Subject Title: ITC **Subject Code:** 17EC54

Question Number | **Solution** P-4/7 | **Marks Allocated**

1 [illegible]

$$P(y_j|x_i)\,P(x_i)=\begin{bmatrix} P(y_1|x_1)P(x_1) & P(y_2|x_1)P(x_1) & \cdots & P(y_m|x_1)P(x_1) \\ P(y_1|x_2)P(x_2) & P(y_2|x_2)P(x_2) & \cdots & P(y_m|x_2)P(x_2) \\ \vdots & \vdots & & \\ P(x_1|x_n)P(x_n) & P(y_2|x_n)P(x_n) & \cdots & P(y_m|x_n)P(x_n) \end{bmatrix}$$ – 2M

Property ① Add column wise of JPM to get the probability of o/p symbols

② Add row wise of JPM to get the prob. of $P(x_i)$ } 2M

③ Sum of all elements of JPM = 1

b) $P(y|x)=\begin{bmatrix}0.8 & 0.2\\ 0.3 & 0.7\end{bmatrix} \Rightarrow P(x,y)=\begin{bmatrix}0.48 & 0.12\\ 0.12 & 0.28\end{bmatrix}$ – 1M

$P(x)=[0.6, 0.4]$ $P(y)=[0.6, 0.4]$

$H(y)=0.6\log_2\frac{1}{0.6}+0.4\log_2\frac{1}{0.4}=0.9709$ bits/sym

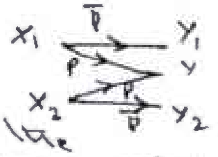
$H(y/x)=\sum\sum P(x_i,y_j)\log\frac{1}{P(y_j/x_i)}=0.7855$ bits/sym } – 1M

$I(x,y)=H(y)-H(y/x)=0.1854$ bits/sym – 2M

$R_t=\{I(x,y)\}r_s=0.1854\times1000=185.4$ bits/sec – 2M

(6M)

c) When ever error occurs, sym y will be received and no decision will be made about the information, but an immediate request will be made (ie ARQ) through a reverse channel for retransmission of the transmitted signal till a correct signal is received. This ensure 100% correct data recovery – 2M



$P(y|x)=\begin{bmatrix}\bar{p} & p & 0\\ 0 & p & \bar{p}\end{bmatrix} \Rightarrow P(x,y)=\begin{bmatrix}\bar{p}w & pw & 0\\ 0 & p\bar{w} & \bar{p}\bar{w}\end{bmatrix}$

$P(x_1=0)=w$ & $P(x_2=1)=\bar{w}$, but $w+\bar{w}=1$ & $p+\bar{p}=1$

$P(y)=[\bar{p}w \quad p \quad \bar{p}\bar{w}]$

$P(x|y)=\begin{bmatrix}1 & w & 0\\ 0 & \bar{w} & 1\end{bmatrix}$

$H(x|y)=\sum\sum P(x_i,y_j)\log\frac{1}{P(x_i|y_j)}$

$\boxed{H(x|y)=p\,H(x)}$

$I(x,y)=H(x)-H(x|y)=(1-p)H(x)=\bar{p}H(x)$

$C=\text{Max}\{I(x,y)\}r_s=\{\bar{p}\,H(x)_{max}\}r_s=\bar{p}\log_2^2\cdot r_s$

$\boxed{C=\bar{p}\,r_s}$ bits/sec – 6M

(8M)

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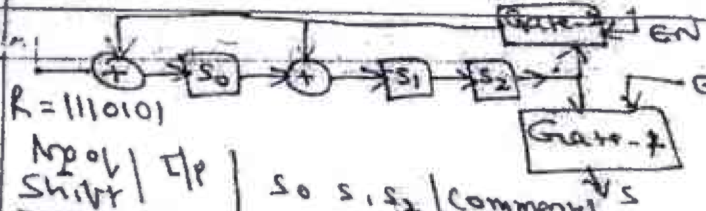
Question Number	Solution P-5/7	Marks Allocated
6 a)	$I(X,Y) = H(X) - H(X \vert Y)$ with Expn	3M
	<u>Properties</u> :- $I(X,Y) = I(Y,X)$ - $I(X,Y) \geq 0$ $I(X,Y) = H(Y) - H(Y \vert X)$ $I(X,Y) = H(X) + H(Y) - H(X,Y)$	3M 6M
b)	$P(Y \vert X) = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$	
	$\begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 0.8\log 0.8 + 0.1\log 0.1 + 0.1\log 0.1 \\ 0.2\log 0.2 + 0.6\log 0.6 + 0.2\log 0.2 \\ 0.2\log 0.2 + 0.2\log 0.2 + 0.6\log 0.6 \end{bmatrix} = \begin{bmatrix} -0.922 \\ -1.371 \\ -1.371 \end{bmatrix}$	2M
	Solving above Matrix, we get $Q_1 = -0.7723$, $Q_2 = -1.5207$, $Q_3 = -1.5207$	2M
	$C = \log_2 [2^{Q_1} + 2^{Q_2} + 2^{Q_3}] \times r_s = 718$ bits/sec	2M 6M
c)	$C = B\log(1 + \frac{S}{N})$ bits/sec B = Channel BW (Hz), N = Noise power (W) $= \eta B$. S = Signal power (W)	2M
	Proof of $C = B\log(1 + S/N)$	6M 8M
7 a)	<u>Adv</u>: Improves Data Quality, Reduction in E_b/N, Reduction in Transmitted power <u>Disadv</u>: Increased BW, System becomes more Complex	3M
	<u>Methods of controlling Errors</u>: - forward acting error correction - Error detection Method with Expn	3M 6M
b)	$C = [d_1, d_2, d_3, d_4, c_5, c_6, c_7]$	
	$C = DG = [d_1\ d_2\ d_3\ d_4] \underbrace{\left[\begin{array}{cccc:ccc} 1&0&0&0&1&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&1&1&1 \\ 0&0&0&1&1&0&1 \end{array}\right]}_{G}$	2M
	$H = [P^T \vdots I_{n-k}] = \left[\begin{array}{cccc:ccc} 1&0&1&1&1&0&0 \\ 1&1&1&0&0&1&0 \\ 0&1&1&1&0&0&1 \end{array}\right]$	2M
	$GH^T = \begin{bmatrix} 1000 & 110 \\ 0100 & 011 \\ 0010 & 111 \\ 0001 & 101 \end{bmatrix} \begin{bmatrix} 1&1&0 \\ 0&1&1 \\ 1&1&1 \\ 1&0&1 \\ 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix} = 0$	2M 6M

Question Number	Solution	Marks Allocated

7 c)



$R = 1110101$ — 3M

No of Shift	I/P	$S_0\ S_1\ S_2$	Comments
Initialization G_2-ON & G_1-OFF		0 0 0	← Clear the content of Register
1	1	1 0 0	
2	0	0 1 0	
3	1	1 0 1	
4	0	1 0 0	
5	1	1 1 0	
6	1	1 1 1	
7	1	0 0 1	← Indicate the error

5M

8M

$S = 001 \Rightarrow S(X) = x^2$

8 a)

$C = DG = [d_1 d_2 d_3]\begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & 1 \\ 0 & 1 & 0 & \vdots & 0 & 1 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 1 & 0 \end{bmatrix}$ — 1M

	code	wt
0 -	000 000	- 0
1	001 110	- 3
2	010 011	- 3
3	011 101	- 4
4	100 101	- 3
5	101 011	- 4
6	110 110	- 4
7	111 000	- 3

— 3M

$d_{min} = 3$

Error dection $= d_{min} - 1 = 2$ — 1M

Error Correction $= \frac{d_{min} - 1}{2} = 1$ — 1M

6M

b) $D = 1101 \Rightarrow D(x) = 1 + X + X^3$

Non-systematic:

$V(x) = D(x)\, g(x)$

$\boxed{V(x) = 1 + X^2 + X^6 \quad \text{OR} \quad V = 1010001}$ 2M

Systematic

$\frac{X^{n-k} D(x)}{g(x)} = \frac{X^3 + X^4 + X^6}{1 + X + X^3}$

$$\begin{array}{r} X^3 \\ X^3 + X + 1 \,\big)\, \overline{X^6 + X^4 + X^3} \\ X^6 + X^4 + X^3 \\ \hline 0 \end{array}$$

$\therefore V(x) = X^{n-k} D(x) + R(x)$

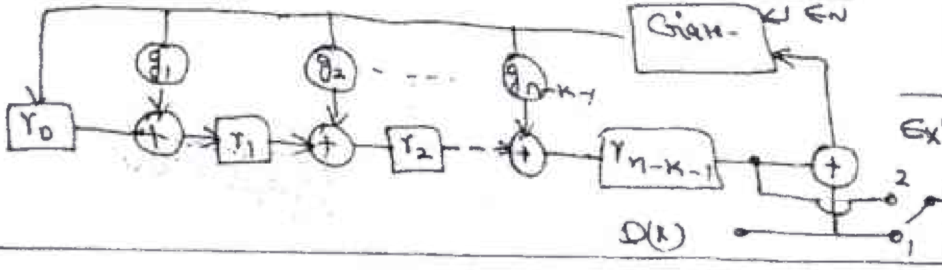
$\boxed{V(x) = X^3 + X^4 + X^6 \quad \text{OR} \quad V = 000 \vdots 1101}$

4M

2+4 = 6M

c) $R(X) = \frac{X^{n-k} D(x)}{g(x)} + Q(x)$

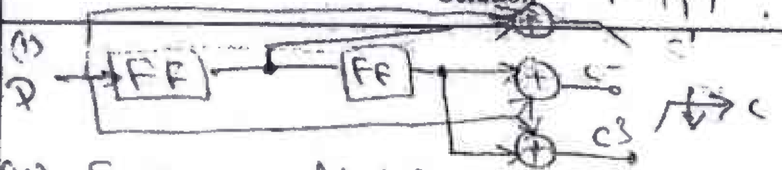
$\Downarrow$



— 3M

Exn — 5M

8M

Question Number	Solution	Marks Allocated
9 a)	(i) 	2M
	(ii) Generator Matrix $G = \begin{bmatrix} 111 & 101 & 011 & 000 & 000 & 000 & 000 \\ 000 & 111 & 101 & 011 & 000 & 000 & 000 \\ 000 & 000 & 111 & 101 & 011 & 000 & 000 \\ 000 & 000 & 000 & 111 & 101 & 011 & 000 \\ 000 & 000 & 000 & 000 & 111 & 101 & 011 \end{bmatrix}$ $C = DG = [11101][G]$ $= [111, 010, 001, 110, 100, 101, 011]$	7M
	Transform domain: $c^1(x) = 1 + x^3 + x^4 + x^5 \Rightarrow 1001110$ $c^2(x) = 1 + x + x^3 + x^6 \Rightarrow 1101001$ $c^3(x) = 1 + x^2 + x^5 + x^6 \Rightarrow 1010011$ $\therefore C = [111, 010, 001, 110, 100, 101, 011]$	7M
b)	BCH codes	4M
10 a)	(i) $g^{(1)} = 1011 \quad g^{(2)} = 1111$ $g^1(x) = 1 + x^2 + x^3, \quad g^2(x) = 1 + x + x^2 + x^3$	2M
	(ii) Time domain: $L = 5, \ l = 8,$ Using $c_i^d = \sum_{i=0} d_{l-i}\, g_{i+1}^d$ $c^1 = 10000001, \ c^2 = 11011101$ $C = [11, 01, 00, 01, 01, 01, 00, 11]$	7M
	Transform domain. $c^d(x) = D(x)\, g^d(x)$ $c^1(x) = 1 + x^7 \Rightarrow 10000001$ $c^2(x) = D(x)\, g^2(x) = 11011101$ $C = [11, 01, 00, 01, 01, 01, 00, 11]$	7M
b)	Golay code	4M