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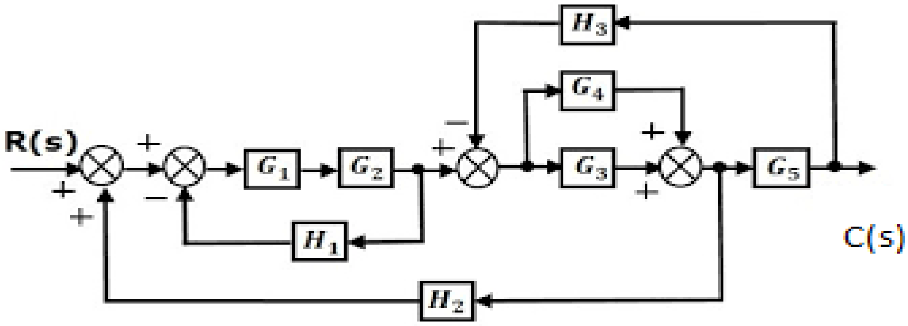
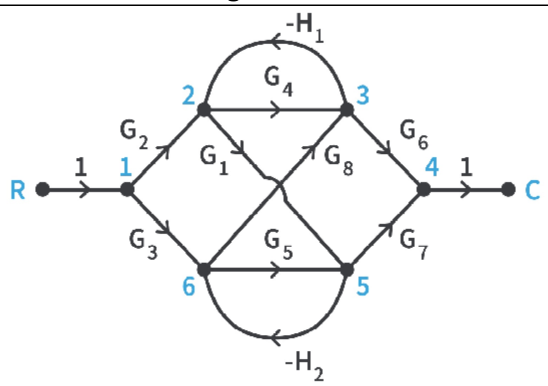
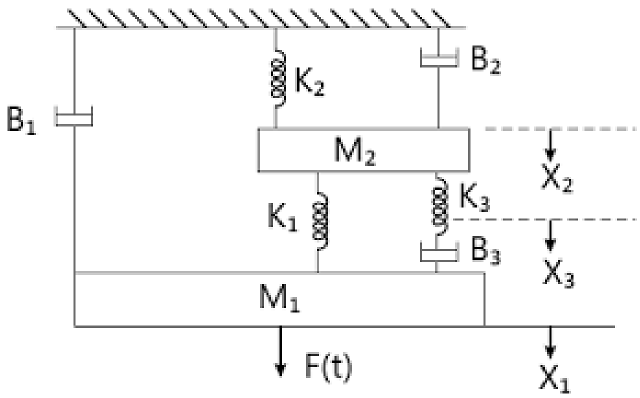
Internal Assessment Test 4 –Feb. 2022

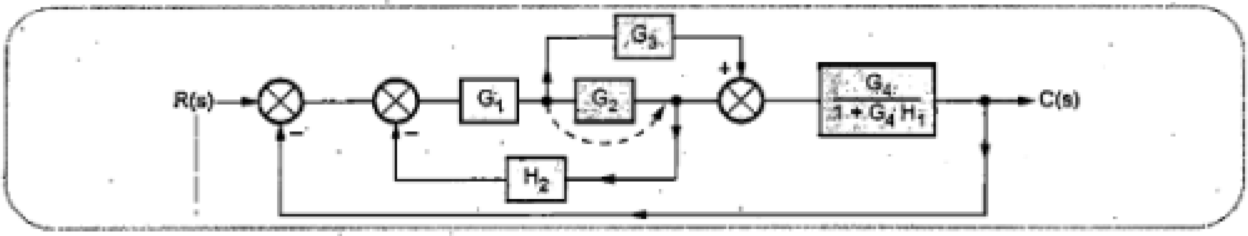
Sub:	Control Engineering					Sub Code:	18ME71/17ME73/ 15ME73	Branch:	ME
Date:	01.02.22	Duration:	90 min's	Max Marks:	50	Sem/Sec:	VII/A&B		OBE

	Answer All the Questions	MARKS	CO	RBT
1	 Reduce the block diagram and determine transfer function of the given system.	[10]	CO2	L3
2	 Determine the transfer function of the given system using Mason's Gain Formula.	[10]	CO2	L3
3	 Draw equivalent mechanical system of the given system. Hence write the set of equilibrium equations for it and obtain electrical analogous circuits using FV and FI analogy.	[10]	CO1	L3

4.	Obtain the expression for peak time and rise time for a second order control system in terms of damping factor and natural frequency.	[10]	CO3	L3
5.	A unity feedback system is characterised by open loop transfer function $$G(s) = \frac{16}{s(s+2)}$$ If the system is subjected to unit step input, determine: i) Undamped natural frequency ii) Damping ratio iii) Peak overshoot iv) Peak time v) Settling time	[10]	CO4	L3

Faculty Signature CCI Signature HOD Signature

1)



Shift take off point of G_3 after the block G_2.

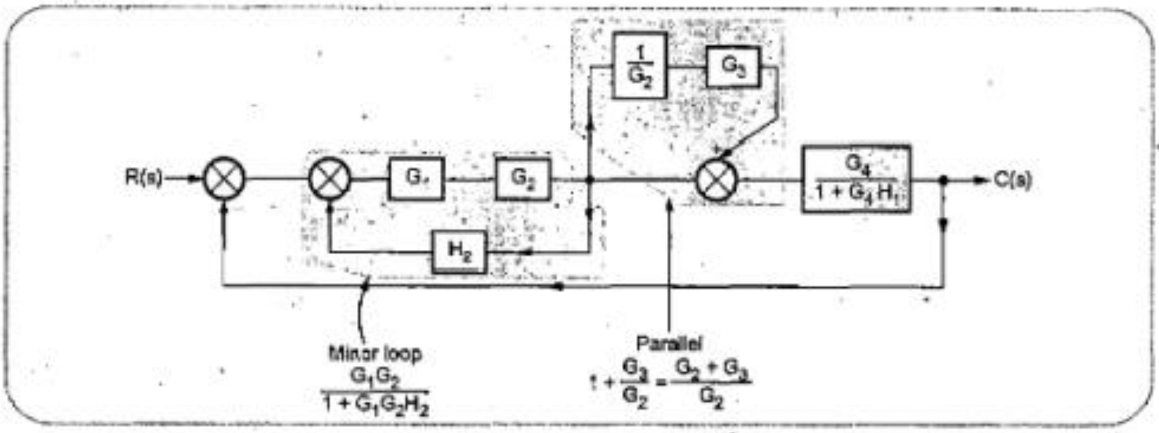


Fig. 5.3.37 (b)

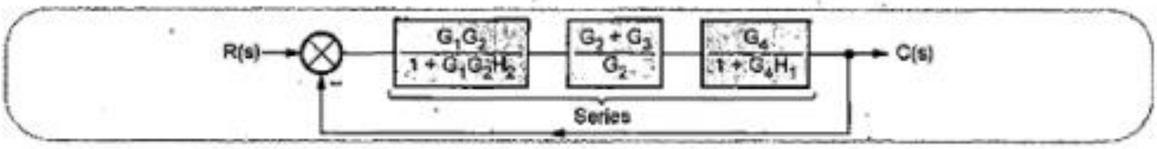


Fig. 5.3.37 (c)

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_4(G_2+G_3)}{(1+G_1G_2H_2)(1+G_4H_1)}}{1+\dfrac{G_1G_4(G_2+G_3)}{(1+G_1G_2H_2)(1+G_4H_1)}}$$

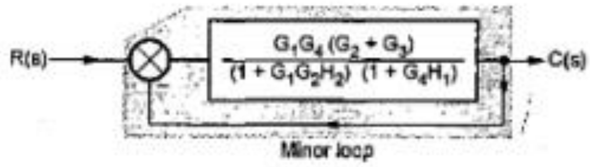


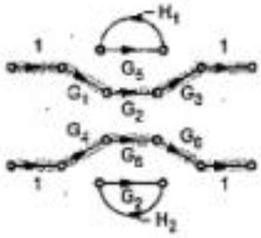
Fig. 5.3.37 (d)

$$\therefore \quad \boxed{\frac{C(s)}{R(s)} = \frac{G_1G_4(G_2+G_3)}{1+G_1G_2H_2+G_4H_1+G_1G_2G_4H_1H_2+G_1G_2G_4+G_1G_3G_4}}$$

2)

$-H_2$

Sol : The number of forward paths are K = 6.



The forward path gains are,

$$T_1 = G_1G_2G_3, \qquad T_2 = G_4G_5G_6$$

$$T_3 = G_1G_7G_6, \qquad T_4 = G_4G_8G_3$$

$$T_5 = G_4G_8(-H_2)G_7G_6, \; T_6 = G_1G_7(-H_1)G_8G_3$$

The feedback loop gains are,

$$L_1 = -G_5H_1, \; L_2 = -G_2H_2, \; L_3 = +G_7H_1G_8H_2$$

The two nontouching loops are $L_1 L_2$.

$$\therefore \quad L_1L_2 = +G_2G_5H_1H_2$$

$$\therefore \quad \Delta = 1-[L_1+L_2+L_3]+[L_1L_2] = 1+G_5H_1+G_2H_2-G_7G_8H_1H_2+G_2G_5H_1H_2$$

For T_1, L_1 is nontouching.

$$\therefore \quad \Delta_1 = 1-L_1 = 1+G_5H_1$$

For T_2, L_2 is nontouching.

$$\therefore \quad \Delta_2 = 1-L_2 = 1+G_2H_2$$

For T_3 to T_6 all loops are touching to all forward paths.

$$\therefore \quad \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$

$$\therefore \quad \text{Gain} = \frac{\Sigma T_K \Delta_K}{\Delta} = \frac{T_1\Delta_1+T_2\Delta_2+T_3\Delta_3+T_4\Delta_4+T_5\Delta_5+T_6\Delta_6}{\Delta}$$

$$\therefore \quad \boxed{\text{Gain} = \frac{G_1G_2G_3\,(1+G_5H_1)+G_4G_5G_6\,(1+G_2H_2)+ G_1G_7G_6+G_4G_8G_3-G_4G_8G_7G_6H_2-G_1G_3G_7G_8H_1}{1+G_5H_1+G_2H_2-G_7G_8H_1H_2+G_2G_5H_1H_2}}$$

... **Ans.**

3)

$$f(t) = M_1 \frac{d^2x_1}{dt^2} + K_1(x_1 - x_2) + B_1 \frac{d(x_1 - x_2)}{dt}$$

Fig. 4.8.9

$$0 = K_1(x_2 - x_1) + B_1 \frac{d(x_2 - x_1)}{dt} + M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2$$

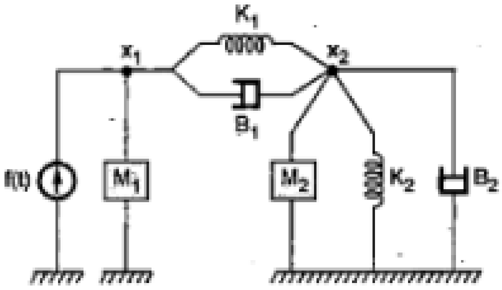


Fig. 4.8.9 (a)

i) F-V analogy : Use $F \to V$, $M \to L$, $B \to R$, $K \to \frac{1}{C}$, $x \to q$,

$\frac{dx}{dt} \to i$, $x \to \int i\,dt$, $\frac{d^2x}{dt^2} \to \frac{di}{dt}$

$$v(t) = L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int (i_1 - i_2)\,dt + R_1(i_1 - i_2)$$

$$0 = R_1(i_2 - i_1) + \frac{1}{C_1}\int (i_2 - i_1)\,dt + L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt$$

Based on loop basis, F-V network is shown in the Fig. 4.8.9 (b).

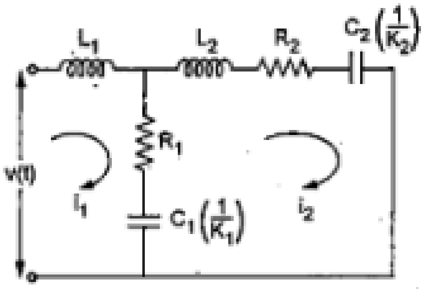


Fig. 4.8.9 (b) F-V analogy

ii) F-I analogy : Use $F \to I$, $M \to C$, $B \to \frac{1}{R}$, $K \to \frac{1}{L}$, $x \to \phi$,

$\frac{dx}{dt} \to v(t)$, $x \to \int v(t)\,dt$, $\frac{d^2x}{dt^2} \to \frac{dv(t)}{dt}$

$$i(t) = C_1 \frac{dv_1(t)}{dt} + \frac{1}{L_1}\int (v_1 - v_2)\,dt + \frac{1}{R_1}(v_1 - v_2)$$

$$0 = \frac{1}{L_1}\int (v_2 - v_1)\,dt + \frac{1}{R_1}(v_2 - v_1) + C_2 \frac{dv_2}{dt} + \frac{1}{R_2} v_2 + \frac{1}{L_2}\int v_2\,dt$$

4)

Derivation of Peak Time T_p

Transient response of second order underdamped system is given by,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \quad \text{where} \quad \theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

As at $t = T_p$, c(t) will achieve its maxima. According to Maxima theorem,

$$\left.\frac{dc(t)}{dt}\right|_{t=T_p} = 0$$

So differentiating c(t) w.r.t. 't' we can write,

$$\text{i.e.} -\frac{e^{-\xi\omega_n t}(-\xi\omega_n)\sin(\omega_d t+\theta)}{\sqrt{1-\xi^2}} - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\omega_d \cos(\omega_d t+\theta) = 0$$

Substituting $\omega_d = \omega_n \sqrt{1-\xi^2}$

$$\frac{\xi\omega_n e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t+\theta) - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\omega_n \sqrt{1-\xi^2}\cos(\omega_d t+\theta) = 0$$

$$\xi\sin(\omega_d t+\theta) - \sqrt{1-\xi^2}\cos(\omega_d t+\theta) = 0 \quad \text{i.e.} \quad \tan(\omega_d t+\theta) = \frac{\sqrt{1-\xi^2}}{\xi}$$

Now $\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$ i.e. $\frac{\sqrt{1-\xi^2}}{\xi} = \tan\theta$

$$\tan(\omega_d t+\theta) = \tan\theta$$

From trigonometric formula,

$$\tan(n\pi+\theta) = \tan\theta$$

$$\omega_d t = n\pi \qquad \text{where } n = 1, 2, 3$$

But T_p, time required for first peak overshoot. $\therefore$ n = 1

$$\omega_d T_p = \pi$$

$$\boxed{T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \text{ sec}}$$

$$\{c(t)|_{t=T_r}\} = 1 \text{ for unit step input}$$

$$\therefore \quad 1 = 1 - \frac{e^{-\xi\omega_n T_r}}{\sqrt{1-\xi^2}}\sin(\omega_d T_r + \theta)$$

$$\therefore \quad -\frac{e^{-\xi\omega_n T_r}}{\sqrt{1-\xi^2}}\sin(\omega_d T_r + \theta) = 0$$

Equation will get satisfied only if,

$$\sin(\omega_d T_r + \theta) = 0$$

Trigonometrically this is true only if,

$$\omega_d T_r + \theta = n\pi \quad \text{where } n = 1, 2 \ldots$$

As we are interested in first attempt use n = 1

$$\therefore \quad \omega_d T_r + \theta = \pi$$

$$\therefore \quad \boxed{T_r = \frac{\pi-\theta}{\omega_d} \text{ sec}}$$

5)

Sol. : From given G(s), the closed loop T.F. is,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{10}{s^2+2s+16}$$

Comparing denominator with $s^2 + 2\xi\omega_n s + \omega_n^2$,

$$\omega_n^2 = 16,\ \omega_n = 4,\ 2\xi\omega_n = 2,\ \xi = 0.25$$

i) $\omega_n = 4$ rad/sec

ii) ξ = Damping ratio = 0.25

iii) $\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = \mathbf{44.43\ \%}$

iv) $\omega_d = \omega_n\sqrt{1-\xi^2} = 3.873$ rad/sec

$\therefore$ $T_p = \frac{\pi}{\omega_d} = \mathbf{0.811\ sec}$

v) $T_s = \frac{4}{\xi\omega_n} = \mathbf{4\ sec}$