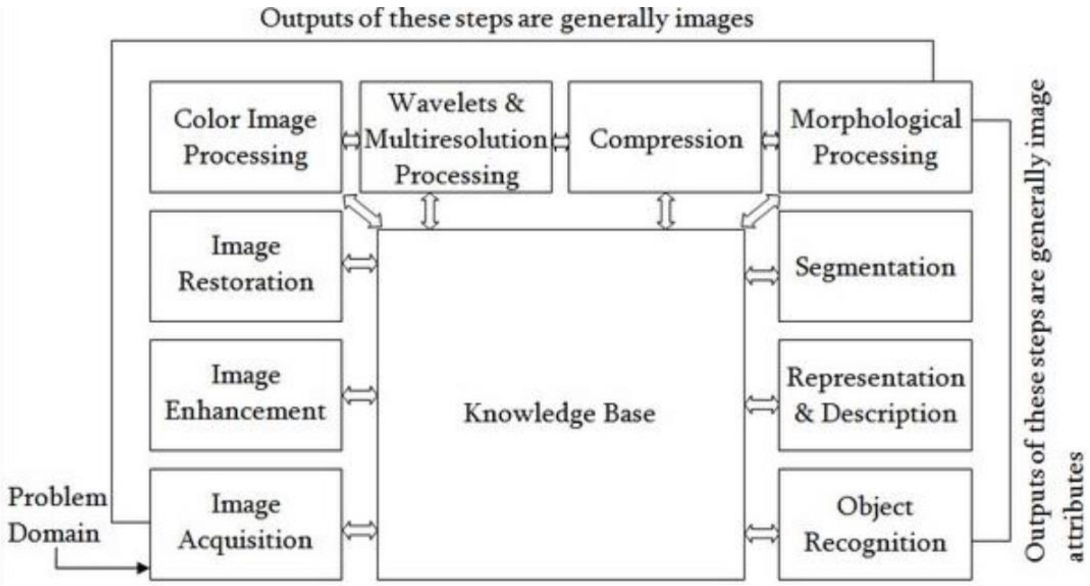
Solution and Scheme for VTU exam
Digital Image Processing 18EC733

Questions and Answers		Marks
1 a) Explain with a block diagram the fundamental steps used in DIP		**[10]**

Tere are two categories of the steps involved in the image processing –

1.Methods whose outputs are input are images.

2.Methods whose outputs are attributes extracted from those images



- Image acquisition: It could be as simple as being given an image that is already in digital form. Generally the image acquisition stage involves processing such scaling.
- Image Enhancement: It is among the simplest and most appealing areas of digital image processing. The idea behind this is to bring out details that are obscured or simply to highlight certain features of interest in image. Image enhancement is a very subjective area of image processing.
- Image Restoration: It deals with improving the appearance of an image. It is an objective approach, in the sense that restoration techniques tend to be based on mathematical or probabilistic models of image processing. Enhancement, on the other hand is based on human subjective preferences regarding what constitutes a "good" enhancement result.
- Color image processing: It is an area that is been gaining importance because of the use of digital images over the internet. Color image processing deals with basically color models and their implementation in image processing applications.
- Wavelets and Multiresolution Processing: These are the foundation for representing image in various degrees of resolution.

- Compression: It deals with techniques reducing the storage required to save an image, or the bandwidth required to transmit it over the network. It has to major approaches a) Lossless Compression b) Lossy Compression
- Morphological processing: It deals with tools for extracting image components that are useful in the representation and description of shape and boundary of objects. It is majorly used in automated inspection applications.
- Representation and Description: It always follows the output of segmentation step that is, raw pixel data, constituting either the boundary of an image or points in the region itself. In either case converting the data to a form suitable for computer processing is necessary.
- Recognition: It is the process that assigns label to an object based on its descriptors. It is the last step of image processing which use artificial intelligence of software.

1b) Explain image acquisition using sensor strips and sensor arrays **[10]**

Image acquisition: The types of images in which we are interested are generated by the combination of an "illumination" source and the reflection or absorption of energy from that source by the elements of the "scene" being imaged

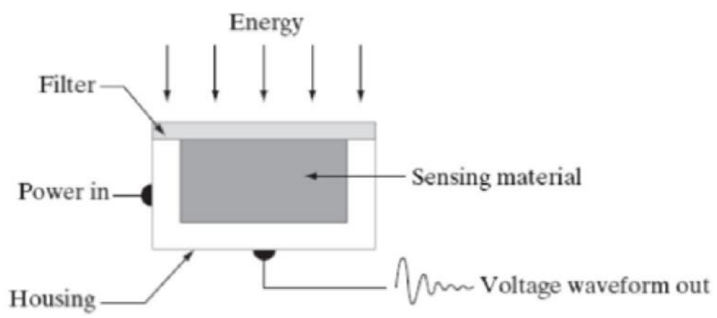


Fig:Single Image sensor

Fig: Line Sensor

Incoming energy is transformed into a voltage by the combination of input electrical power and sensor material that is responsive to the particular type of energy being detected. The output voltage waveform is the response of the sensor(s), and a digital quantity is obtained from each sensor by digitizing its response.

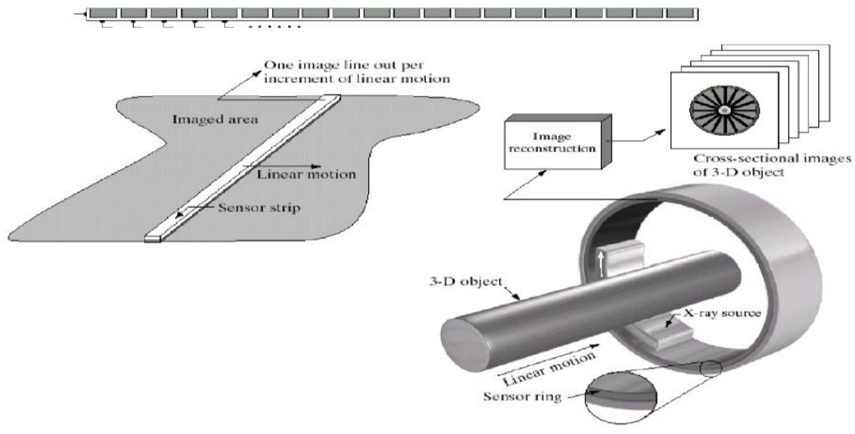
Image acquisition using a single sensor:

- Photodiode: made of silicon and output voltage waveform is proportional to light
- Filter improves selectivity
- To generate a 2D image using a single sensor, there has to be displacement in both x-axis and y-axis between sensors and area to be imaged.

- Example: Microdensitometers. Mechanical arrangement using a flat bed, the sensor moving in two linear directions.
- Example: Place a laser source coincident with the sensor. Moving mirrors are used to control the outgoing beam in a scanning pattern and to direct the reflected laser signal onto the sensor.

Image acquisition using sensor strip:

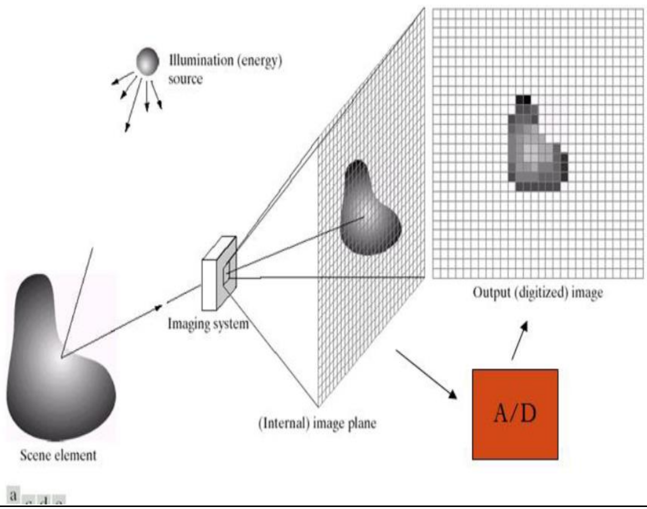
- The strip provides imaging elements in one direction. Motion perpendicular to the strip provides imaging in the other direction.



a b
FIGURE 2.14 (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

Image acquisition using Sensor arrays:

- No motion is required here.
- Applications: Numerous EM and some ultrasonic sensing devices are arranged in this manner, digital cameras, astronomical image acquisition
- CCD array (4000X4000 elements) is typically used in digital cameras.
- The response of each sensor is proportional to the integral if the light energy projected onto the surface of the sensor



2 a) What is DIP? Explain the applications of DIP **[6]**

Digital image processing refers to processing digital images by means of digital computer. Digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are called picture elements, image elements, pels and pixels. Pixel is the term used most widely to denote the elements of digital image.

What is the need of Digital Image Processing:

- For Improvement of pictorial information for human perception
- For autonomous application
- Efficient storage and transmission

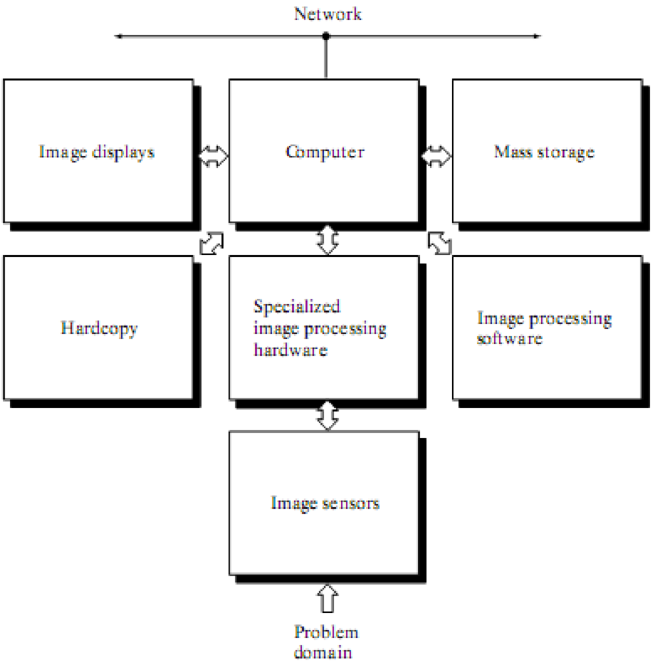
APPLICATIONS OF DIGITAL IMAGE PROCESSING:

1. Remote sensing via satellites and other spacecrafts

2. Image transmission and storage for business applications
3. Medical processing,
4. (Radio Detection and Ranging)
5. SONAR(Sound Navigation and Ranging) and
6. Acoustic image processing (The study of underwater sound is known as underwater acoustics or hydro acoustics.)
    a. Robotics and automated inspection of industrial parts.

2b) With the help of a neat diagram, explain the components of a general purpose image processing **[08]**

Figure 1 shows the basic components of a general-purpose image processing system. The function of each component is discussed in the following paragraphs, starting with image sensing



1. **Image sensing**: Two elements are required to acquire digital images, the first is a physical device that is sensitive to the energy radiated by the object we wish to image. The second, called a digitizer, is a device for converting the output of the physical sensing device into digital form.

2. **Specialized image processing hardware**: Specialized image processing hardware usually consists of the digitizer just mentioned, plus hardware that performs other primitive operations, such as an arithmetic logic unit (ALU), which performs arithmetic and logical operations in parallel on entire images. One example of how an ALU is used is in averaging images as quickly as they are digitized, for the purpose of noise reduction. This type of hardware sometimes is called a front-end subsystem, and its most distinguishing characteristic is speed.

3. **Computer:** The computer in an image processing system is a general-purpose computer and can range from a PC to a supercomputer. In dedicated applications, sometimes specially designed computers are used to achieve a required level of performance.

4. **Image processing software:** Software for image processing consists of specialized modules that perform specific tasks. A well-designed package also includes the capability for the user to write code that, as a minimum,utilizes the specialized modules. More sophisticated software packages allow the integration of those modules and general-purpose software commands from at least one computer language.

5. Mass storage: Mass storage capability is a must in image processing applications. An image of size 1024*1024 pixels, in which the intensity of each pixel is an 8-bit quantity, requires one megabyte of storage space if the image is not compressed. When dealing with thousands, or even millions, of images, providing adequate storage in an image processing system can be a challenge. Digital storage for image processing applications falls into three principal categories: (1) short-term storage for use during processing, (2) on-line storage for relatively fast re-call, and (3) archival storage, characterized by infrequent access. Storage is measured in bytes (eight bits), Kbytes (one thousand bytes), Mbytes (one million bytes), Gbytes (meaning giga, or one billion, bytes), and Tbytes (meaning tera, or one trillion, bytes). One method of providing short-term storage is computer memory. Another is by specialized boards, called frame buffers, that store one or more images and can be accessed rapidly, usually at video rates (e.g., at 30 complete images per second). The latter method allows virtually instantaneous image zoom, as well as scroll (vertical shifts) and pan (horizontal shifts). Frame buffers usually are housed in the specialized image processing hardware unit shown in Fig.1. Online storage generally takes the form of magnetic disks or optical-media storage. The key factor characterizing on-line storage is frequent access to the stored data. Finally, archival storage is characterized by massive storage requirements but infrequent need for access. Magnetic tapes and optical disks housed in –jukeboxes are the usual media for archival applications.

6. **Image Display:** Image displays in use today are mainly color (preferably flat screen) TV monitors. Monitors are driven by the outputs of image and graphics display cards that are an integral part of the computer system. Seldom are there requirements for image display applications that cannot be met by display cards available commercially as part of the computer system. In some cases, it is necessary to have stereo displays, and these are implemented in the form of headgear containing two small displays embedded in goggles worn by the user.

7. **Hardcopy:** Hardcopy devices for recording images include laser printers, film cameras, heat-sensitive devices, inkjet units, and digital units, such as optical and CD-ROM disks. Film provides the highest possible resolution, but paper is the obvious medium of choice for written material. For presentations, images are displayed on film transparencies or in a digital medium if image projection equipment is used. The latter approach is gaining acceptance as the standard for image presentations.
8. **Networking:** Networking is almost a default function in any computer system in use today. Because of the large amount of data inherent in image processing applications, the key consideration in image transmission is bandwidth. In dedicated networks, this typically is not a problem, but communications with remote sites via the Internet are not always as efficient. Fortunately, this situation is improving quickly as a result of optical fiber and other broadband technologies.

2 c) How image is formed in eye? Explain visual perception of eye

Image Formation in the Eye:

- The principal difference between the lens of the eye and an ordinary optical lens is that the former is flexible
- As illustrated in Fig. 2.3, the radius of curvature of the anterior surface of the lens is greater than the radius of its posterior surface.
- The shape of the lens is controlled by tension in the fibers of the ciliary body.
- To focus on distant objects, the controlling muscles cause the lens to be relatively flattened.
- Similarly, these muscles allow the lens to become thicker in order to focus on objects near the eye

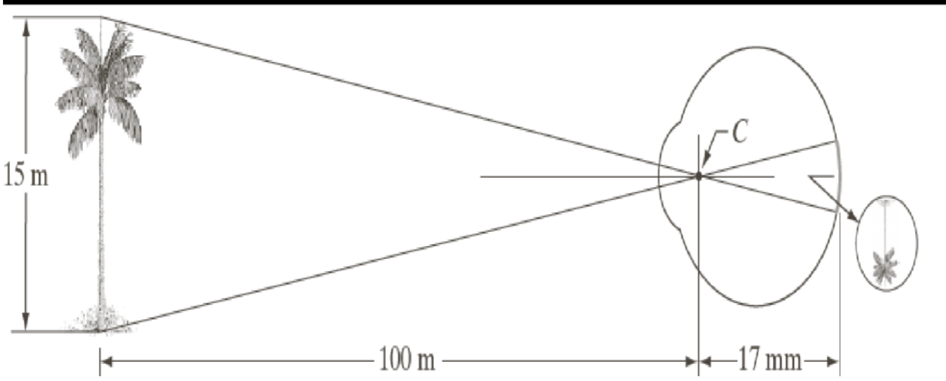


FIGURE 2.3 Graphical representation of the eye looking at a palm tree. Point C is the optical center of the lens.

- The distance between the center of the lens and the retina (called the focal length) varies from approximately 17 mm to about 14 mm, as the refractive power of the lens increases from its minimum to its maximum.

- When the eye focuses on an object farther away than about 3 m, the lens exhibits its lowest refractive power. When the eye focuses on a nearby object, the lens is most strongly refractive. This information makes it easy to calculate the size of the retinal image of any object
- The retinal image is reflected primarily in the area of the fovea. Perception then takes place by the relative excitation of light receptors, which transform radiant energy into electrical impulses that are ultimately decoded by the brain.

3 a) Explain the process of image sampling nad quantization in digital image formulation **[8]**

An image may be continuous w.r.t x and y co-ordinate and also in amplitude. To convert it to digital form, we have to sample the function in both co-ordinate and amplitude.

Sampling :

- The process of digitizing the co-ordinate values is called Sampling.
- A continuous image f(x, y) is normally approximated by equally spaced samples arranged in the form of a NxM array where each elements of the array is a discrete quantity.

$$f(x,y) = \begin{bmatrix} f(0,0) & \cdots & f(0,M-1) \\ \vdots & \ddots & \vdots \\ f(N-1,0) & \cdots & f(N-1,M-1) \end{bmatrix}$$

- The sampling rate of digitizer determines the spatial resolution of digitized image.
- Finer the sampling (i.e. increasing M and N), the better the approximation of continuous image function f(x, y).

Quantization :

- The process of digitizing the amplitude values is called Quantization.
- Magnitude of sampled image is expressed as the digital values in Image processing.
- No of quantization levels should be high enough for human perception of the fine details in the image.
- Most digital IP devices uses quantization into k equal intervals.
- If b-bits are used,

3b) With necessary graphs explain the log and power law transformation used for spatial image enhancement **[8]**

Log Transformations:

- The general form of log transformation is:
  - s=c log(1+r)
- Where c is a constant and it is assumed that $r \geq 0$
- This transformation maps a narrow range of low intensity values in the input into a wider range of output levels. The opposite is true for higher values of input levels.
- We use transformation of this type to expand the values of dark pixels in an image while compressing the higher level values.
- The opposite of this is inverse log transformation.
- Compresses the dynamic range of images with large variations in pixel values

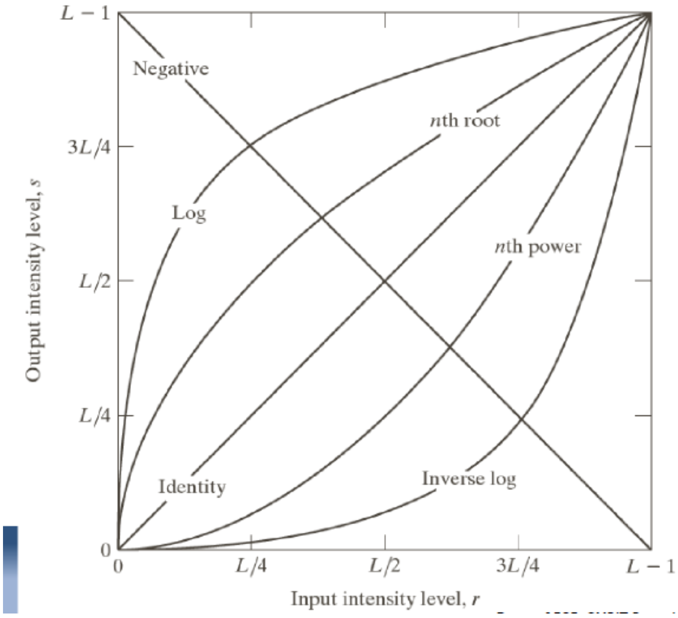
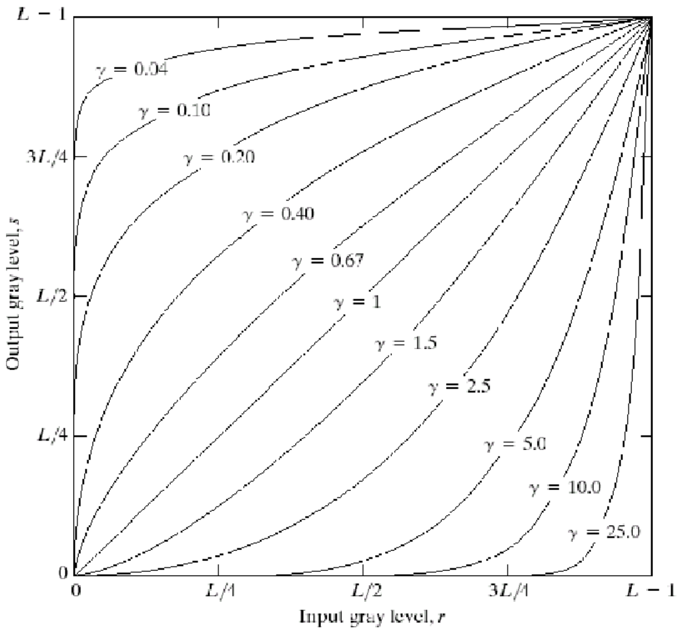


FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

Power law (Gamma) transformation:

- Achieves the spreading/compression of intensity values as log transformation but Power law transformation is more versatile.
- Power law transformation has the basic form
  - $s = cr^{\gamma}$
- Where c and γ are positive constants.
- Unlike log transformation, a family of curves are obtained simply by changing γ
- $c = \gamma = 1$: Identity transformation
- $\gamma > 1$ has an opposite effect as $\gamma < 1$



- A lot of devices used for image capture, printing, and display respond to power law.
- The process used to correct these power law response phenomenon is called gamma correction.
- Example: CRT (cathode ray tubes) have an intensity-to-voltage response that is power function, with exponents varying from approximately 1.8 to 2.5
- Therefore, the display system tends to produce images that are darker than intended
- For colored images, gamma correction does not only change the intensity but also the ratios of red, green and blue

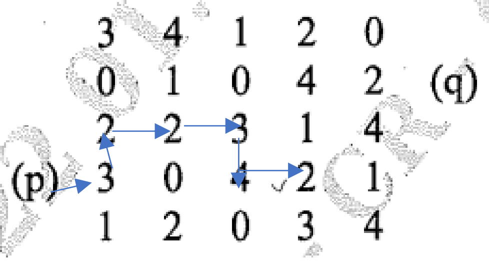
Compute the lengths of the shortest 4, 8 and M path between p and q in the image segment shown in Table Q3(c) by considering v = {2, 3, 4}. [4]

	3	4	1	2	0	
	0	1	0	4	2	(q)
	2	2	3	1	4	
(p)	3	0	4	2	1	
	1	2	0	3	4	

Table Q3(c)

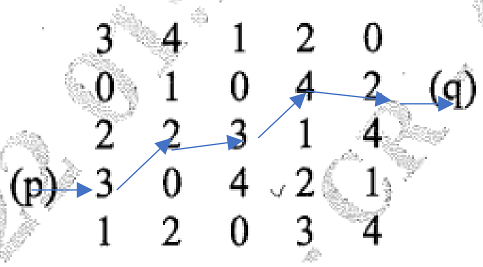
(04 Marks)

4-path

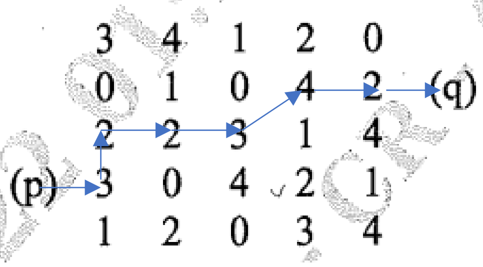


No 4 path exist

8-Path



M-path



4 a) Explain the adjacency, connectivity, regions and boundaries between pixels with examples

Two pixels are said to be connected if they are adjacent in some sense:
- they are neighbors (N_4, N_D, N_8) and
-their intensity values (gray-scale) are similar.

a) Let V be the set of gray –level values used to define adjacency, in a binary image, V={1}
b) If the possible intensity values 0 – 255, *V* set can be any subset of these 256 values.

Two pixels are said to be adjacent if they are connected.

Three types of adjacency

1. 4- Adjacency – two pixel P and Q with value from V are 4 –adjacency if P is in the set $N_4\ (Q)$
2. 8- Adjacency
3. M-adjacency-two pixel P and Q with value from V are m – adjacency if (i) Q is in $N_4\ (P)$ or (ii) Q is in $N_D(P)$ and the set $N_4\ (P) \cap N_4\ (Q)$ has no pixel whose values are from V.

Mixed adjacency is a modification of 8-adjacency. It is introduced to eliminate the ambiguities that often arise when 8-adjacency is used

c) Let R be a subset of pixels in an image. We call R a region of the image if R is a connected set
d) Regions that are not adjacent are said to be disjoint

4b. What do you mean by histogram processing

e) Gives a global information about the appearance of the image
f) Histogram of a digital image with intensity levels in the range [0, L-1] is a discrete function
   a) $h(r_k) = n_k$
g) It is a graph between various gray levels on x-axis and the number of times a grey level has occurred on y-axis.
h) Normalized histogram
i) Used for
j) Image enhancement
k) Compression
l) Segmentation

Histogram Equalization Operation:

m) To enhance the image
n) Assume 'r' represent the gray levels in an image
o) 'r' is in the range [0 L-1]
p) Transformation

$$s = T(r), \qquad 0 \le r \le L-1$$

That produce an output intensity level s for every pixel in the input image having intensity r. Assume that:

q) T(r) is a monotonically increasing function in the interval $0 \le r \le L-1$, and

r) $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$

s) In some formulations to be discusses later, we use the inverse

$$r = T^{-1}(s), \qquad 0 \leq s \leq L-1$$

In which case we change condition a) to

a') T(r) is a strictly monotonically increasing function in the interval $0 \leq r \leq L-1$

The requirement in condition(a) that T(r) be monotonically increasing guarantees that output intensity values will never be less than corresponding input values, thus preventing artifacts created by reversal of intensity

Condition (b) guarantees that the range of output intensities is the same as the input intensities.

Condition (a') guarantees that the mapping from s to r will be one-to-one, thus preventing ambiguities

- The intensity levels in an image may be viewed as random variables in the interval [0, L-1]. Let $p_r(r)$ and $p_s(s)$ denote the PDFs (probability density functions) of r and s respectively
- If $p_r(r)$ and T(r) are known, and T(r) is a continuous and differentiable over the range of values in interest, then the PDF of the transformed(mapped) variables s can be obtained as :

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

PDF of output intensity variable s is obtained using PDF of r and the transformation function

- A transformation function of particular importance in image processing has the form:

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Explain image smoothing in frequency domain using ideal low pass filter, Butterworth low pass filter and Gaussian low pass filtering

The edges and other sharp transitions (such as noise) in the gray levels of an image contribute significantly to the high-frequency content of its Fourier transform. Hence blurring (smoothing)is achieved in the frequency domain by attenuating high frequencies in the transform of a given image.

$$G(u, v) = H(u, v) F(u, v)$$

where F (u, v) is the Fourier transform of an image to be smoothed. The problem is to select a filter transfer function H (u, v) that yields G (u, v) by attenuating the high-frequency components of F (u, v). The inverse transform then will yield the desired smoothed image g (x, y).

Ideal Filter:

A 2-D ideal lowpass filter (ILPF) is one whose transfer function satisfies the relation

$$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_0 \\ 0 & \text{if } D(u, v) > D_0 \end{cases}$$

where D is a specified nonnegative quantity, and D(u, v) is the distance from point (u, v) to the origin of the frequency plane; that is,

$$D(u, v) = (u^2 + v^2)^{1/2}.$$

Figure 1(a) shows a 3-D perspective plot of H (u, v) as a function of u and v. The name ideal filter indicates that all frequencies inside a circle of radius Do are passed with no attenuation, whereas all frequencies outside this circle are completely attenuated.

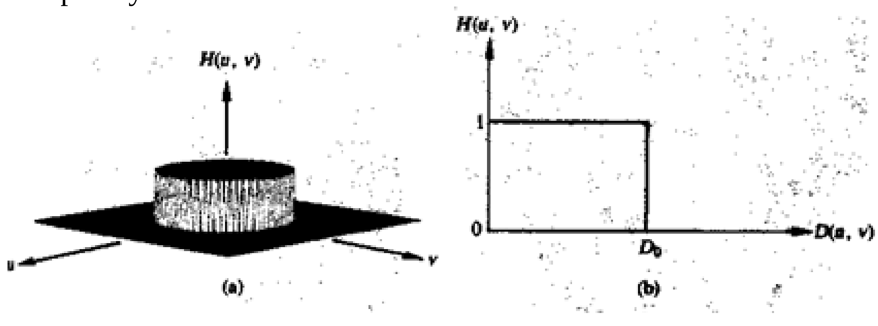


Fig. 1a) Perspective plot of an ideal lowpass filter transfer function; (b) filter crosssection.

The lowpass filters are radially symmetric about the origin. For this type of filter, specifying a cross section extending as a function of distance from the origin along a radial line is sufficient, as Fig. 1 (b) shows. The complete filter transfer function can then be generated by rotating the cross section 360 about the origin. Specification of radially symmetric filters centered on the N x N frequency square is based on the assumption that the origin of the Fourier transform has been centered on the square.

For an ideal lowpass filter cross section, the point of transition between H(u, v) = 1 and H(u, v) = 0 is often called the cutoff frequency. In the case of Fig.1 (b), for example, the cutoff frequency is Do. As the cross section is rotated about the origin, the point Do traces a circle giving a locus of cutoff frequencies, all of which are a distance Do from the origin.

Butterworth low pass filter

The transfer function of the Butterworth lowpass (BLPF) of order n and with cutoff frequency locus at a distance Do, from the origin is defined by the relation

$$H(u, v) = \frac{1}{1 + [D(u, v)/D_0]^{2n}}$$

A perspective plot and cross section of the BLPF function are shown in figure 2.

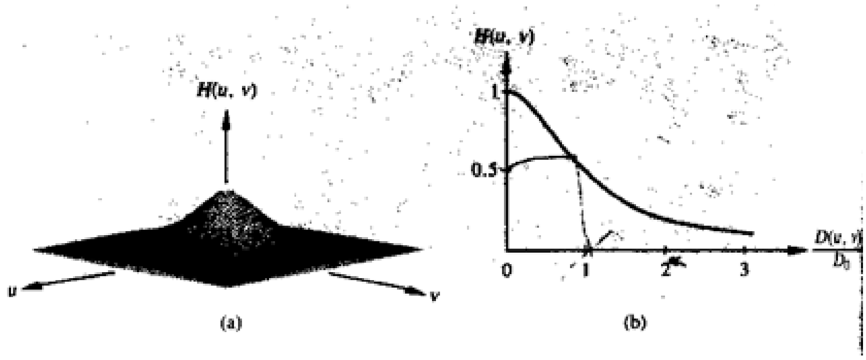


Fig.2 (a) A Butterworth lowpass filter (b) radial cross section for n = 1.

Unlike the ILPF, the BLPF transfer function does not have a sharp discontinuity that establishes a clear cutoff between passed and filtered frequencies. For filters with smooth transfer functions, defining a cutoff frequency locus at points for which H (u, v) is down to a certain fraction of its maximum value is customary. In the case of above Eq. H (u, v) = 0.5 (down 50 percent from its maximum value of 1) when D (u, v) = Do. Another value commonly used is 1/√2 of the maximum value of H (u, v). The following simple modification yields the desired value when D(u, v) = Do:

$$H(u, v) = \frac{1}{1 + [\sqrt{2} - 1][D(u, v)/D_0]^{2n}} = \frac{1}{1 + 0.414[D(u, v)/D_0]^{2n}}.$$

It also serves as a common base for comparing the behavior of different types of filters.

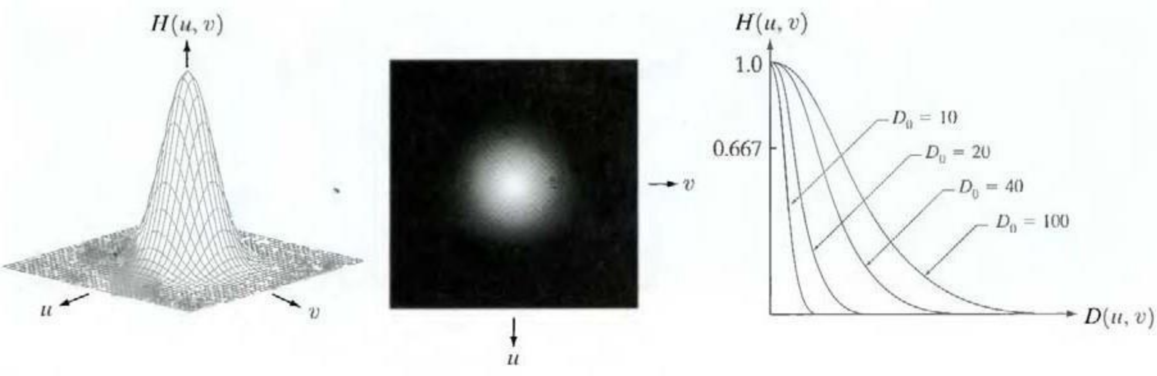
The sharp cutoff frequencies of an ideal lowpass filter cannot be realized with electronic components, although they can certainly be simulated in a computer.

Gaussian Lowpass Filters:

The form of these filters in two dimensions is given by

$$H(u, v) = e^{-D^2(u,v)/2\sigma^2}$$

where, D(u, v) is the distance from the origin of the Fourier transform.



a b c

Fig.3 (a) Perspective plot of a GLPF transfer function, (b) Filter displayed as an image, (c) Filter radial cross sections for various values of Do.

σ is a measure of the spread of the Gaussian curve. By letting σ = Du, we can express the filter ina more familiar form in terms of the notation:

$$H(u, v) = e^{-D^2(u,v)/2D_0^2}$$

where Do is the cutoff frequency. When D (u, v) = Do, the filter is down to 0.607 of itsmaximum value.

5**b) Explain the properties of 2D DFT**

For an M × N 2D image f(x,y), 2D DFT is defined as follows:

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)},$$

Where u=0,1,2,…,M-1, and v=0,1,2,….,N-1

Inverse 2D-DFT :

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

1. Periodicity
   Where $k_1 and\ k_2$ are integers

   Spatial domain: $f(x,y) = f(x+k_1M,y) = f(x,y+k_2N)$
   $= f(x+k_1M,y+k_2N)$
   Freq. domain: $F(u,v) = F(u+k_1M,v) = F(u,v+k_2N)$
   $= F(u+k_1M,v+k_2N)$

2. Convolution
   Spatial domain: $f(x,y) * h(x,y)$
   Frequency domain: $F(u,v)H(u,v)$

3. Multiplication
   Spatial domain: $f(x,y)h(x,y)$
   Frequency domain: $F(u,v) * H(u,v)$

4. *Correlation*
   Spatial domain: $Corr(f(x,y),h(x,y))$
   Frequency domain: $F^*(u,v)H(u,v)$

5. Linearity
   Spatial domain: $af(x,y) + bg(x,y)$
   Frequency domain: $aF(u,v) + bG(u,v)$

$$Where\ DFT\big(f(x,y)\big) = F(u,v)$$
$$DFT\big(g(x,y)\big) = G(u,v)$$

6 a) Explain the basic steps of filtering in frequency domain. Explain one method of sharpening frequency domain filters **[10]**

1. Given an input image $f(x,y)$ of size MXN, obtain the padding parameters P and Q using the following equations:

$$P \geq M + C - 1 \; and \quad Q \geq N + D - 1$$

where the mask used for filtering is of the size CXD.
Typically, we select P=2Mand Q=2N.
2. Form a padded image $f_p(x,y)$ of size PXQ by appending the necessary number of zeros to $f(x,y)$
3. Multiply $f_p(x,y)$ by $(-1)^{x+y}$ to center its transform.
4. Compute the DFT, F(u,v), of the image from step 3.
5. Generate a real, symmetric filter function H(u,v) of size PXQ with senter at co-ordinates (P/2,Q/2). Form the product:

G(u,v)=F(u,v)H(u,v)

Using array multiplication, i.e. G(i,k)=F(i,k)H(i,k)
6. Obtain the processed image:

$$g_p(x,y) = \{real[\Im^{-1}[G(u,v)]]\}(-1)^{x+y}$$

Where the real part is selected in order to ignore parasitic complex components resulting from the computational inaccuracies and the subscript p indicates that we are dealing with padded arrays.
7. Obtain the final processes result $g(x,y)$, by extracting the MXN region from the top, left quadrant of $g_p(x,y)$

IMAGE SHARPENING USING FREQUENCY DOMAIN FILTERS:

An image can be smoothed by attenuating the high-frequency components of its Fourier transform.

Because edges and other abrupt changes in intensities are associated with high-frequency components,

image sharpening can be achieved in the frequency domain by high pass filtering, which attenuates the low-frequency components without disturbing high frequency information in the Fourier transform.

The meaning of sharpening is

- Edges and fine detail characterized by sharp transitions in image intensity
- Such transitions contribute significantly to high frequency components of Fourier transform
- Intuitively, attenuating certain low frequency components and preserving high frequency components result in sharpening.

A high pass filter is obtained from a given low pass filter using the equation.

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

Where $H_{LP}(u,v)$ is the transfer function of the low-pass filter. That is when the lowpass
filter attenuates frequencies, the high-pass filter passed them, and vice-versa.

BUTTER-WORTH HIGH-PASS FILTERS:

Butter-worth high-pass filter to behave smoother than IHPFs

As before, we see that the Butter-worth filter represents a transition between the sharpness of the ideal filter and the broad smoothness of the Gaussian filter.

$$H(u,v) = \frac{1}{1+\left[D_0 / D(u,v)\right]^{2n}}$$

6b) Discuss homomorphic filtering approach for image enhancement

An image f(x,y) can be expressed as the product of its illumination i(x,y) and reflectance r(x,y), components:

$$f(x,y) = i(x,y)r(x,y)$$

This equation cannot be used directly to operate on the frequency components of illumination and reflectance because the Fourier transform of a product is not the product of the transforms:

$$\Im[f(x,y)] \neq \Im[i(x,y)]\Im[r(x,y)]$$

However, we can define:

$$z(x,y) = \ln\big(f(x,y)\big) = \ln\big(i(x,y)\big) + \ln\,(r(x,y))$$

Then,

$$\Im[z(x,y)] = \Im[\ln\,(f(x,y))]$$
$$= \Im[\ln\,\{i(x,y)\}] + \Im[\ln\,\{r(x,y)\}]$$

Or

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

Where $F_i(u,v)$ and $F_r(u,v)$ are the Fourier transform of $\ln\{i(x,y)\}\, and \ln\,\{r(x,y)\}$, respectively.

We can filter Z(u,v) using a filter H(u,v), such that:

$$S(u,v) = H(u,v)Z9u,v) = H(u,v)F_i(u,v) + H(u,v)F_r(u,v)$$

The filtered image in the spatial domain is

$$s(x,y) = \Im^{-1}\{S(u,v)\} \qquad = \Im^{-1}\{H(u,v)F_i(u,v) + H(u,v)F_r(u,v)\}$$

By defining

$i'(x,y) = \Im^{-1}\{H(u,v)F_i(u,v)\}$, and
$r'(x,y) = \Im^{-1}\{H(u,v)F_r(u,v)\}$,
Therefore

$$s(x,y) = i'(x,y) + r'(x,y)$$

Finally because z(x,y) was formed by taking natural logarithm of the input image, we reverse the process by taking the exponential of the filtered result to form the output image:

$$\boldsymbol{g(x,y) = e^{s(x,y)}}$$
$$\boldsymbol{= e^{i'(x,y)} e^{r'(x,y)}}$$
$$\boldsymbol{= i_0(x,y) r_0(x,y)}$$

Where,

$$\boldsymbol{i_0(x,y) = e^{i'(x,y)}\ and\ r_0(x,y) = e^{r'(x,y)}}$$

Are the illumination and reflectance components of the output (processed) image.
The filter H(u,v) is called the homomorphic filter.
The key approach is the separation of the illumination and reflectance components.
The illumination component of an image generally is characterized by the slow spatial variation

While the reflectance component tends to vary abruptly particularly at the junctions of dissimilar objects These characteristics lead to associating the low frequencies of the Fourier transform of the logarithmic of an image with illumination and high frequencies with reflectance Better control can be gained over the illumination and reflectance components with a homomorphic filter This control requires specification of a filter function H(u,v) that affects the low- and high- frequency components of the Fourier transform in different, controllable ways