oint \vec{H} \cdot \vec{dl}}{\Delta x \Delta y} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$ $\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$ $= \hat{a}_x \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) + \hat{a}_y \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) + \hat{a}_z \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$ $\therefore$ combining ①, ② and ③ we get, $\text{curl } \vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\hat{a}_z$ $= J_x \hat{a}_x + J_y \hat{a}_y + J_z \hat{a}_z = \vec{J}$ or $\boxed{\vec{\nabla} \times \vec{H} = \vec{J}}$ → Point form of Ampere's law.			
8.	Verify Stoke's theorem for the field $\mathbf{H} = 6xy\mathbf{a}_x - 3y^2\mathbf{a}_y$ (A/m) and the rectangular path around the region $2 \le x \le 5$, $-1 \le y \le 1$, $z=0$. Let the positive direction of $\mathbf{ds}$ be $\mathbf{a}_z$.	[10]	CO3	L3

$$\oint \vec{H}\cdot\vec{dl} = \int_A^B + \int_B^C + \int_C^D + \int_D^A$$



Along the path A-B,

$$\int \vec{H}\cdot\vec{dl} = \int_{y=-1}^{1} \left(6xy\,\hat{a}_x - 3y^2\,\hat{a}_y\right)\cdot\left(dy\,\hat{a}_y\right)$$

$$= -\int_{y=-1}^{1} 3y^2\,dy = -\cancel{3}\left[\frac{y^3}{\cancel{3}}\right]_{-1}^{1} = -2$$

Along the path B-C

$$\int \vec{H}\cdot\vec{dl} = \int_{x=5}^{2} \left(6xy\,\hat{a}_x - 3y^2\hat{a}_y\right)\cdot dx\,\hat{a}_x$$

$$= \int_{x=5}^{2} 6xy\,dx = 6y\int_{x=5}^{2} x\,dx = 6\cdot 1\cdot\left[\frac{x^2}{2}\right]_5^2 = -63.$$

Along the path C-D ;-

$$\int \vec{H}\cdot\vec{dl} = \int_{y=1}^{-1} \left(6xy\,\hat{a}_x - 3y^2\hat{a}_y\right)\cdot dy\,\hat{a}_y = 2.$$

Along the path D-A

$$\int \vec{H}\cdot\vec{dl} = \int_{x=2}^{5} \left(6xy\,\hat{a}_x - 3y^2\hat{a}_y\right)\cdot dx\,\hat{a}_x = \int_{x=2}^{5} 6xy\,dx = -\cancel{6}^{3}\left[\frac{x^2}{\cancel{2}}\right]_2^5 = -63$$

$$\oint \vec{H}\cdot\vec{dl} = -63 - 2 + 2 - 63$$
$$= -126$$

$$\iint (\vec{\nabla}\times\vec{H})\cdot\vec{ds}$$

$$(\vec{\nabla}\times\vec{H}) = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$\left[\vec{H} = 6xy\,\hat{a}_x - 3y^2\,\hat{a}_y \text{ given}\right]$$

$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy & -3y^2 & 0 \end{vmatrix}$$

$$= \hat{a}_z(-6x)$$

$$\iint (\vec{\nabla}\times\vec{H})\cdot\vec{ds} = \int_{x=2}^{5}\int_{y=-1}^{1} -6x\,\hat{a}_z \cdot dx\,dy\,\hat{a}_z$$

$$= \int_{x=2}^{5}\int_{y=-1}^{1} -6x\,dx\,dy = -\cancel{6}^{\,3}\left[\frac{x^2}{\cancel{2}}\right]_2^5 \left[y\right]_{-1}^{1}$$

$$= -3\left[25-4\right]\left[1-(-1)\right]$$
$$= -3\times 21\times 2$$

$$\therefore \boxed{\oint \vec{H}\cdot\vec{dl} = \iint (\vec{\nabla}\times\vec{H})\cdot\vec{ds}} = -126$$
(verified)