



**Third Semester B.E. Degree Examination, July/August 2022**  
**Advanced Mathematics – I**

Time: 3 hrs.

Max. Marks: 100

**Note: Answer any FIVE full questions.**

- 1 a. Find the modulus and amplitude of  $\frac{1}{1 - \cos \alpha + i \sin \alpha}$ . (07 Marks)
- b. Express  $\frac{(3+i)(1-3i)}{2+i}$  in the form of  $x + iy$ . (06 Marks)
- c. Find the cube roots of  $1-i$ . (07 Marks)
  
- 2 a. Find the  $n^{\text{th}}$  derivative of  $\frac{x}{(x+1)(2x+1)}$ . (07 Marks)
- b. Find the  $n^{\text{th}}$  derivative of  $\cos x \cos 3x \cos 5x$ . (06 Marks)
- c. If  $y = a \cos(\log x) + b \sin(\log x)$  show that  $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$ . (07 Marks)
  
- 3 a. Show that the following pairs of curves intersect orthogonally  $r_1 = a(1 + \sin \theta)$  and  $r_2 = a(1 - \sin \theta)$ . (07 Marks)
- b. With usual notations prove that  $\tan \phi = r \frac{d\theta}{dr}$ . (06 Marks)
- c. Expand  $\sqrt{1 + \sin 2x}$  by Maclaurin's series upto the term containing  $x^4$ . (07 Marks)
  
- 4 a. State and prove Euler's theorem on homogeneous functions. (07 Marks)
- b. If  $u = f(x - y, y - z, z - x)$  show that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ . (06 Marks)
- c. If  $u = x + 3y^2 - z^3, v = 4x^2yz, w = 2z^2 - xy$  find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$  at  $(1, -1, 0)$ . (07 Marks)
  
- 5 a. Obtain a reduction formula for  $I_n = \int_0^{\pi/2} \cos^n x \, dx$ ,  $n$  being a positive integer. (07 Marks)
- b. Evaluate  $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) \, dy \, dx$ . (06 Marks)
- c. Evaluate  $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) \, dy \, dx \, dz$ . (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 6 a. Prove that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ . (07 Marks)
- b. Express  $I = \int_0^1 x^{3/2}(1-x)^{1/2} dx$  in terms of beta function. (06 Marks)
- c. Show that  $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_0^{\pi/2} \sqrt{\sin \theta} d\theta = \pi$ . (07 Marks)
- 7 a. Solve:  $(2x + y + 1) dx + (x + 2y + 1) dy = 0$ . (07 Marks)
- b. Solve  $\frac{dy}{dx} + y \cot x = \cos x$ . (06 Marks)
- c. Solve:  $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$ . (07 Marks)
- 8 a. Solve:  $6 \frac{d^2 y}{dx^2} + 17 \frac{dy}{dx} + 12y = e^{-x}$ . (07 Marks)
- b. Solve:  $4 \frac{d^4 y}{dx^4} - 8 \frac{d^3 y}{dx^3} - 7 \frac{d^2 y}{dx^2} + 11 \frac{dy}{dx} + 6y = 0$ . (06 Marks)
- c. Solve:  $\frac{d^3 y}{dx^3} - y = 3 \cos 2x$ . (07 Marks)

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