VTU Question Paper Solution

18EC53, Principles of Communication Systems

FEB/MAR 2022

CBCS SCHEME

USN 1CR19EC071 **18EC53**

Fifth Semester B.E. Degree Examination, Feb./Mar. 2022

Principles of Communication Systems

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Write an AM wave expression in time domain and in frequency domain. Draw AM waveform. (07 Marks)

b. With neat diagram, explain the demodulation of AM wave using envelope detector. (08 Marks)

c. An audio frequency signal $M(t) = 5 \sin 2\pi (10^3)t$ is used to amplitude modulate a carrier of $C(t) = 100 \sin 2\pi (10^6)t$. Assume modulation index $\mu = 0.4$. Find: i) Sideband frequencies ii) Amplitude of each sideband iii) Bandwidth iv) Total power delivered to a load of 100μ v) Find efficiency of AM wave, assume $R = 1\Omega$. (05 Marks)

OR

2 a. Explain the generation of DSBSC wave using a Ring modulator. (10 Marks)

b. Explain with a neat diagram, the working of Quadrature Carrier Multiplexing (QAM). (08 Marks)

c. An AM signal with a carrier of 1kW has 200W in each sideband. What is the percentage of modulation? (02 Marks)

Module-2

3 a. Define angle modulation. Derive the FM wave expression in time domain. (08 Marks)

b. Define the following terms:
i) Modulation index
ii) Frequency deviation
iii) Bandwidth (07 Marks)

c. A FM wave is represented by the equation $V = 10 \sin [5 \times 10^8 t + 4 \sin 1250t]$. Find: i) Carrier frequency and modulating frequency ii) Modulation index and frequency deviation iii) Bandwidth using Carson's rule. (05 Marks)

OR

4 a. Write the basic block diagram of PLL. Derive the expression for non-linear model of PLL. (10 Marks)

b. Explain the direct method of generating FM wave using Hartley oscillator with relevant equations and diagram. (06 Marks)

c. Write the Narrowband FM and wideband FM expression. (04 Marks)

Module-3

5 a. Derive the expression for figure of merit of an AM receivers using envelope detection. (10 Marks)

b. Explain the noisy receiver model with neat diagram. Explain briefly the figure of merit. (06 Marks)

c. Explain the noise equivalent bandwidth with relevant equation. (04 Marks)

OR

6 a. Derive the expression for Figure Of Merit (FOM) for DSBSC receiver. (10 Marks)

b. Explain the use of pre-emphasis and de-emphasis circuit in an FM system. (06 Marks)

c. Define the white noise. Briefly explain the power spectral density and autocorrelation function of white noise. (04 Marks)

Module-4

7 a. State sampling theorem. Write the mathematical form of sampled signal and explain the steps to reconstruct the signal g(t) from the sequence of sample value. (10 Marks)

b. Explain the concept of TDM with a neat block diagram. (06 Marks)

c. What is aperture effect? Briefly explain how to overcome this effect. (04 Marks)

OR

8 a. Briefly explain the following pulse modulation with waveform:
i) PAM ii) PWM iii) PPM. (09 Marks)

b. With neat block diagram, explain the generation of PPM wave. (05 Marks)

c. Explain the following terms:
i) Under sampling
ii) Over sampling
iii) Nyquist rate (06 Marks)

Module-5

9 a. Derive the expression of output signal to noise ratio of a uniform quantizer. (08 Marks)

b. With neat block diagram, explain the transmitter, transmission path and receiver of a PCM system. (08 Marks)

c. An audio signal digitalized using PCM. Assume the audio signal bandwidth to be 20kHz.
i) What is the Nyquist rate and Nyquist period of the audio signal?
ii) If the samples are quantized to L = 4096 levels and then binary coded, determine the number of bits required to encode a sample. (04 Marks)

OR

10 a. Draw the line codes for given binary representation 01101001
i) Unipolar NRZ signaling
ii) Polar NRZ signaling
iii) Unipolar RZ signaling
iv) Bipolar RZ signaling
v) Manchester code. (10 Marks)

b. Explain granular noise and slope overload distortion in delta modulation. (04 Marks)

c. With neat diagram explain delta modulation system. (06 Marks)

* * * * *

1a)

Time and Frequency domain description of AM-signal:

Q) Define Amplitude Modulation. Obtain the Expression for AM by both time domain and frequency domain representation with necessary waveforms.

→ Amplitude Modulation:-

Defⁿ:- It is a process of altering the amplitude of carrier signal in accordance with the instantaneous values of message signal by keeping frequency and phase of carrier signal constant.

Expression for AM signal:-

- The instantaneous value of message signal is given by,

$$m(t) = A_m \cos(2\pi f_m t) \quad \text{—— (1)}$$

where, $A_m \Rightarrow$ Amplitude of message signal.

$f_m \Rightarrow$ frequency @ Bandwidth of message signal.

- The instantaneous value of carrier signal is given by,

$$c(t) = A_c \cos(2\pi f_c t) \quad \text{—— (2)}$$

where, $A_c \Rightarrow$ Amplitude of carrier signal.

$f_c \Rightarrow$ Frequency of carrier signal.

- We know that the standard equation of AM signal is given by,

$$s(t) = A_c\left[1 + k_a m(t)\right] \cos(2\pi f_c t) \quad \text{—— (3)}$$

where, k_a = Amplitude Sensitivity parameter.

Substitute $m(t) = A_m \cos 2\pi f_m t$ in equation (3)

$$s(t) = A_c\left[1 + \underbrace{k_a A_m}_{} \cos(2\pi f_m t)\right] \cos(2\pi f_c t)$$

$$\therefore \; s(t) = A_c\left[1 + \mu \cos(2\pi f_m t)\right] \cos(2\pi f_c t) \quad \text{—— (4)}$$

Where $\mu = K_a A_m \Rightarrow$ Modulation Index for AM-signal

$$S(t) = [A_c + \mu A_c \cos 2\pi f_m t] \cos 2\pi f_c t$$

$$S(t) = A_c \cos 2\pi f_c t + \mu A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t$$

We know that, $\cos A \cdot \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$$\therefore S(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos 2\pi (f_c - f_m) t + \frac{\mu A_c}{2} \cos 2\pi (f_c + f_m) t$$

$$\boxed{S(t) = A_c \underbrace{\cos 2\pi f_c t}_{\text{carrier}} + \frac{\mu A_c}{2} \underbrace{\cos 2\pi (f_c - f_m) t}_{\text{LSB}} + \frac{\mu A_c}{2} \underbrace{\cos 2\pi (f_c + f_m) t}_{\text{USB}}} \longrightarrow (5)$$

Equation (5) gives the simplified expression of AM-signal. It consists of three frequency components

- $f_c \longrightarrow$ carrier frequency with amplitude 'A_c', which does not contains any message signal
- $f_c - f_m \rightarrow$ Lower side band (LSB) with amplitude $\frac{\mu A_c}{2}$
- $f_c + f_m \rightarrow$ upper side band (USB) with amplitude $\frac{\mu A_c}{2}$

Taking Fourier transformation on both sides of equation (5), we get-

$$\boxed{\begin{aligned} S(f) = {} & \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \\ & \frac{\mu A_c}{4}[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))] + \\ & \frac{\mu A_c}{4}[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))] \end{aligned}} \longrightarrow (6)$$

Equation (6) gives the fourier transform of $S(t)$.

Figure 1(b) shows the spectrum of AM wave s(t).

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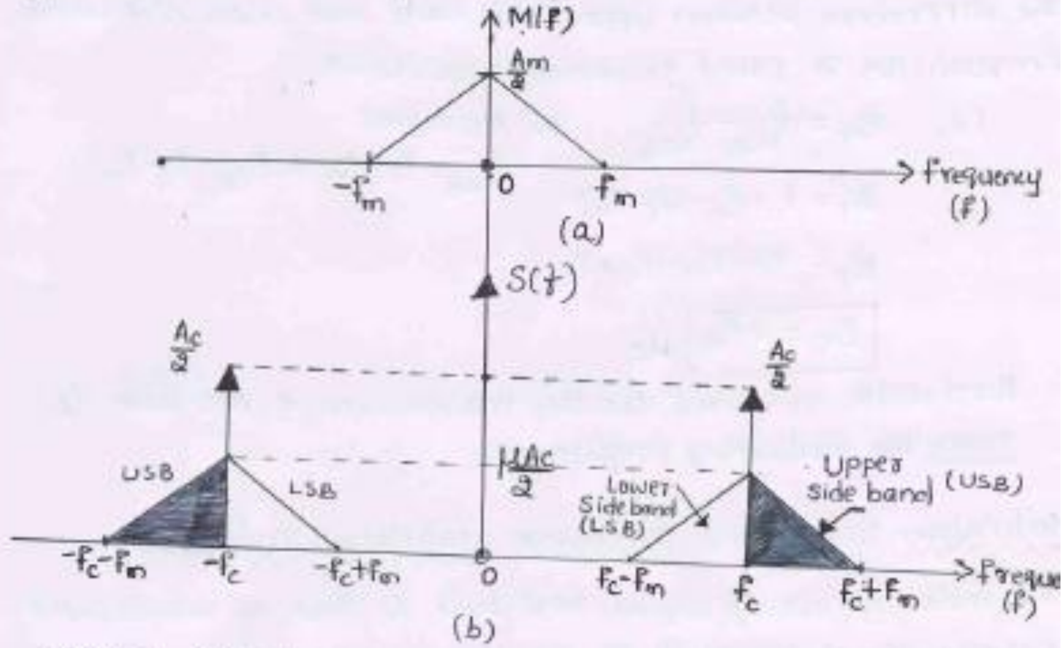


Figure 1. (a) Spectrum of m(t) (b) spectrum of AM signal.

Figure 2, shows the time domain signal waveforms.

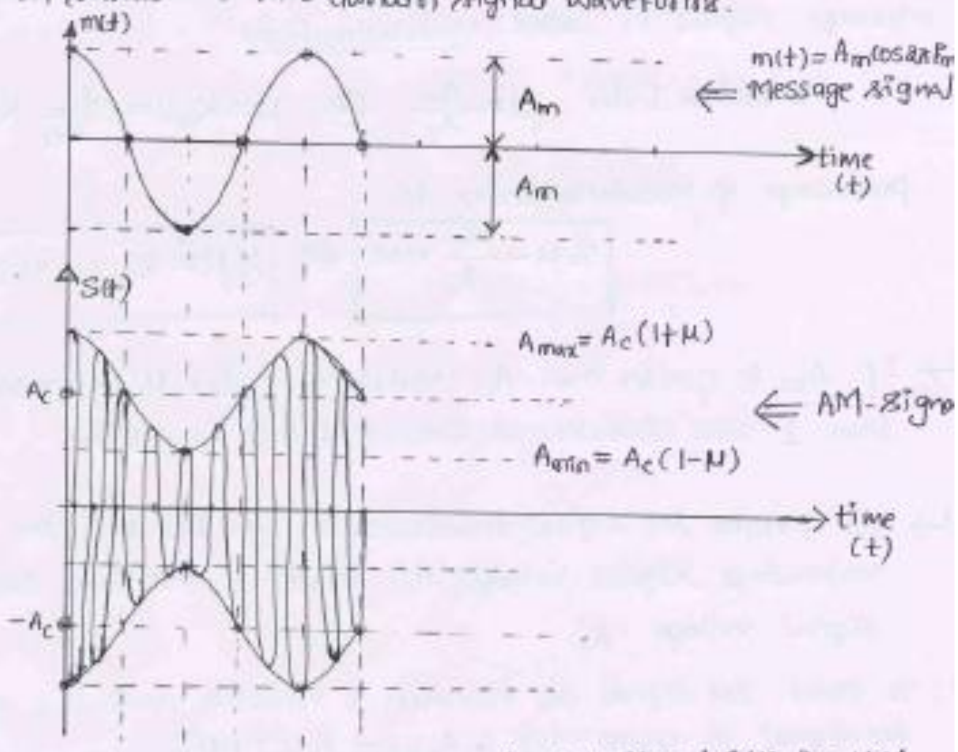


Figure 2: (a) Message signal m(t) (b) AM-Signal s(t) for $\mu<1$

1B)

Q> Explain the operation of envelope detector with neat diagrams and waveforms. Also mention the significance of RC-time constant.

June- July 2017

➡ Demodulation (or) Detection is the process of recovering the original message signal from the modulated wave at the receiver.

Envelope Detector : It is a simple and highly effective diode circuit. which is commonly used for demodulation of AM-signal.

Circuit diagram:-

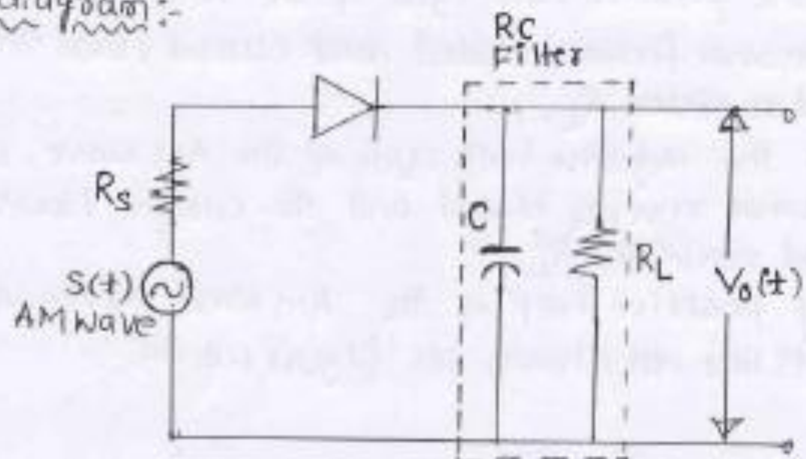


Figure 1.4(a): Circuit diagram of Envelope Detector:-

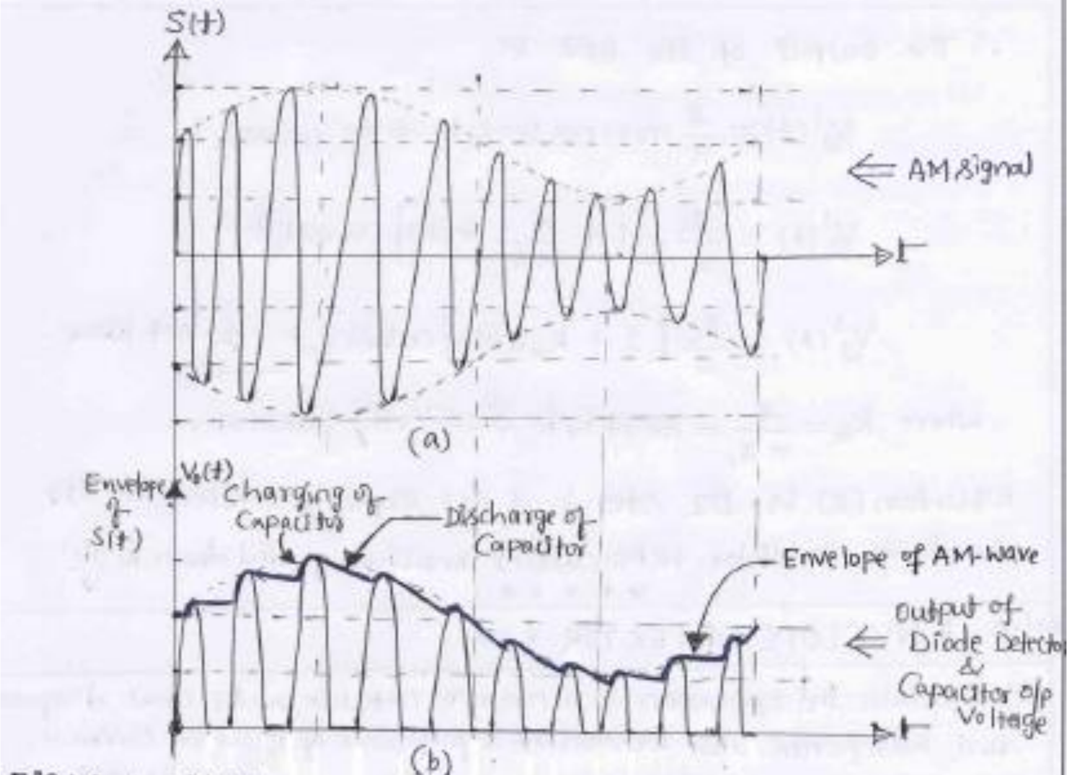


Figure 1.4(b): (a) AM Signal S(t), input to the Envelope Detector
(b) Envelope of AM Signal and voltage across Capacitor

Figure 4(a) shows the envelope detector circuit. It consists of a diode and a RC-filter. This circuit is also known as "Diode Detector".

Circuit Operation:-

↳ In the positive half cycle of the AM-signal, Diode 'D' becomes forward biased and current flows through load resistor 'R_L'.

↳ In the Negative half cycle of the AM-wave, Diode 'D' becomes reverse biased and No-current flows through load resistor 'R_L'.

∴ Only positive half of the AM wave appears across RC-Filter as shown in figure 1.4(b).

Working of RC-Filter:-

↳ During the +ve half cycle of AM wave, the capacitor 'C' charges up rapidly towards the peak value of the input signal. When the input signal falls below this value, the Diode becomes Reverse biased and the Capacitor 'C' discharges slowly through the load resistor 'R_L'.

↳ The Discharging process Continues Until the next positive half cycle of AM-Wave. When the input signal becomes greater than the voltage across Capacitor, the diode starts conducting again and the process is repeated

↳ This Continuous process of charging and Discharging of Capacitor, gives the Envelope of AM signal as shown in figure 1.4(b). which is in same shape as that of message Signal.

Selection of RC Constant:-

* The charging time Constant '$R_S C$' must be very much less than the Carrier period '$1/f_c$'.

$$\therefore \quad R_S C \ll \frac{1}{f_c} \;\Longrightarrow$$ To ensure Capacitor Charges up rapidly.

* the Discharging Time Constant '$R_L C$' should be long enough to ensure that the capacitor discharges slowly through the load resistor 'R_L' between positive peaks of the Carrier Wave.

i.e., $$\boxed{\frac{1}{f_c} \ll R_L C \ll \frac{1}{f_m}} \Longrightarrow$$ To ensure Slow Discharge of Capacitor

— * ** — *** —

1C) I) 110KhZ, 900Khz II)20 iii)2kHz

2a)

1.6. RING MODULATOR ***

Q) Explain the generation of DSBSC Wave using Ring Modulator and also Sketch the necessary Waveforms.

↳ Ring Modulator is a product Modulator used for Generating DSBSC-Modulated signal.

Circuit diagram:

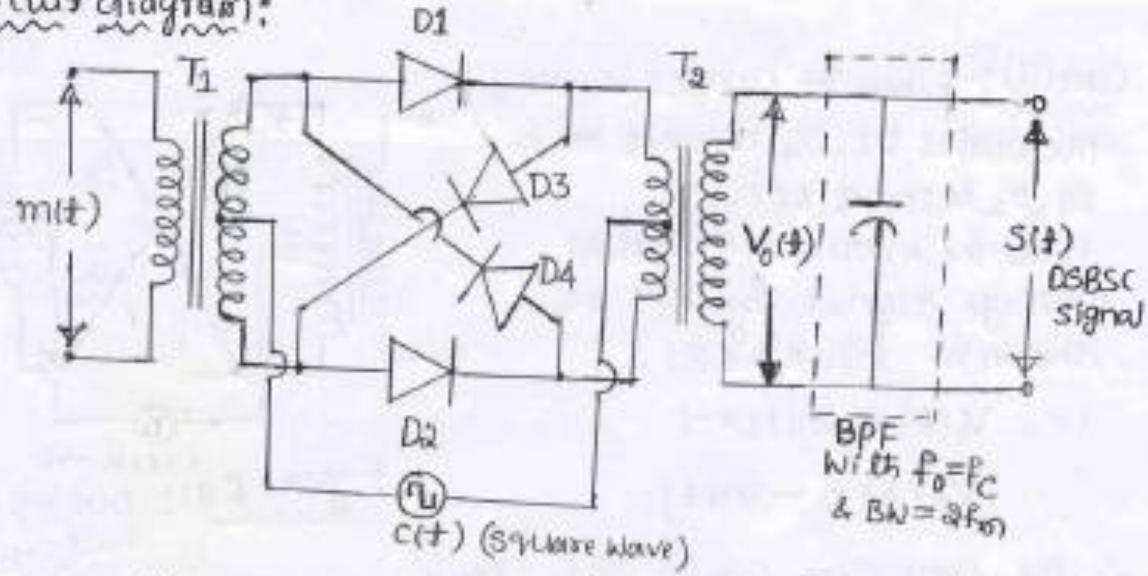


Figure 1.6(a): Circuit diagram of Ring Modulator

↳ The Circuit diagram of Ring modulator is shown in figure 1.6(a) consists of two Center-tapped transforms T1, T2 and Four diodes D1, D2, D3 and D4 Connected in bridge circuit and a BPF with Center frequency 'f_c', $BW = 2f_m$.

↳ The carrier signal is applied to the center taps of the input (T_1) and output (T_2) transformers. Modulating signal is applied to the input transformer T_1.

↳ The output voltage appears across the secondary of the transformer, T_2 (After passing through BPF).

↳ The Diodes connected in the bridge circuit (Ring) acts like switches and their switching is controlled by carrier signal (square wave).

Circuit operation :-

Case(i) : When the carrier is +ve, the Diodes D1, D2 becomes ON & Diodes D3, D4 becomes OFF.
Hence the Modulator Multiplies message signal m(t) by +1.

i.e., $V_0(t) = m(t) \times (+1) = m(t)$

Equivalent circuit is shown in Figure 1.6(b)

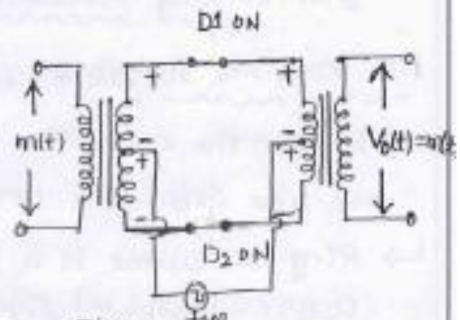


Figure 1.6(b): During +ve half cycle of C(t)

Case(ii) :- When the carrier is -ve, the Diodes D3, D4 becomes ON & D1, D2 becomes OFF.
Hence the modulator multiplies message signal by "-1" as shown in figure 1.6 (c).

i.e., $V_0(t) = m(t) \times -1$

$$V_0(t) = -m(t)$$

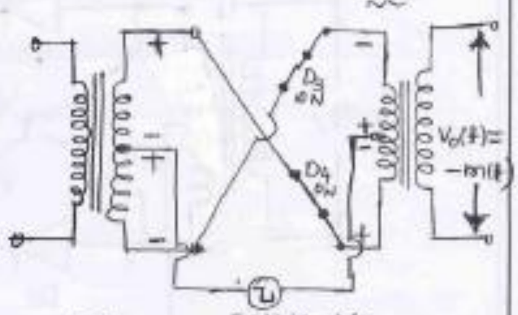


Figure 1.6 (c): During -ve half cycle of C(t)

∴ By combining Case(i) and Case(ii)
The Ring Modulator output at the secondary of transformer T_2 is given by

$$V_0(t) = m(t) \times C(t) \quad \text{—(1)}$$

The square wave Carrier $C(t)$ can be represented by a fourier Series as:

$$C(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t (2n-1)]$$

$$\therefore C(t) = \frac{4}{\pi}\left[\cos 2\pi f_c t - \frac{1}{3}\cos 6\pi f_c t + \cdots\right] \quad (2)$$

$\therefore$ Substitute equation (2) in $V_0(t)$ equation (1) we get

$$V_0(t) = m(t) \times \frac{4}{\pi}\left[\cos 2\pi f_c t - \frac{1}{3}\cos 6\pi f_c t + \cdots\right] \quad (3)$$

When $V_0(t)$ is passed through BPF having Center frequency 'f_c' and Bandwidth '$2f_m$' we get DSBSC Signal,

$$\boxed{S(t) = \frac{4}{\pi}\, m(t) \cos 2\pi f_c t} \Leftarrow \text{DSBSC Wave generated from RING Modulator}$$

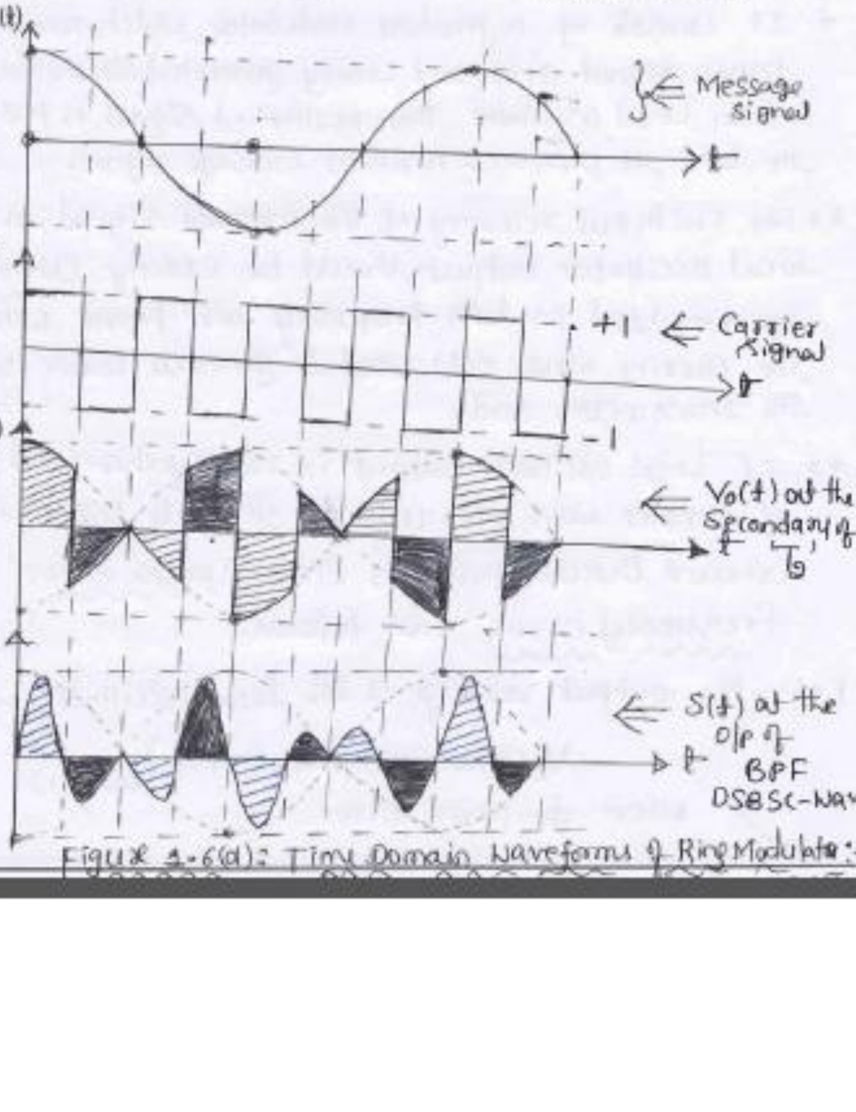


Figure 1.6(d): Time Domain Waveforms of Ring Modulator:-

2b)

1.9. Quadrature Carrier Multiplexing:

⟨Q⟩ With relevant diagrams, explain the operation of the quadrature carrier multiplexing transmitter scheme and receiver scheme. Dec 2016/Jan 2017 8M.

↳ Quadrature carrier multiplexing is a technique in which we can transmit more number of signals (DSBSC-wave) within the same channel bandwidth. This technique is also named as Quadrature Amplitude Modulation (QAM).

QAM transmitter:

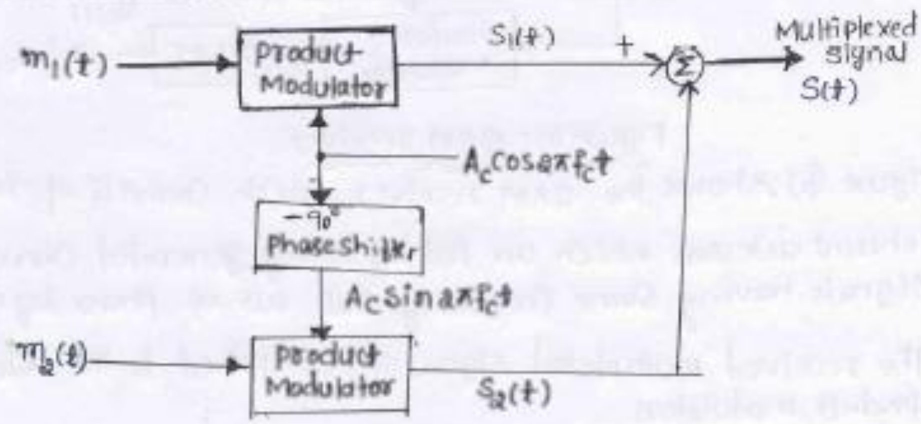


Fig (a) Quadrature Carrier Multiplexing (or) QAM Transmitter

- Figure (a) shows QAM transmitter. It consists of two product modulators that are supplied with carriers which differ in phase by 90° (phase quadrature)
- The output of the two product modulators are summed to produce multiplexed signal. S(t).

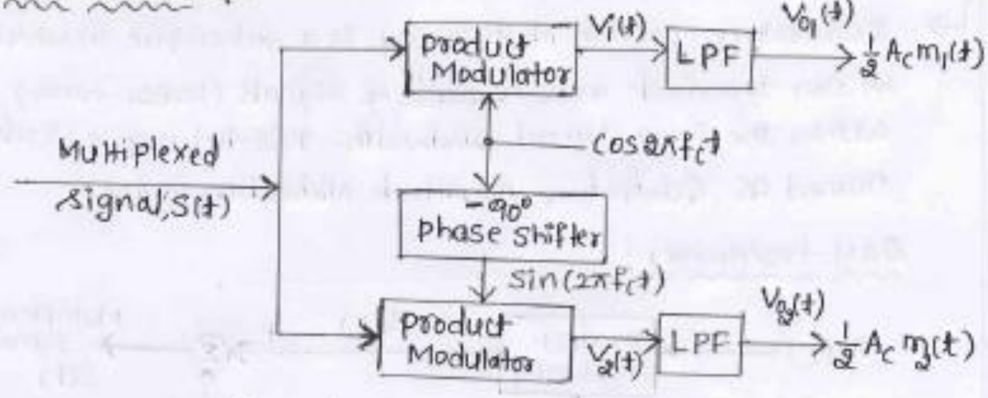
i.e., $S(t) = S_1(t) + S_2(t)$ -

$$S(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t$$

∴ QAM transmitter allows two modulated (DSBSC) waves to occupy the same transmission channel bandwidth.

∴ The Multiplexed Signal $s(t)$, occupies a channel Bandwidth of $BW = 2W$ ∴ $W = \text{Maximum}(f_{m_1}, f_{m_2})$ Centered at the Carrier frequency 'f_c'.

QAM - Receiver:-



Figure(b): QAM receiver

- Figure (b) shows the QAM receiver, which Consists of two Coherent detectors which are fed by locally generated Carrier Signals having Same frequency but out-of-phase by 90°.
- The received multiplexed signal $s(t)$ is applied to the two product modulators.

↳ The output of top product Modulator is

$$V_1(t) = s(t) \times \cos 2\pi f_c t.$$

↳ The top LPF removes the high frequency terms and allows only $\frac{A_c}{2} m_1(t)$.

$$\therefore V_{o_1}(t) = \frac{A_c}{2} \cdot m_1(t)$$

Similarly the output of bottom product modulator is

$$V_2(t) = s(t) \times \sin 2\pi f_c t$$

↳ The bottom LPF removes the high frequency terms and allows only $\frac{A_c}{2} \cdot m_2(t)$

$$\therefore V_{o_2}(t) = \frac{A_c}{2} m_2(t)$$

Application: used in color TV.

2c) 89%

3A)

Angle Modulation, is a process of altering either Frequency (or) Phase of carrier signal in accordance with the instantaneous values of message signal, by keeping amplitude of carrier constant.

↳ General equation of Angle Modulated wave is given by

$$S(t) = A_c \cos \theta_i(t) \quad \text{—— (1)}$$

Where, A_c = Amplitude of Carrier signal

$\theta_i(t)$ = Angle of the modulated signal.

Angle Modulation techniques are further divided into two types

→ Frequency Modulation (FM)

→ Phase Modulation (PM)

- Frequency Modulation :- It is a process of altering frequency of carrier signal in accordance with the instantaneous values of message signal by keeping amplitude, phase of carrier constant.

  ↳ The General equation of FM-signal is given by

$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int m(t)\,dt\right] \quad \text{—(2)}$$

where, K_f = Frequency Sensitivity parameter in Hz/volt.

$m(t)$ = Message signal

- Phase Modulation :

It is a process of altering phase of carrier signal in accordance with the instantaneous values of message signal.

↳ The General equation of PM signal is

$$S(t) = A_c \cos\left[2\pi f_c t + K_p m(t)\right] \quad \text{—(3)}$$

Where K_p = Phase Sensitivity parameter.

1.2 Frequency Modulation:-

Q) Define Frequency Modulation. Derive the time domain expression for frequency modulated wave & also sketch necessary waveforms.

→ Frequency Modulation is a process of altering the frequency of carrier signal in accordance with the instantaneous values of message signal by keeping amplitude & phase of carrier constant.

Time domain expression:-

- Let the instantaneous value of carrier signal is

$$c(t) = A_c \cos 2\pi f_c t \longrightarrow (1)$$

- Let the instantaneous value of message signal is

$$m(t) = A_m \cos 2\pi f_c t \longrightarrow (2)$$

- We know that the standard equation of Angle modulated wave is given by,

$$s(t) = A_c \cos \theta_i(t) \longrightarrow (3)$$

where $\theta_i(t)$ = Angle of FM wave (modulated wave)

- We know that the instantaneous frequency $f_i(t)$ of FM signal is given by

$$f_i(t) = f_c + k_f m(t) \quad \text{---} (4)$$

where, k_f = frequency sensitivity

$m(t)$ = message signal

- We know that the Angular frequency,

$$\omega_i(t) = \frac{d}{dt}\theta_i(t)$$

$$\Downarrow$$

$$2\pi f_i(t) = \frac{d}{dt}\theta_i(t)$$

$$\therefore f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t) \quad \text{---} (5)$$

Substitute $f_i(t) = f_c + k_f m(t)$ in equation (5) we get,

$\therefore\ f_c + k_f m(t) = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t)$

$$\therefore\ \frac{d\,\theta_i(t)}{dt} = 2\pi f_c + 2\pi k_f m(t) \quad —(6)$$

⇒ Apply Integral on both sides of equation (6) we get

$$\int \frac{d\,\theta_i(t)}{dt} = \int [2\pi f_c + 2\pi k_f m(t)]\,dt$$

$$\Downarrow$$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int m(t)\,dt \quad —(7)$$

∴ The General equation of FM signal is

$$S(t) = A_c \cos \theta_i(t) \quad \text{using equation (7)}$$

$$\boxed{S(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int m(t)dt]} \quad —(8)$$

Equation (8) is the general equation of FM signal for any message signal $m(t)$.

for, $m(t) = A_m \cos 2\pi f_m t$

$$\int m(t)\,dt = \int A_m \cos 2\pi f_m t\,dt \qquad \left(\because \int \cos mx\,dx = \frac{\sin mx}{m}\right)$$

$$= \frac{A_m}{2\pi f_m} \cdot \sin 2\pi f_m(t) \quad —(9)$$

∴

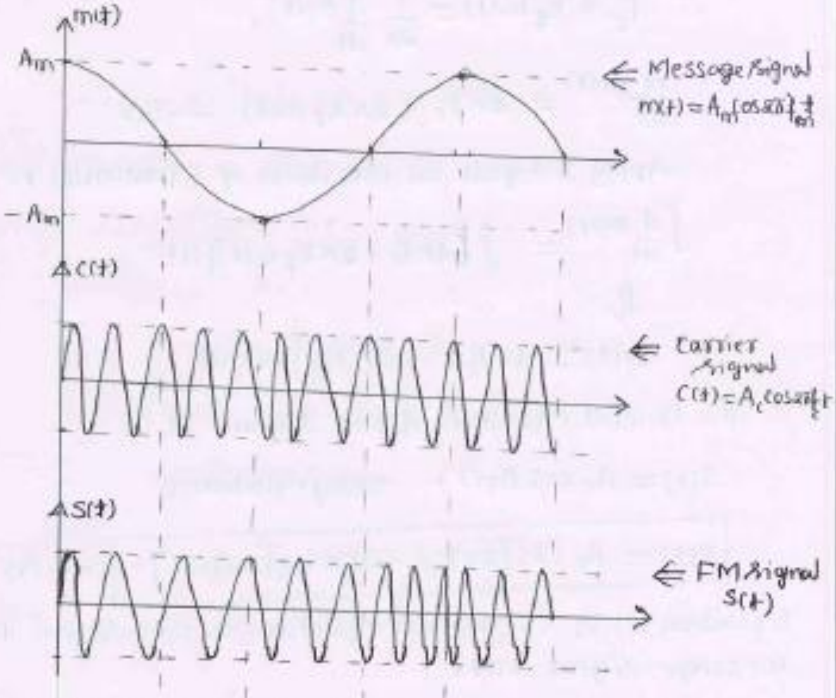
$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \times \frac{A_m}{2\pi f_m} \cdot \sin(2\pi f_m t)\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

$$\boxed{S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]} \quad —(10)$$

Equation (10) is the standard equation of FM signal for $m(t) = A_m \cos 2\pi f_m t$ ∴ where $\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f_{max}}{f_m}$ ⇐ Modulation Index of FM signal.

The required waveforms of $m(t)$, $C(t)$ & $S(t)$ are shown in figure (2).



Figure(2): (a) Message Signal m(t) (b) Carrier Signal c(t)
(c) Frequency Modulated (FM) Signal.

Figure 2(c) shows the time domain representation of FM-Signal S(t) shown in equation (10). The Frequency of S(t) linearly varies with respect to message signal m(t). i.e. $\left[f_i(t) = f_c + k_f m(t)\right]$.

3b)

Modulation Index : (β)

It is the ratio of Maximum frequency deviation to that of frequency of message signal. It is denoted by Symbol 'β'.

i.e., Modulation Index in FM $\Rightarrow \boxed{\beta = \dfrac{\Delta f_{max}}{f_m}}$ No units

4a)

c) Maximum Frequency deviation $[\Delta f_{max}]$:-

It is the difference between maximum frequency of FM signal to that of unmodulated carrier frequency.

It is denoted by Δf_{max}.

i.e, $\boxed{\Delta f_{max} = K_f A_m}$

Proof: From the definition of frequency deviation,

$$\Delta f_{max} = \text{Max. Frequency of FM-signal} - \text{Frequency of Carrier Signal}$$

$$\Delta f_{max} = f_i(t)_{Max} - f_c$$

From figure 1, $f_i(t)_{max} = f_c + K_f A_m$

$$\therefore \Delta f_{max} = f_c + K_f A_m - f_c$$

$\boxed{\Delta f_{max} = K_f A_m}$ Hz $\Leftarrow$ Indicated in figure 1.

The Bandwidth of wide band FM signal can be calculated from Carson's Rule shown in equation (1)

$\boxed{BW_T = 2f_m + 2\Delta f_{max}}$ $\Leftarrow$ CARSON'S Rule to find Bandwidth of wide band FM-Signal.

3 c)

i) Fc=79.6M Hz

Fm= 190Hz

ii)modulation index=4, deviation= 760

iii)1900

4a)

phase Locked Loop (PLL) is a negative feedback system that consists of three major components

⟨i⟩ A Multiplier used as a phase detector (or) phase comparator.

⟨ii⟩ A - voltage controlled oscillator ⟨VCO⟩

⟨iii⟩ A - Loop filter, which is a Low pass filter ⟨LPF⟩.

The Block diagram of PLL is shown in Fig.1.

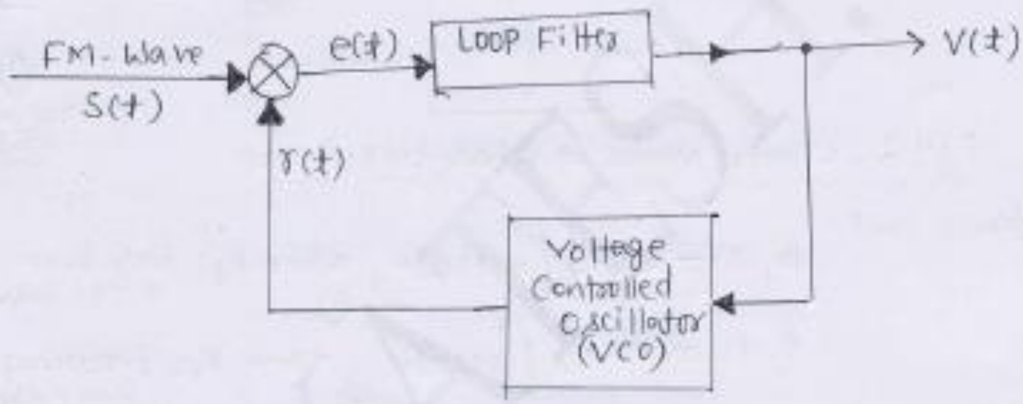


Fig.1: Block diagram of PLL

↳ The VCO output is defined as

$$r(t) = A_v \cos(2\pi f_c t + \phi_2(t)) \quad \text{— ①}$$

$$\text{Where } \phi_2(t) = 2\pi K_v \int_0^t v(t)\, dt.$$

↳ Then, the incoming signal (FM) (S(t)) and the VCO output r(t) are applied to the multiplier, then it gives error signal,

$$e(t) = r(t) \cdot S(t) \quad \text{—②}$$

$$\text{Where } S(t) = A_c \sin[2\pi f_c t + \phi_1(t)] \quad \text{—③}$$

$$\text{Where } \phi_1(t) = 2\pi K_f \int_0^t m(t)\, dt \quad \text{—④}$$

3.9: Non-Linear effects in FM-Wave :- <VTU Q.P>

Q) Write a short note on Non-linear effects in FM-system.

↳ Non-linear effects can be of two-types

(i) Strong (ii) Weak.

* Non-linearity is said to be strong, if it is intentionally introduced into the circuit in a controlled manner.

  Ex: square law devices.

* Non linearity is said to be weak, when it is inherently present in the circuit.

The effect such non-linearities will limit m(t) levels in the system.

In FM-generation system, weak non-linearity is present. the effect of weak non-linearity in FM-systems can be by considering the input and output relation of the memoryless - non linear device used in the frequency multiplier.

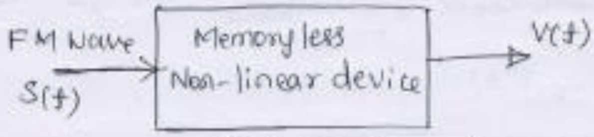


Fig 1: Non-linear device used in FM system:-

Consider a memoryless Non-linear device as shown in fig. 1.
W.K.T, the relation between input & output signal is

$$V(t) = a_1 S(t) + a_2 S^2(t) + a_3 S^3(t) + \cdots + a_n S^n(t) \quad —(1)$$

Let us consider upto 3rd order

i.e. $V(t) = a_1 S(t) + a_2 S^2(t) + a_3 S^3(t)$ —(2)

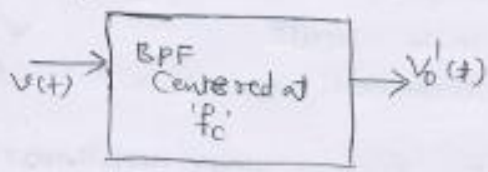
W.K.T the expression for FM-wave is

$$S(t) = A_c \cos[2\pi f_c t + \phi(t)] \quad \text{where } \phi(t) = 2\pi k_f \int_0^t m(t)\,dt$$

$$\therefore V(t) = a_1 A_c \cos[2\pi f_c t + \phi_1(t)] + a_2 A_c^2 \cos^2[2\pi f_c t + \phi_1(t)] + a_3 A_c^3 \cos^3[2\pi f_c t + \phi_1(t)] \quad —(3)$$

$\therefore$ The the output voltage consists of DC component and three FM-signals with carrier frequencies f_c, $2f_c$ & $3f_c$ having frequency deviations Δf, $2\Delta f$ and $3\Delta f$ respectively.

The Desired FM-signal can be separated using a BPF as shown in fig. 2.



We get the FM-system o/p after passing through BPF is

$$V_0'(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] \quad —(4)$$

Equation (4) is same as that of FM-input signal

$S(t) = A_c \cos[2\pi f_c t + \phi(t)]$ Except for change in amplitude.

$\therefore$ Amplitude Non-linearities of the FM-S/m does not affect FM-signal

4b)

56. Generation of FM-Waves :-

There are two basic methods of generating FM-Waves,

⟨i⟩ Direct Method.

⟨ii⟩ Indirect Method ⟨Armstrong Modulator⟩

*** Imp**

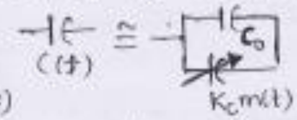
⟨i⟩ Generation of frequency modulated signal using DIRECT-METHOD :-

⟨Q⟩ Explain generation of frequency modulated signal using direct method. V.T.U June/July-2017 (5M)

↳ The Direct method uses a sinusoidal oscillator, with one of the reactive elements (example: Capacitive element) in the tank circuit of the oscillator being directly controlled by the message signal, m(t).

↳ In direct method of FM-signal generation, the instantaneous frequency of the carrier wave is varied directly in accordance with the message signal.

↳ Fig.1, shows a Hartly oscillator in which the Capacitive Component of the tank circuit is,



$$C(t) = C_0 + K_c m(t) \quad —— (*)$$

where, C_0 = Total Capacitance in the absence of modulation.

K_c = Variable Capacitor Sensitivity to voltage change.

$m(t)$ = message signal $= A_m \cos(2\pi f_m t)$.

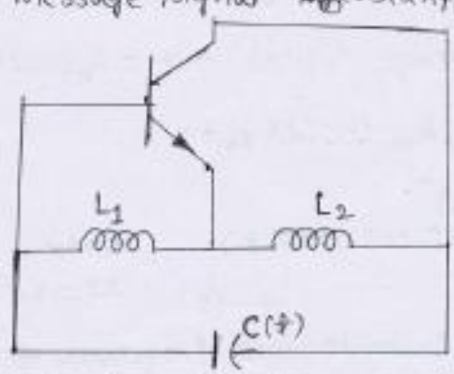


Fig.1: Hartley oscillator

The frequency of the Hartley oscillator is given by

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,c(t)}} \quad ; \text{where } c(t) = C_0 + K_c m(t)$$

$$\therefore \quad f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)[C_0 + K_c m(t)]}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_0\left[1+\frac{K_c}{C_0}m(t)\right]}}$$

$$f_i(t) = \frac{f_0}{\sqrt{1+\frac{K_c}{C_0}m(t)}} \quad ; \text{where } f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}}$$

$$f_i(t) = f_0\left(1+\frac{K_c}{C_0}m(t)\right)^{-1/2} \quad \text{---} (1)$$

Using Binomial theorem, $(1+x)^{-1/2} = \left(1-\frac{x}{2}\right)$

$$\therefore \quad \left(1+\frac{K_c}{C_0}m(t)\right)^{-1/2} = \left(1-\frac{K_c\,m(t)}{2C_0}\right) \quad \text{---} (2)$$

Using equation (2) in (1) we get

$$f_i(t) = f_0\left(1-\frac{K_c\,m(t)}{2C_0}\right) \quad \text{---} (3)$$

Let us assume, $\frac{-K_c}{2C_0} = \frac{K_f}{f_0}$, where K_f = frequency sensitivity parameter ;

$$\therefore \quad f_i(t) = f_0\left(1+\frac{K_f}{f_0}m(t)\right)$$

$$f_i(t) = f_0 + K_f m(t) \quad \text{---} (4)$$

for sinusoidal message signal, $m(t) = A_m \cos 2\pi f_m t$

$$\therefore \quad f_i(t) = f_0 + \underbrace{K_f A_m}_{\Downarrow} \cos(2\pi f_m t)$$

$$\therefore \quad f_i(t) = f_0 + \Delta f \cos(2\pi f_m t) \quad \text{---} (5)$$

; where $\Delta f = K_f A_m$ = Maximum frequency deviation.

Equation (5) gives, the instantaneous frequency of FM-wave generated by using direct method.

4c)

3. Classification of Frequency modulated Signals:-

Depending on the value of modulation index 'β' and channel Bandwidth FM-signals are classified into two two types

i) Narrow Band FM

ii) Wide Band FM

i) Narrow Band FM:

In Narrow band FM signal

↳ The value of modulation index, $\beta < 1$ (Less than 1)

↳ only two side bands are present

↳ The transmission channel Bandwidth, $BW_T = 2W$

↳ The message signal frequency, f_m is in between 30HZ to 3KHZ.

↳ Maximum frequency deviation is 15KHZ

Application:- Narrow band FM-technique is mainly used for Speech signal transmission
Example: Mobile communication

ii) Wide-band FM:-

In Wide-band signal

↳ The Value of modulation index, $\beta \gg 1$. (Greater than 1)

↳ Infinite number of side bands are present.

** ↳ The message signal frequency, f_m is in between 30HZ to 15KHZ.

*** The Bandwidth of Wide band FM signal can be calculated from Carson's Rule shown in equation (1)

$$\boxed{BW_T = 2f_m + 2\Delta f_{max}} \Leftarrow$$ CARSON'S Rule to find Bandwidth of wide band FM-Signal.

↳ The Maximum frequency deviation is 75 KHZ.

Applications of Wide band FM:-

* Wide-band FM technique is mainly used in High Quality Music signal transmission.

Example: FM-channels

Narrow band FM signal

$$\boxed{S(t) = A_c \cos 2\pi f_c t - A_c \beta \cdot \sin 2\pi f_c t \cdot \sin 2\pi f_m t}$$

Wideband expression:

$$\boxed{S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]}$$

5 a)

2.3) NOISE IN AM RECEIVERS

➢ An AM signal is given by

$$s(t) = A_c(1 + k_a m(t))\cos 2\pi f_c t \quad (1)$$

where,

$m(t)$ = message signal and let us assume that the message signal power is 'P' watts

$c(t) = A_c\cos 2\pi f_c t$ = carrier signal and

the average power of the carrier signal is $\frac{A_c^2}{2}$

$$s(t) = A_c\cos 2\pi f_c t + A_c k_a m(t)\cos 2\pi f_c t \quad (2)$$

So average power in $s(t) = \frac{A_c^2}{2} + \frac{A_c^2 k_a^2 P}{2} = \frac{A_c^2}{2}(1 + k_a^2 P)$

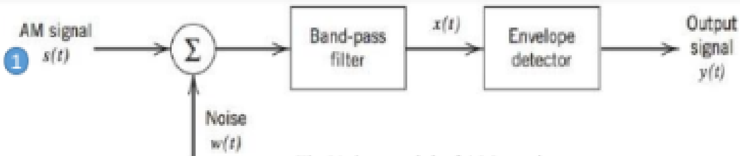


Fig Noisy model of AM receiver.

➢ The combination $s(t) + w(t)$ is applied to a bandpass filter, the BPF is actually a narrow- BPF such that $f_c \gg B_T$,

➢ After passing from BPF, wideband noise $w(t)$ gets converted into narrowband noise $n(t)$

➢ The filtered signal $x(t)$ available for demodulation is defined by

$$x(t) = s(t) + n(t)$$

$$x(t) = A_c\cos 2\pi f_c t + A_c k_a m(t)\cos 2\pi f_c t + n(t) \quad (3)$$

➢ The power of the noise $n(t)$ is given by $N_o W$, where W is the bandwidth of message signal

➢ we define **the *channel signal-to-noise ratio*,**

$$(SNR)_C = \frac{\text{average power of the modulated signal}}{\text{the average power of noise in the message bandwidth}}$$

$$(SNR)_C = \frac{\frac{A_c^2}{2}(1 + k_a^2 P)}{N_o W} = \frac{A_c^2(1 + k_a^2 P)}{2N_o W} \quad (4)$$

From equation (3)

$$x(t) = A_c cos2\pi f_c t + A_c k_a m(t) cos2\pi f_c t + n(t)$$

$$x(t) = A_c cos2\pi f_c t + A_c k_a m(t) cos2\pi f_c t + n_I(t) cos2\pi f_c t - n_Q(t) sin2\pi f_c t$$

$$x(t) = A_c(1 + k_a m(t) + n_I(t)) \, cos2\pi f_c t - n_Q(t) sin2\pi f_c t \quad (5)$$

After passing from envelope detector, the envelope of output $x(t)$ is given by

$y(t)$ = **Envelope of** $\mathbf{x(t)}$

$$y(t) = \left\{ \left(A_c(1 + k_a m(t) + n_I(t)\right)^2 + \left(n_Q(t)\right)^2 \right\}^{1/2} \quad (6)$$

However, when the average carrier power is large compared with the average noise power, so that the receiver is operating satisfactorily, then the signal term $A_c(1 + k_a m(t))$ will be large compared with the noise terms $n_I(t))$ and $n_Q(t)$, at least most of the time. Then we may approximate the output $y(t)$ *as*

$$y(t) \simeq A_c(1 + k_a m(t) + n_I(t))$$

After passing from LPF

$$y(t) \simeq \underbrace{A_c k_a m(t)}_{\text{Demodulated Signal}} + \underbrace{n_I(t))}_{\text{Noise}} \quad (7)$$

The average power of the demodulated signal is= $\frac{A_c^2 k_a^2 P}{2}$ (8)

- The power of the noise $n_I(t)$ is given by $N_o W$, where W is the bandwidth of message signal

- The *output signal-to-noise ratio,*

$$(SNR)_0 = \frac{\textit{average power of the demodulated message signal}}{\textit{the average power of the noise}}$$

$$(SNR)_0 = \frac{A_c^2 k_a^2 P}{2N_o W} \quad (9)$$

Finally we need to find out 'Figure of Merit' of AM as

$$FOM = \frac{(SNR)_0}{(SNR)_C}$$

$$FOM = \frac{\frac{A_c^2 k_a^2 P}{2N_o W}}{\frac{A_c^2(1 + k_a^2 P)}{2N_o W}}$$

$$\boxed{FOM = \frac{k_a^2 P}{1 + k_a^2 P}} \quad (10)$$

FOM of AM receiver is given by

$$FOM = \frac{k_a^2 P}{1 + k_a^2 P}$$

Suppose, if the message signal is a **singletone** waveform then,

$\boldsymbol{m(t)} = A_m cos2\pi f_m t,$

Then average message signal power is

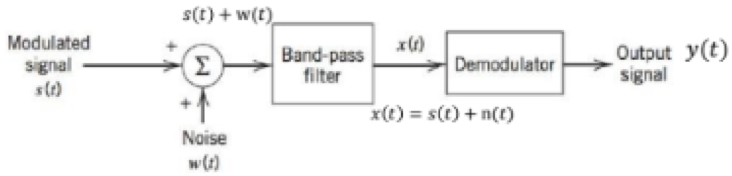
$$P = \frac{A_m^2}{2} \quad (11)$$

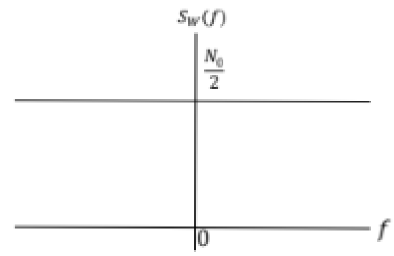
So,

$$FOM = \frac{k_a^2\left(\frac{A_m^2}{2}\right)}{1 + k_a^2\left(\frac{A_m^2}{2}\right)} = \frac{\mu^2}{2 + \mu^2} \quad (\because \mu = k_a A_m) \quad (12)$$

5b)

2.1) Receiver Model





- To make receiver model analysis convenient the kind of noise affecting the channel is chosen to be Additive White Gaussian Noise (AWGN)
- AWGN has following characteristics

(i) It is additive in nature it gets added on the top of information or message signal

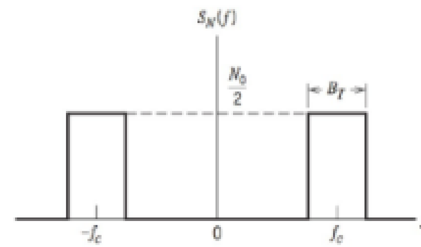
(ii) It is prevalent in all frequency hence it is known as white

(iii) The distribution of this noise is Gaussian in nature

- Here, $s(t)$=modulated signal coming from transmitter
  $w(t)$ = wideband noise
- The combination $s(t) + w(t)$ is applied to a bandpass filter, the BPF is actually a narrow- BPF such that $f_c \gg B_T$,

where, f_c = carrier frequency and B_T =Transmission BW

- After passing from BPF, wideband noise $w(t)$ gets converted into narrowband noise $\mathrm{n}(t)$



- Here,

PSD of wideband noise , $S_W(f) = \frac{N_0}{2}$; $for\ all\ frequency$ 1

PSD of narrowband noise , $S_N(f) = \frac{N_0}{2}$; $for\ {-B_T}/_2 < |f| < {B_T}/_2$ 2

Where, $N_0 = Average\ noise\ per\ unit\ bandwidth$

- Mathematically a narrowband noise equation can be written as

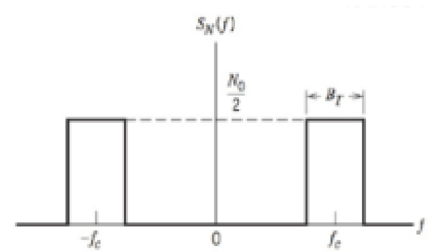
$$n(t) = n_I(t)cos2\pi f_c t - n_Q(t)cos2\pi f_c t \quad 3$$

where $n_I(t)$ is the in-phase noise component and $n_Q(t)$ is the quadrature noise component, both measured with respect to the carrier wave $\cos 2\pi f_c t$

- The filtered signal $x(t)$ available for demodulation is defined by

$$x(t) = s(t) + n(t) \quad 4$$

- The details of $s(t)$ depend on the type of modulation used, in any event, the average noise power at the demodulator input is equal to the total area under the curve of the power spectral density $S_N(f)$
- From figure we readily see that this **average noise power** is equal to $\boldsymbol{N_0 B_T}$



Given the format of $s(t)$, we also have to determine the average **signal power** at the demodulator input.

- we define **the *channel signal-to-noise ratio*,**

$$(SNR)_C = \frac{average\ power\ of\ the\ modulated\ signal}{the\ average\ power\ of\ noise\ in\ the\ message\ bandwidth}, \quad 5$$

measured at the receiver input.

- The *output signal-to-noise ratio,*

$$(SNR)_0 = \frac{average\ power\ of\ the\ demodulated\ message\ signal}{the\ average\ power\ of\ the\ noise}$$

measured at the receiver output. 6

Finally we need to find out 'Figure of Merit' as

$$FOM = \frac{(SNR)_0}{(SNR)_C} \quad (7)$$

Clearly, the higher the value of the figure of merit, the better will the noise performance of the receiver be. The figure of merit may equal one, be less than one, or be greater than one, depending on the type of modulation used.

5 c)

2.9)NOISE EQUIVALENT BANDWIDTH

The average output noise power of the any filter is proportional to the bandwidth.

Noise equivalent bandwidth is a term that basically compares the noise power of any arbitrary filter response with the ideal filter response.

Consider the case of a LPF, suppose that we have a source of white noise of zero mean and power spectral density $N_0/2$ connected to the input of an arbitrary lowpass filter of transfer function $H(f)$. The resulting average output noise power is therefore

$$N_{out} = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df$$

$$N_{out} = N_0\int_{0}^{\infty} |H(f)|^2 df \quad (1)$$

Consider next the same source of white noise connected to the input of an *ideal* lowpass filter of zero-frequency response $H(0)$ and bandwidth B.

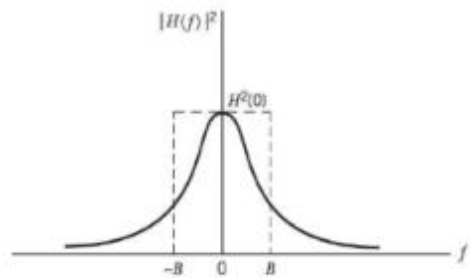
In this case, the average output noise power is

$$N_{out} = N_0 B H^2(0) \quad (2)$$

Therefore, equating this average output noise power to that in (1) we may define the *noise equivalent bandwidth* as

$$B = \frac{\int_0^{\infty} |H(f)|^2 df}{H^2(0} \quad (3)$$

Thus, the procedure for calculating the noise equivalent bandwidth consists of replacing the arbitrary low-pass filter of transfer function $H(f)$ by an equivalent ideal low-pass filter of zero frequency response $H(0)$ and bandwidth B, as illustrated in Figure



6 a)

2.2) NOISE IN DSB-SC RECEIVERS

- A DSB-SC signal is given by

$$s(t) = m(t)c(t) \quad (1)$$

where,

$\boldsymbol{m(t)}$ = message signal and let us assume that the message signal power is **'P'** watts

$\mathbf{c(t)} = A_c cos2\pi f_c t$ =carrier signal and the power of the carrier signal is $\frac{A_c^2}{2}$

Hence,

$$s(t) = A_c m(t) cos2\pi f_c t \quad (2)$$

- The combination $s(t) + w(t)$ is applied to a bandpass filter, the BPF is actually a narrow- BPF such that $f_c \gg B_T$,
- After passing from BPF, wideband noise $w(t)$ gets converted into narrowband noise $\mathrm{n}(t)$
- The filtered signal $x(t)$ available for demodulation is defined by

$$x(t) = s(t) + n(t)$$

$$x(t) = A_c m(t) cos2\pi f_c t + n(t) \quad (3)$$

- The power of the noise $n(t)$ is given by $N_o W$, where W is the bandwidth of message signal

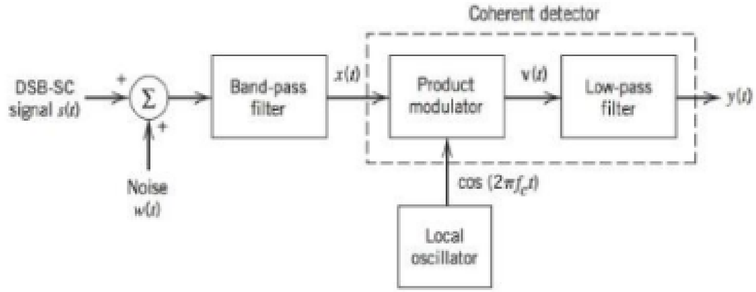
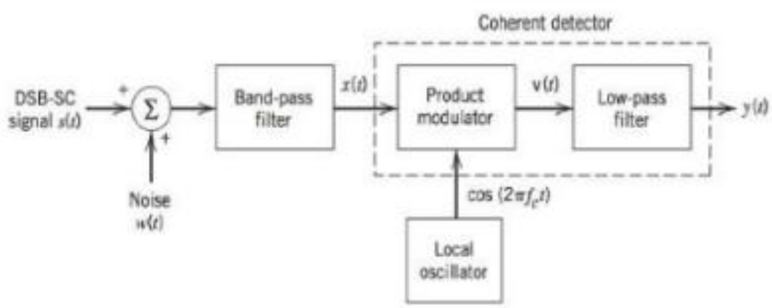


Fig. Model of DSB-SC receiver using coherent detection.

- we define **the** ***channel signal-to-noise ratio***,

$$(SNR)_C = \frac{\textit{average power of the modulated signal}}{\textit{the average power of noise in the message bandwidth}}$$

$$(SNR)_C = \frac{\frac{A_c^2}{2}P}{N_o W} = \frac{A_c^2 P}{2N_o W} \quad (4)$$



In the coherent detecter the incoming signal x(t) is multiplied by the locally generated carrier signal to produce v(t) which is given by

$$\Rightarrow v(t) = x(t) cos2\pi f_c t$$

$$\Rightarrow v(t) = (s(t) + n(t)) cos2\pi f_c t \quad (5)$$

$$\because n(t) = n_I(t) cos2\pi f_c t - n_Q(t) sin2\pi f_c t \quad (6)$$

$$\Rightarrow v(t) = (A_c m(t) cos2\pi f_c t + n_I(t) cos2\pi f_c t - n_Q(t) cos2\pi f_c t) cos2\pi f_c t \quad (7)$$

After passing from a LPF, all the higher frequency terms will be eliminated, the output is given by

$$y(t) = \underbrace{\frac{A_c m(t)}{2}}_{\text{Demodulated Signal}} + \underbrace{\frac{n_I(t)}{2}}_{\text{Noise}} \quad (8)$$

The power of the demodulated signal $\frac{A_c m(t)}{2}$ is $\frac{A_c^2 P}{4}$

The power of the noise $\frac{n_I(t)}{2}$ is $\frac{N_o W}{2}$

- The *output signal-to-noise ratio*,

$$(SNR)_0 = \frac{\textit{average power of the demodulated message signal}}{\textit{the average power of the noise}}$$

measured at the receiver output.

$$(SNR)_0 = \frac{\frac{A_c^2 P}{4}}{\frac{N_o W}{2}}$$

$$(SNR)_0 = \frac{A_c^2 P}{2N_o W} \quad (9)$$

Finally we need to find out 'Figure of Merit' of DSB-SC as

$$FOM = \frac{(SNR)_0}{(SNR)_C}$$

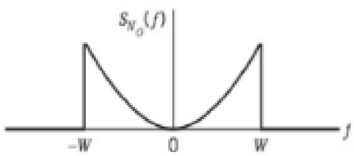
$$\boxed{FOM = 1} \quad (10)$$

6 b)

2.8) PRE-EMPHASIS AND DE-EMPHASIS IN FM

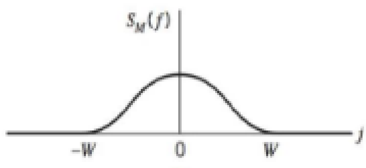
- The power spectral density of output noise is given by

$$S_{N0}(f) = \begin{cases} \frac{f^2}{A_c} N_0 ; |f| \le B_T = W \\ 0; otherwise \end{cases} \quad (1)$$

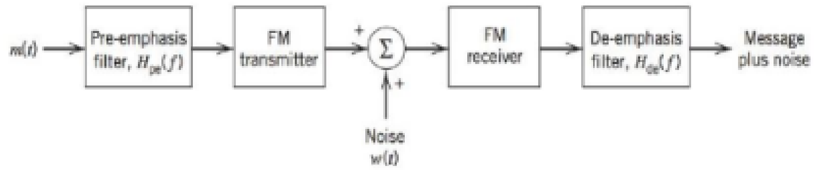


(a)

- The power spectral density of a typical message source; audio and video signals typically have spectra of this form, shown in figure (b),



- Near the cut-off frequency noise becomes more dominant compared to message signal, obviously SNR will go down.
- A more satisfactory approach to the efficient utilization of the allowed frequency band is based on the use of *pre-emphasis* in the transmitter and *de-emphasis* in the receiver



- In this method, we artificially emphasize the high-frequency components of the message signal prior to modulation in the transmitter
- Then, at the discriminator output in the receiver, we perform the inverse operation by de-emphasizing the high-frequency components, so as to restore the original signal-power distribution of the message
- In order to produce an undistorted version of the original message at the receiver output, the pre-emphasis filter in the transmitter and the de-emphasis filter in the receiver must ideally have transfer functions that are the inverse of each other.

$$H_{de}(f) = \frac{1}{H_{pe}(f)} \quad (2)$$

- Simple pre-emphasis filter that emphasizes high frequencies and is commonly used in practice is defined by the transfer function

$$H_{pe}(f) = 1 + \frac{jf}{f_0} \quad (3)$$

Hence,

$$H_{de}(f) = \frac{1}{1+\frac{jf}{f_0}} \quad (4)$$

Average output noise power

$$\text{with de-emphasis} = |H_{de}(f)|^2 \int_{-w}^{W} S_{N0}(f) df$$

$$= \frac{N_0}{A_c^2} \int_{-w}^{W} |H_{de}(f)|^2 f^2 df \quad (5)$$

The improvement in output signal-to-noise ratio produced by the use of pre-emphasis in the transmitter and de-emphasis in the receiver is defined by

$$I = \frac{\text{average output noise power without pre-emphasis and de-emphasis}}{\text{average output noise power with pre-emphasis and de-emphasis}}$$

$$I = \frac{\frac{2N_0 W^3}{3A_c^2}}{\frac{N_0}{A_c^2}\int_{-w}^{W} |H_{de}(f)|^2 f^2 df} = \frac{2W^3}{3\int_{-w}^{W} |H_{de}(f)|^2 f^2 df}$$

$$I = \frac{2W^3}{3\int_{-w}^{W} |H_{de}(f)|^2 f^2 df}$$

$$I = \frac{2W^3}{3\int_{-w}^{W} \left|\frac{1}{1+\frac{jf}{f_0}}\right|^2 f^2 df}$$

$$I = \frac{\left(W/f_0\right)^3}{3\left[\left(W/f_0\right) + \tan^{-1}\left(W/f_0\right)\right]} \quad (7)$$

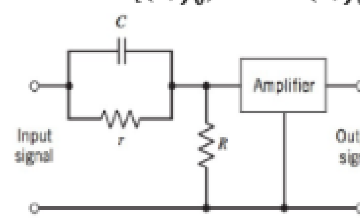


Fig.(c) Pre-emphasis filter

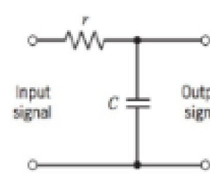


Fig.(d) de-emphasis filter

6 c)

1.3) WHITE NOISE

- In signal processing, white noise is a random signal having equal intensity at different frequencies, giving it a constant power spectral density
- White noise refers to a statistical model for signals and signal sources, rather than to any specific signal. White noise draws its name from white light for the matter of fact that white light contains equal amounts of all frequencies within the visible band of electromagnetic radiation
- We express the power spectral density of white noise, with a sample function denoted by $w(t)$,as

$$S_W(f) = \frac{N_0}{2} \quad (1)$$

- Since the autocorrelation function is the inverse Fourier transform of the power spectral density, it follows that for white noise

$$R_W(\tau) = \frac{N_0}{2}\delta(\tau) \quad (2)$$

- The dimensions of N_o are in watts per Hertz. The parameter N_o is usually referenced to the input stage of the receiver of a communication system. It may be expressed as

$$N_o = kT_e \quad (3)$$

where *k* is Boltzmann's constant andT_e is the *equivalent noise temperature* of the receiver.

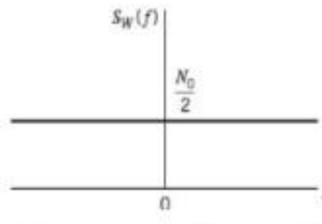


Fig. power spectral density of white noise

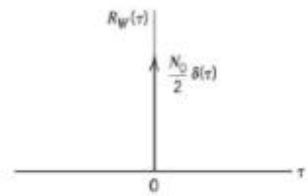


Fig.autocorrelation function

7a)

Sampling Theorem

Sampling theorem states that any continuous time signal can be completely represented in its periodic samples and can be recovered back if the sampling frequency is greater than or equal to twice the highest frequency component of base band (message)signal.

$$f_s \geq 2W$$

$f_s = Samping\ frequency$ and W= bandwidth of message signal

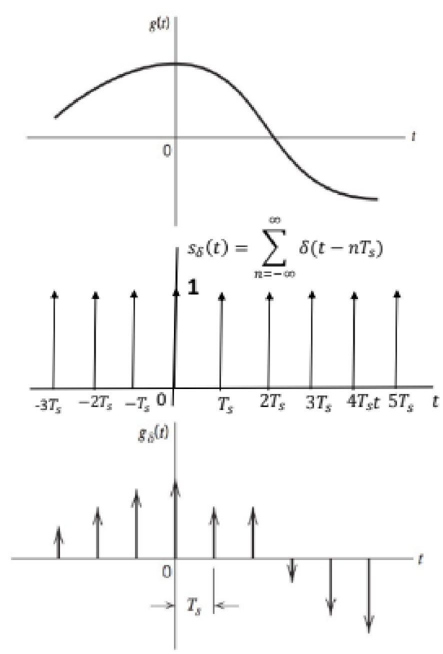
Consider an arbitrary signal $g(t)$ of finite energy, which is specified for all time.
Consider a train of unit impulses separated at a distance T_s and represented by $s_\delta(t)$
Multiplying $g(t)$ with $s_\delta(t)$, yields a ideal sampled signal $g_\delta(t)$

Ideal Sampled signal thus can be written as

$$g_\delta(t) = g(t)s_\delta(t) \quad (1)$$

$$g_\delta(t) = g(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s)$$

$$\boxed{g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s)} \quad (2)$$



From equation (1)
$g_\delta(t) = g(t)s_\delta(t)$

F.T.

$G_\delta(f) = [G(f)] * [S_\delta(f)]$

$G_\delta(f) = [G(f)] * [f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)]$

($\because$ Fourier Transform of periodic impulse train is a periodic impulse train with change in amplitude)

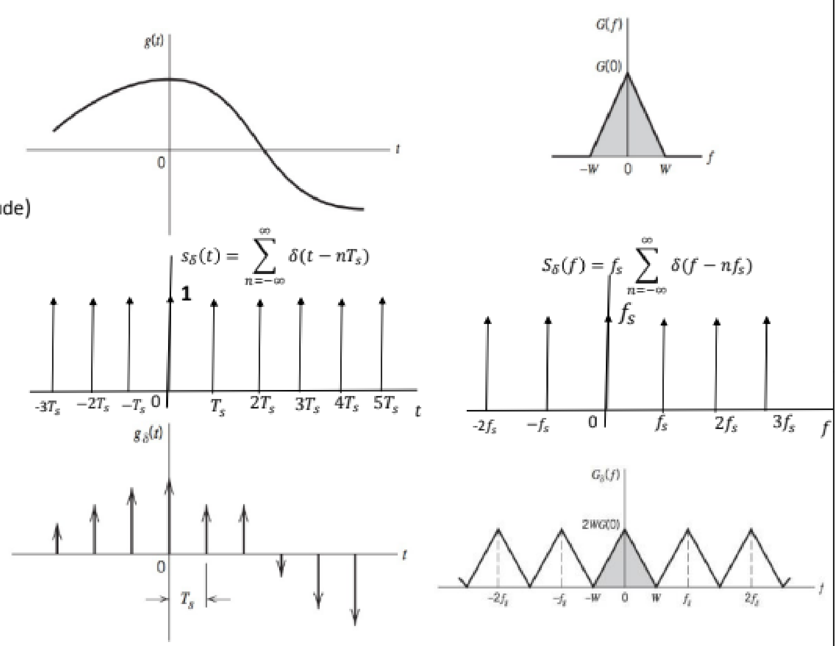
$$G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s) \quad (3)$$

($\because$ by convolution property of impulse $G(f) * \delta(f - nf_s) = G(f - nf_s)$)

We can rewrite equation (3) as

$$G_\delta(f) = \underset{n=0}{f_s G(f)} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} G(f - nf_s) \quad (4)$$

$where\ f_s = 2W$

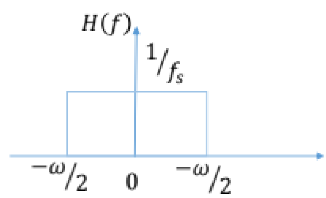


Reconstruction of original signal through samples

Signal $g(t)$ can be reconstructed from ideal sampled signal $g_\delta(t)$ using a reconstruction filter $h(t)$.
The characteristic of reconstruction filter is given as

1) The amplitude of the reconstruction filter must be $^1/_{f_s}$
2) It's bandwidth must be equivalent to ω Hz

$$H(f) = \begin{cases} ^1/_{f_s}; & ^{-\omega}/_2 \le f \le {}^{-\omega}/_2 \\ 0; elsewhere \end{cases} \quad (5)$$



Taking inverse Fourier transform of equation (5)

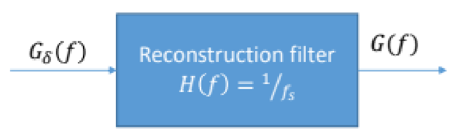
$$h(t) = sinc(2\omega t) \quad (6)$$

Passing equation (4) from a LPF

$$G_\delta(f) = f_s G(f)$$

$$G(f) = \frac{1}{f_s} G_\delta(f) \quad (7)$$

It is like passing $G_\delta(f)$ from reconstruction flter



$$G(f) = H(f)G_\delta(f) \quad (8)$$

Taking inverse Fourier transform

$$g(t) = h(t) * g_\delta(t)$$

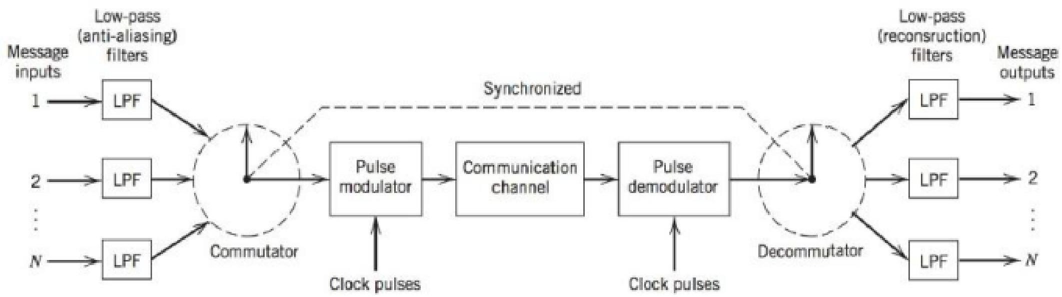
$$g(t) = sinc(2\omega t) * \left[\sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s)\right]$$

$$g(t) = \left[\sum_{n=-\infty}^{\infty} g(nT_s) sinc(2\omega(t - nT_s))\right] \quad (9)$$

Equation (9) is known as interpolation formula

7b) TDM

- An important feature of the sampling process is a **conservation of time.**
- The transmission of the message samples engages the communication channel for only a fraction of the sampling interval on a periodic basis
- So, some of the time interval between adjacent samples is cleared for use by other independent message sources on a time-shared basis
- A *time-division multiplex* (TDM) *system,* enables the joint utilization of a common communication channel by a transmission of multiple independent messages without mutual interference among them



Transmitter

- The concept of TDM is illustrated by the block diagram, each input message signal is first restricted in bandwidth by a low-pass pre-alias filter
- The lowpass filter outputs are then applied to a *commutator,* which is usually implemented using electronic switching circuitry
- The function of the commutator is twofold:

1) to take a narrow sample of each of the N input messages at a rate f_s that is slightly higher than $2W$,
where W is the cutoff frequency of the pre-alias filter, and
(2) to sequentially interleave these N samples inside the sampling interval T_s

- Following the commutation process, the multiplexed signal is applied to a pulse modulator, the purpose of which is to transform the multiplexed signal into a form suitable for transmission over the common channel

Receiver

- At the receiving end of the system, the received signal is applied to a *pulse demodulator,* which performs the reverse operation of the pulse modulator.
- The narrow samples produced at the pulse demodulator output are distributed to the appropriate low-pass reconstruction filters by means of a *decommutator,* which operates in *synchronism* with the commutator in the transmitter.

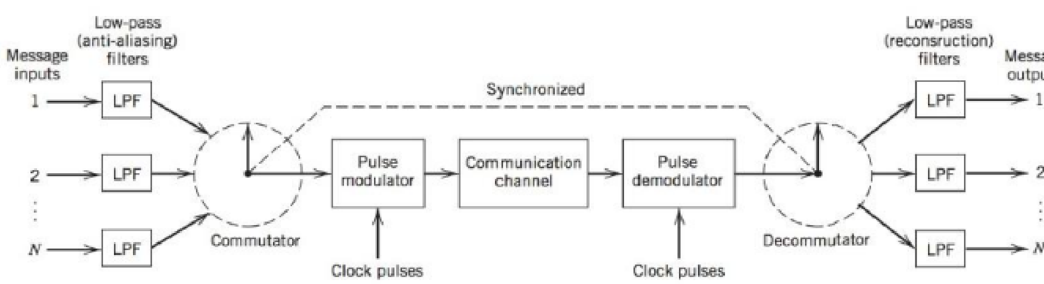


Fig. Block diagram of TDM system

8a)

Pulse modulation is a type of modulation in which train of pulses are used as the carrier wave and one of its parameters such as amplitude are modulated in order to carry information. Pulse modulation is divided into two types as **analog and digital modulation**. The analog pulse modulation techniques are further classified into Pulse Amplitude Modulation (PAM), Pulse Width Modulation (PWM) and Pulse Position Modulation (PPM).

Pulse Amplitude Modulation (PAM) is an analog modulating technique in which the amplitude of the pulse carrier varies proportionally to the instantaneous amplitude of the message signal.

Pulse Width Modulation (PWM) is an analog modulating technique in which the width of the pulse carrier varies proportionally to the instantaneous amplitude of the message signal.

Pulse Position Modulation (PPM) is an analog modulating technique in which the amplitude and width of the pulses are kept constant, and the position of each pulse, concerning the position of a reference pulse, varies according to the instantaneous sampled value of the message signal.

8b)PPM

* PULSE – POSITION MODULATION :

* In pulse-duration modulation (PDM), the samples of the message signal are used to vary the duration of the individual pulses. This form of modulation is also referred to as Pulse-width modulation or pulse-length modulation.
* In PPM, the position of a pulse relative to its unmodulated time of occurance is varied in accordance with the message signal as shown in fig(6)(d) for the case of sinusoidal modulation.

* GENERATION OF PPM WAVES :

The PPM signal which is generated is shown in fig(7)(a). The message signal m(t) is first converted into a PAM signal by means of a sample and Hold circuit, generating a staircase waveform u(t), which is shown in 8(b) for the message signal m(t) shown in fig 8(a).

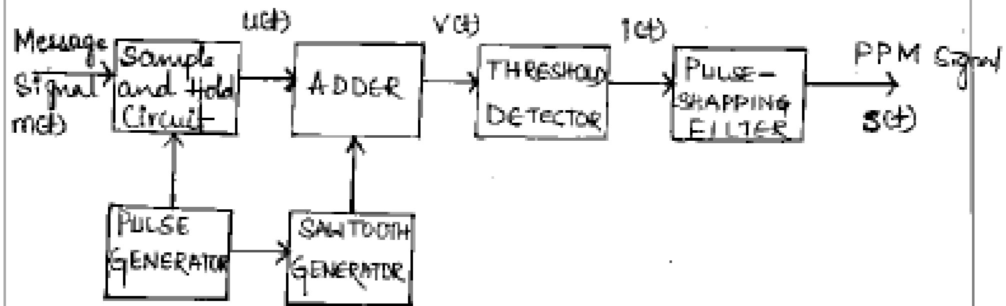


Fig 7(a) : Block diagram of PPM generator.

* Next, the signal u(t) is added to a sawtooth wave, yielding the combined signal v(t). The combined signal v(t) is applied to a threshold detector that produces a very narrow pulse each time v(t) crosses zero in the -ve going direction. The resulting sequence of "impulses" i(t) is shown in fig (8)(e). Finally, the PPM signal s(t) is generated by using this sequence of impulses to excite a filter whose impulse response is defined by the standard pulse g(t).

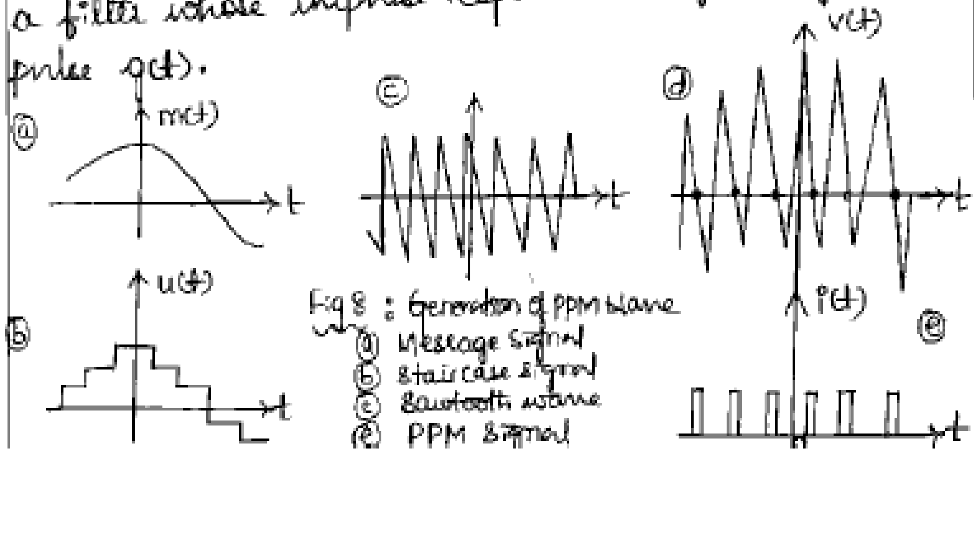


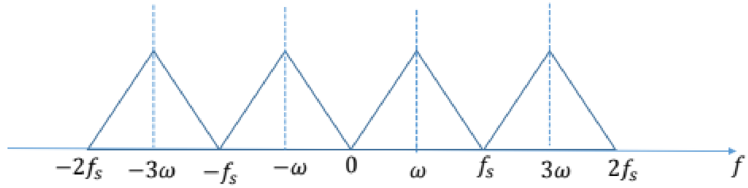
Fig 8 : Generation of PPM wave
(a) Message Signal
(b) Staircase signal
(c) Sawtooth wave
(e) PPM Signal

8c)

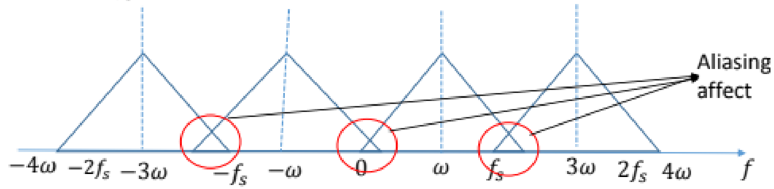
Concept of under-sampling and over-sampling

Rahul Tiwar

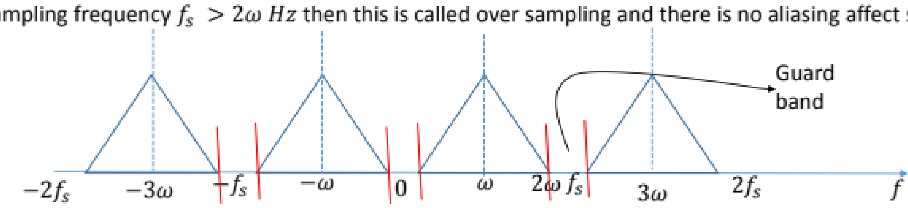
1) When sampling frequency $f_s = 2\omega\ Hz$ then this is called correct sampling and there is no aliasing affect seen at this rate.



2) When sampling frequency $f_s < 2\omega\ Hz$ then this is called under sampling and there will be aliasing affect seen at this rate.



3) When sampling frequency $f_s > 2\omega\ Hz$ then this is called over sampling and there is no aliasing affect seen at this rate.



Sampling Theorem

Sampling theorem states that any continuous time signal can be completely represented in its periodic samples and can be recovered back if the sampling frequency is greater than or equal to twice the highest frequency component of base band (message)signal.

$$f_s \geq 2W$$

$f_s = Samping\ frequency$ and W= bandwidth of message signal

9a)

2.1) Uniform Quantizer

- In a uniform quantizer, the representation levels are uniformly spaced; otherwise, the quantizer is non-uniform.
- The quantizer characteristic can also be of midtread or midrise type. Midtread and midrise are two types types of uniform quantizers
- The input-output characteristic of a uniform quantizer of the midtread type, which is so called because the origin lies in the middle of a tread of the staircase like graph Fig.A
- The corresponding input-output characteristic of a uniform quantizer of the midrise type, in which the origin lies in the middle of a rising part of the staircase like graph Fig.B

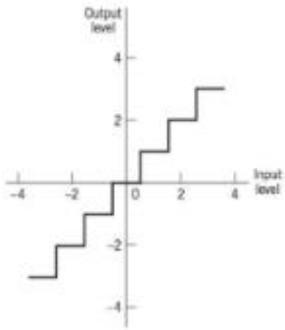


Fig.A Midtread type uniform quantizer

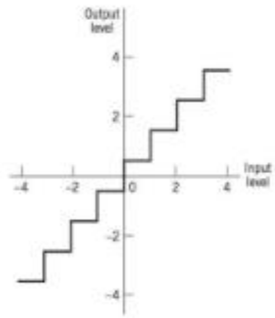


Fig.B Midrise type uniform quantizer

3).QUANTIZATION NOISE

The use of quantization introduces an error defined as the difference between the input signal m and the output signal v. This error is called quantization noise.

Typical variation of the quantization noise as a function of time, assuming the use of a uniform quantizer of the midtread type

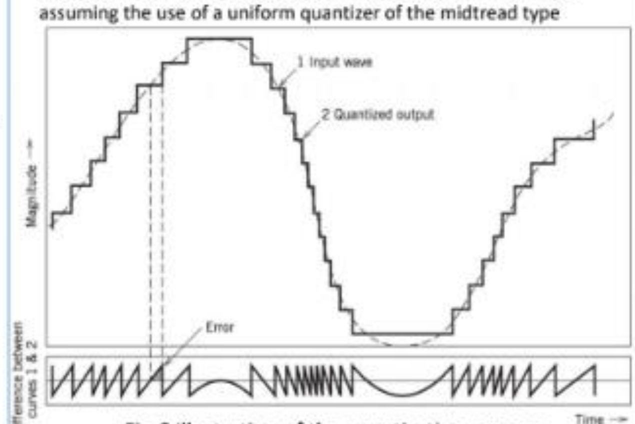


Fig.**C** Illustration of the quantization process

Let the quantization error be denoted by the random variable Q of sample value q.

$$\boxed{q = m - v} \quad (1)$$

or, correspondingly,

$$Q = M - V \quad (2)$$

m be the sample value of a zero-mean random variable M.

v be the sample value of a zero-mean discrete random variable V

Quantization error be denoted by the random variable Q of sample value q

Consider then an input m of continuous amplitude is in the range $(-m_{max}, m_{max})$

The step-size of the quantizer is given by

$$\Delta = \frac{2m_{max}}{L} \quad (3)$$

where L is the total number of representation levels

Let R denote the number of bits per sample used in the construction of the binary code

then, $L = 2^R$ (4)

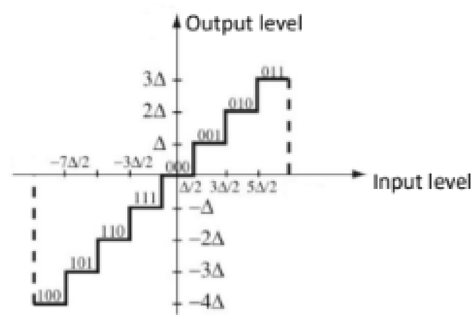
equivalently, $R = \log_2 L$ (5)

For a uniform quantizer, the quantization error Q will have its sample values bounded by $-\Delta/2 \leq q \leq \Delta/2$

Assuming that the quantization error Q is a uniformly distributed random variable,we may thus express the probability density function of the quantization error Q as follows:

$$f_Q(q) = \begin{cases} 1/_\Delta; & -\Delta/2 \leq q \leq \Delta/2 \\ 0; & elsewhere \end{cases} \quad (5)$$

with the mean of the quantization error being zero, its variance σ_Q^2 is the same as the mean-square value



$$\sigma_Q^2 = \int_{-\Delta/2}^{-\Delta/2} q^2 f_Q(q)dq = E[Q^2] \quad (6)$$

$$\sigma_Q^2 = \int_{-\Delta/2}^{-\Delta/2} q^2\, 1/_\Delta\, dq = E[Q^2] \quad \boxed{\sigma_Q^2 = \Delta^2/_{12}} \quad (7)$$

$$\sigma_Q^2 = \frac{\left(\frac{2m_{max}}{L}\right)^2}{12} \Rightarrow \sigma_Q^2 = \frac{\left(\frac{2m_{max}}{2^R}\right)^2}{12} \boxed{\Rightarrow \sigma_Q^2 = \frac{1}{3} m_{max}^2 2^{-2R}} \quad (8)$$

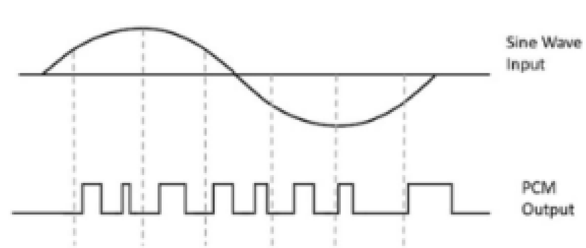
Let P denote the average power of the message signal m(t).We may then express the output signal-to-noise ratio of a uniform quantizer as

$$(SNR)_o = \frac{P}{\sigma_Q^2} = \left(\frac{3P}{m_{max}^2}\right) 2^{2R} \quad (9)$$

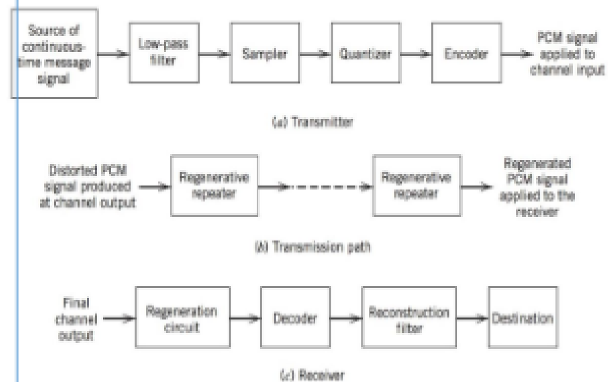
9b)

4).PULSE-CODE MODULATION

- A signal is pulse code modulated to convert its analog information into a binary sequence, i.e., **1s** and **0s**. The output of a PCM will resemble a binary sequence. The following figure shows an example of PCM output with respect to instantaneous values of a given sine wave.



- PCM is the most basic form of digital pulse modulation.In pulse-code modulation (PCM) a message signal is represented by a sequence of coded pulses, which isaccomplished by representing the signal in discrete form in both time and amplitude.
- The basic operations performed in the transmitter of a PCM system are sampling, quantizing, and encoding,
- The basic operations in the receiver are regeneration of impaired signals, decoding, and reconstruction of the train of quantized samples
- The quantizing and encoding operations are usually performed in the same circuit, which is called an analog-to-digital converter.

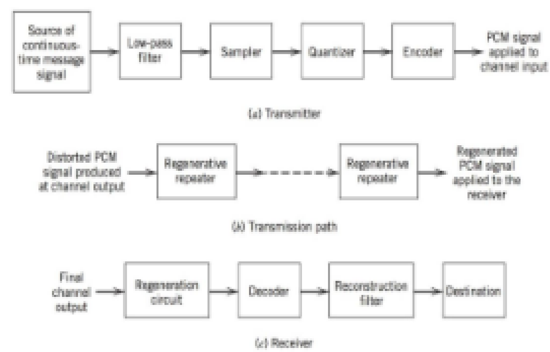


4.1)SAMPLING

The incoming message signal is sampled with a train of narrow rectangular pulses, to ensure perfect reconstruction of the message signal at the receiver, the sampling rate must be greater than twice the highest frequency component W of the message signal in accordance with the sampling theorem.

$$f_s \geq 2W$$

$f_s = Samping\ frequency$ and W= bandwidth of message signal



4.2) QUANTIZATION

The sampled version of the message signal is then quantized, thereby providing a new representation of the signal that is discrete in both time and amplitude.

Quantizers can be of a uniform or non-uniform type.

The uniform quantizer can be characteristise as midtread or midrise type.

The use of a non-uniform quantizer is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer.

The non-uniform quantizer requires to apply **companding**, it is a joint term use for the combination of a compressor and an expander.

4.3) ENCODING

After the processes of sampling and quantization we have limited discrete samples in time and amplitude, but not in the form best suited to transmission over a line or radio path.

Further encoding process is required to covert these discrete set of sample values to a more appropriate form of signal suitable for transmission over the channnel

Encoding process ensures transmitted signal more robust to noise, interference, and other channel degradations

Representing these discrete set of values as a particular arrangement of discrete events is called a code. One of the discrete events in a code is called a code element or symbol. For example, the presence or absence of a pulse is a symbol

Suppose that, in a binary code, each code word consists of R bits:Then, using such a code, we may represent a total of 2^R distinct numbers.

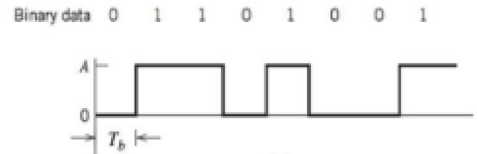
4.3.1) LINE CODES

It is in a line code that a binary stream of data takes on an electrical representation. Any one of several line codes can be used for the electrical representation of a binary data stream

4.3.1) (i) Unipolar Nonreturn-to-Zero (NRZ) Signaling.

In this line code, symbol 1 is represented by transmitting a pulse of amplitude A for the duration of the symbol, and symbol 0 is represented by switching off the pulse, as in fig. A

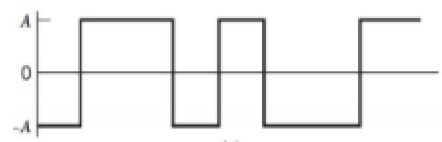
This line code is also referred to as on-off signaling. A disadvantage of on-off signaling is the waste of power due to the transmitted DC level.



(a)

4.3.1) (ii) Polar Nonreturn-to-Zero (NRZ) Signaling.

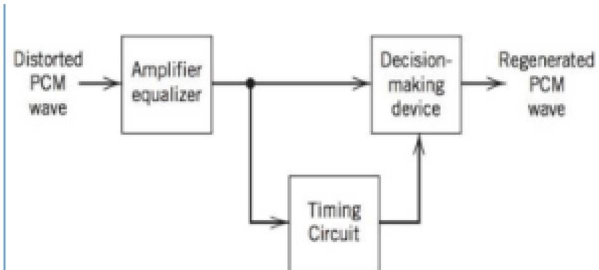
In this second line code, symbols 1 and 0 are represented by transmitting pulses of amplitudes +A and -A, respectively, as illustrated in Figure B This line code is relatively easy to generate and is more power-efficient than its unipolar counterpart.



(b)

4.4) REGENERATION

- The most important feature of any digital system lies in the ability to control the effects of distortion and noise produced by transmitting a digital signal through a channel.
- This capability is accomplished by reconstructing the signal by means of a chain of regenerative repeaters located at sufficiently close spacing along the transmission route.
- As illustrated in the block diagram, three basic functions are performed by a regenerative repeater: equalization, timing , and decision making.
- The equalizer shapes the received pulses so as to compensate for the effects of amplitude and phase distortions produced by the transmission characteristics of the channel.
- The timing circuitry provides a periodic pulse train, derived from the received pulses, for sampling the equalized pulses at the instants of time where the signal-to-noise ratio is a maximum.
- The sample so extracted is compared to a predetermined threshold in the decision-making device. In each bit interval a decision is then made whether the received symbol is a 1 or a 0 on the basis of whether the threshold is exceeded or not. If the threshold is exceeded, a clean new pulse representing symbol 1 is transmitted to the next repeater. Otherwise, another clean new pulse representing symbol 0 is transmitted.
- In this way, the accumulation of distortion and noise in a repeater span is completely removed



In practice, however, the regenerated signal departs from the original signal for two main reasons:

1. The unavoidable presence of channel noise and interference causes the repeater to make wrong decisions occasionally, thereby introducing bit errors into the regenerated signal.
2. If the spacing between received pulses deviates from its assigned value, a jitter is introduced into the regenerated pulse position, thereby causing distortion.

4.5) DECODING

The first operation in the receiver is to regenerate (i.e., reshape and clean up) the received pulses one last time. These clean pulses are then regrouped into code words and decoded (i.e., mapped back) into a quantized PAM signal. The decoding process involves generating a pulse the amplitude of which is the linear sum of all the pulses in the code word, with each pulse being weighted by its place value $(2^0, 2^1 2^2, 2^3, \dots, 2^{R-1})$ in the code, where R is the number of bits per sample.

4.6) FILTERING

The final operation in the receiver is to recover the message signal wave by passing the decoder output through a low-pass reconstruction filter whose cutoff frequency is equal to the message bandwidth W. Assuming that the transmission path is error free, the recovered signal includes no noise with the exception of the initial distortion introduced by the quantization process.

4.8) MULTIPLEXING

- In applications using PCM, it is natural to multiplex different message sources by time division, whereby each source keeps its individuality throughout the journey from the transmitter to the receiver.

- This individuality accounts for the comparative ease with which message sources may be dropped or reinserted in a time-division multiplex system.

- As the number of independent message sources is increased, the time interval that may be allotted to each source has to be reduced, since all of them must be accommodated into a time equal to the reciprocal of the sampling rate.

- This in turn means that the allowable duration of a code word representing a single sample is reduced. However, pulses tend to become more difficult to generate and to transmit as their duratio is reduced.

- Furthermore, if the pulses become too short, impairments in the transmission medium begin to interfere with the proper operatio of the system.

✓ Accordingly, in practice, it is necessary to restrict the number of independent message sources that can be included within a time-division group.

10a)

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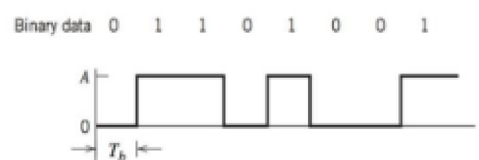
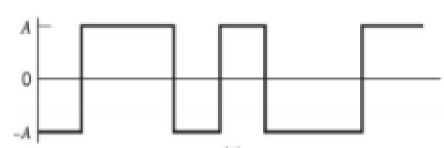
This line code is also referred to as on-off signaling. A disadvantage of on-off signaling is the waste of power due to the transmitted DC level.



(a)

4.3.1) (ii) Polar Nonreturn-to-Zero (NRZ) Signaling.

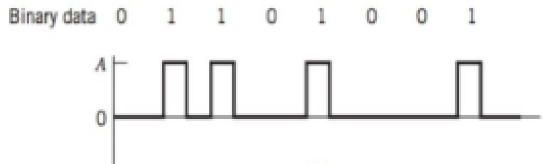
In this second line code, symbols 1 and 0 are represented by transmitting pulses of amplitudes +A and -A, respectively, as illustrated in Figure B This line code is relatively easy to generate and is more power-efficient than its unipolar counterpart.



(b)

4.3.1) (iii) Unipolar Return-to-Zero (RZ) Signaling.

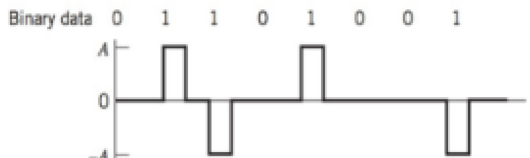
In this other line code, symbol 1 is represented by a rectangular pulse of amplitude A and half-symbol width, and symbol 0 is represented by transmitting no pulse, as illustrated in Fig.(c)



(c)

4.3.1) (iv) Bipolar Retum-to-Zero (BRZ) Signaling.

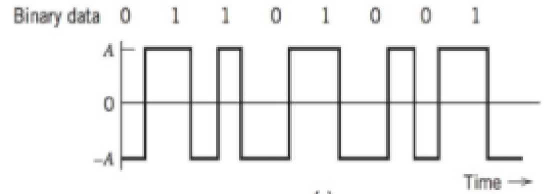
This line code uses three amplitude levels as indicated in Fig(d) Specifically, positive and negative pulses of equal amplitude (i.e., +A and -A) are used alternately for symbol 1, with each pulse having a half-symbol width. No pulse is always used for symbol 0. Advantage of BRZ is that it helps in error detection



(d)

4.3.1) (v) Split-Phase (Manchester Code).

In this method of signaling, illustrated in Fig€, symbol 1 is represented by a positive pulse of amplitude A followed by a negative pulse of amplitude -A, with both pulses being a half-symbol wide. For symbol 0, the polarities of these two pulses are reversed

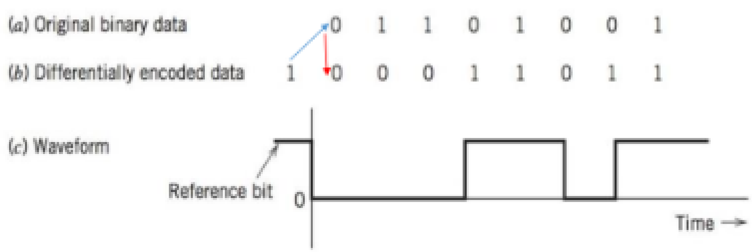


(e)

This line code is also called alternate mark inversion (AMI) signaling.

4.3.1) (vi) DIFFERENTIAL ENCODING

This method is used to encode information in terms of signal transitions. In particular, a transition is used to designate symbol 0 in the incoming binary data stream, while no transition is used to designate symbol 1, as illustrated in Fig

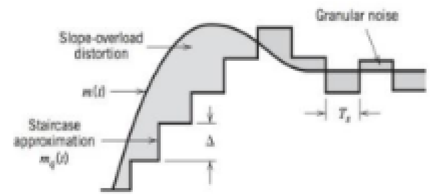


The original binary information is recovered simply by comparing the polarity of adjacent binary symbols to establish whether or not a transition has occurred.

Note that differential encoding requires the use of a reference bit before initiating the encoding process.

10 b)

Delta modulation is subject to two types of quantization error:
(1) slope overload distortion and (2) granular noise



(1) slope overload distortion

- For fair approximation of analog input into digital output through staircase sequence via 1 bit quantization, the step size must be compared with maximum slope in the messahe signal
- If we consider the maximum slope of the original input waveform $m(t)$,it is clear that in order for the sequence of samples $\{m_q(nT_s)\}$ to increase as fast as the input sequence of samples $\{m(nT_s)\}$ in a region of maximum slope of $m(t)$, we require that the condition

$$\frac{\Delta}{T_s} \geq \max \left|\frac{dm(t)}{dt}\right|$$

be satisfied.

- If the above condition is not followed, and

$$\frac{\Delta}{T_s} \ll \max \left|\frac{dm(t)}{dt}\right|$$

we find that the step-size Δ is too small for the staircase approximation m_q(t) to follow a steep segment of the input waveform $m(t)$, with the result that m_q(t) falls behind $m(t)$.

- This condition is called **slope overload**, and the resulting quantization error is called **slope-overload distortion (noise).**

(2) Granular noise

In contrast to slope-overload distortion, granular noise occurs when the step-size Δ is too large relative to the local slope characteristics of the input waveform m(t),

$$\frac{\Delta}{T_s} \gg \max \left|\frac{dm(t)}{dt}\right|$$

thereby causing the staircase approximation m_q(t) to hunt around a relatively flat segment of the input waveform

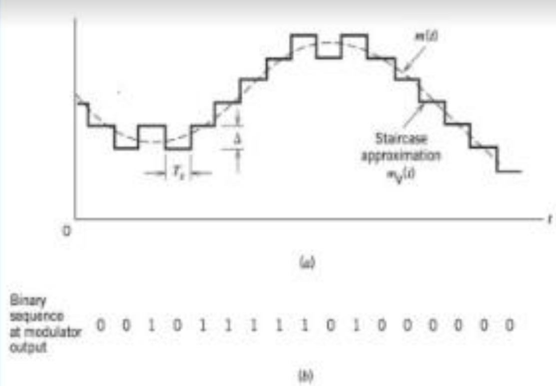
10 c)

6) DELTA MODULATION

- The problem with PCM is congestion, as the number of quantization level increases, number of bits/ sample increases, hence it creates unnecessary load

- In Delta modulation, an analog input is approximated by a staircase function that moves up and down by one Quantization level (Δ) at each signal interval T_s, so it can be referred as **1 bit quantizer**

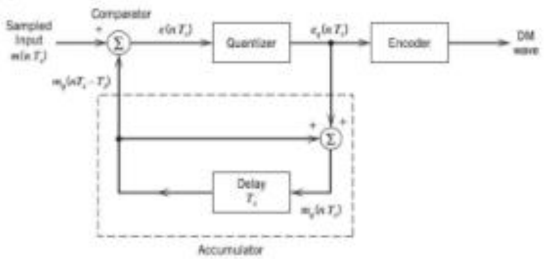
- In delta modulation (DM), an incoming message signal is oversampled (i.e., at a rate much higher than the Nyquist rate) to purposely increase the correlation between adjacent samples of the signal



- The difference between the input $m(t)$, and the approximation $m_q(t)$, is quantized into only two levels, namely, $\pm \Delta$, corresponding to positive and negative differences, respectively.
- Thus, if the approximation falls below the signal at any sampling epoch, it is increased by Δ. If, on the other hand, the approximation lies above the signal, it is diminished by Δ.
- The principal virtue of delta modulation is its simplicity. It may be generated by applying the sampled version of the incoming message signal to a digital encoder that involves a comparator, quantizer, and accumulator interconnected

Transmitter

Denoting the input signal as $m(t)$, and its staircase approximation as $m_q(t)$, the basic principle of delta modulation



$$e(nT_s) = m(nT_s) - m_q(nT_s - T_s)$$

$$e_q(nT_s) = \Delta\,\mathrm{sgn}[e(nT_s)]$$

$$m_q(nT_s) = m_q(nT_s - T_s) + e_q(nT_s)$$

where T_s is the sampling period;
$e(nT_s)$ is an error signal representing the difference between the present sample value $m(nT_s - T_s)$ of the input signal and the latest approximation to it, that is, $m_q(nT_s)$
$e_q(nT_s)$ is the quantized version of $e(nT_s)$.

The quantizer output $e_q(nT_s)$ is finally coded to produce the desired DM signal.

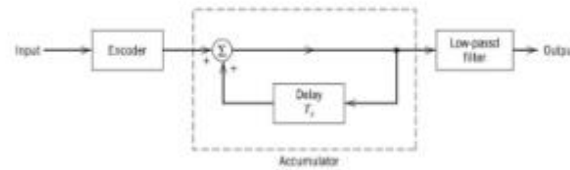
The comparator computes the difference between its two inputs. The quantizer consists of a hard limiter with an input-output relation that is a scaled version of the signum function. The quantizer output is then applied to an accumulator, producing the result

$$m_q(nT_s) = \Delta \sum_{i=1}^{n} sgn(e(iT_s)) = \sum_{i=1}^{n} sgn(e_q(iT_s))$$

Thus, at the sampling instant nT_s,the accumulator increments the approximation by a step Δ in a positive or negative direction, depending on the algebraic sign of the error signal $e(nT_s)$

Receiver

In the receiver shown in figure the staircase approximation mq(t) is reconstructed by passing the sequence of positive and negative pulses, produced at the decoder output, through an accumulator in a manner similar to that used in the transmitter.



The out-of-band quantization noise in the high-frequency staircase waveform $m_q(t)$is rejected by passing it through a low-pass filter, with a bandwidth equal to the original message bandwidth.