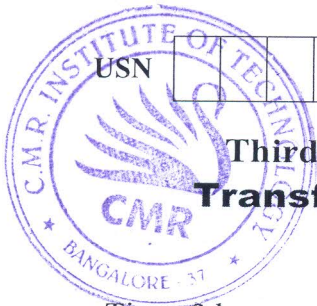


CBCS SCHEME



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18MAT31

Third Semester B.E. Degree Examination, July/August 2022 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the Laplace transform,
 - (i) $e^{-2t}(2\cos 5t - \sin 5t)$ (ii) $\cosh^2 3t$ (06 Marks)
- b. Find the Laplace transform of the full wave rectifier $f(t) = E\sin \omega t$ $0 < t < \frac{\pi}{\omega}$ having a period $\frac{\pi}{\omega}$. (07 Marks)
- c. Find the inverse Laplace transform $\left[\frac{s^2 + 4}{s(s+4)(s-4)} \right]$. (07 Marks)

OR

- 2 a. Find the Laplace transform, $\frac{\cos at - \cos bt}{t}$. (06 Marks)
- b. Solve by using Laplace transform method $y'''(t) + 2y''(t) - y'(t) - 2y(t) = 0$, given $y(0) = y'(0) = 0$ and $y''(0) = 6$ (07 Marks)
- c. Express the function $f(t)$ in terms of unit step function and hence find its inverse LT,

$$f(t) = \begin{cases} \cos t & 0 < t \leq \pi \\ 1 & \pi < t \leq 2\pi \\ \sin t & t > 2\pi \end{cases}$$
 (07 Marks)

Module-2

- 3 a. Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$, in $0 < x < 2\pi$. Hence deduce that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$$
 (06 Marks)
- b. Show that the sine half range series for the function, $f(x) = Lx - x^2$, in $0 < x < L$ is

$$\frac{8L^2}{\pi^3} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} \sin\left(\frac{2n+1}{L}\pi x\right)$$
 (07 Marks)
- c. Obtain the Fourier series of y up to the first harmonics for the following values :

x°	45	90	135	180	225	270	315	360
y	4.0	3.8	2.4	2.0	-1.5	0	2.6	3.4

(07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

OR

- 4 a. Expand the function $f(x) = x \sin x$, as a Fourier series in the interval $-\pi \leq x \leq \pi$. Deduce that $\frac{1}{1,3} - \frac{1}{3,5} + \frac{1}{5,7} \dots = \frac{\pi-2}{4}$ (06 Marks)
- b. Obtain the half range cosine series of $f(x) = x \sin x$, $0 \leq x \leq \pi$. (07 Marks)
- c. Obtain the constant term and the first three coefficients in the Fourier cosine series for y using the following data :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(07 Marks)

Module-3

- 5 a. Find the complex Fourier transform of the function, $f(x) = \begin{cases} 1 & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$

Hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$.

(06 Marks)

- b. If $\overline{f(z)} = \frac{2z^2 + 3z + 12}{(z-1)^4}$ find the value of u_0, u_1, u_2, u_3 (07 Marks)

- c. Solve by using z-transforms, $u_{n+2} + 5u_{n+1} + 6u_n = 2^n$; $u_1 = 0, u_0 = 0$ (07 Marks)

OR

- 6 a. Find the Fourier sine transform of e^{-ax} , $a > 0$. (06 Marks)
- b. Find the Fourier sine and cosine transform of $2e^{-3x} + 3e^{-2x}$. (07 Marks)
- c. Solve by using Z-transforms, $y_{n+2} + 2y_{n+1} + y_n = n$, with $y(0) = 0 = y$ (07 Marks)

Module-4

- 7 a. Use Taylor's series method to find $y(4.1)$ given that $\frac{dy}{dx} = \frac{1}{x^2 + y}$ and $y(4) = 4$. (06 Marks)
- b. Use Fourth order Runge-Kutta method to solve $(x+y)\frac{dy}{dx} = 1$, $y(0.4) = 1$ at $x = 0.5$. Correct to four decimal places. (07 Marks)
- c. The following table gives the solution of $5xy^1 + y^2 - 2 = 0$, find the value of y at $x = 4.5$ using Milne's Predictor and Corrector formulae, use the corrector formulae twice.

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187

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(07 Marks)

OR

- 8 a. Using modified Euler's method find y at $x = 0.2$ given $\frac{dy}{dx} = 3x + \frac{y}{2}$, with $y(0) = 1$ taking $h = 0.1$. (06 Marks)
- b. Using Runge-Kutta method of fourth order find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.2$ (07 Marks)
- c. Apply Adams-Bashforth method to solve the equation $(y^2 + 1)dy - x^2 dx = 0$, at $x = 1$, given $y(0) = 1$, $y(0.25) = 1.0026$, $y(0.5) = 1.0206$, $y(0.75) = 1.0679$. Apply the corrector formulae twice. (07 Marks)

Module-5

- 9 a. Given $\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} - 2xy = 1$, $y(0) = 1$, $y'(0) = 0$, Evaluate $y(0.1)$ using Runge-Kutta method of order 4. (06 Marks)
- b. A necessary condition for the integral $I = \int_x^{x_2} f(x, y, y') dx$ where $y(x_1) = y_1$ and $y(x_2) = y_2$ to be extremum that $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$. (07 Marks)
- c. Show that the extremal of the functional $\int_0^1 y^2 \{ 3x(y'^2 - 1) + yy'^3 \} dx$, subject to the conditions $y(0) = 0$, $y(1) = 2$, is the circle $x^2 + y^2 - 5x = 0$. (07 Marks)

OR

- 10 a. Apply Milne's method to compute $y(0.8)$. Given that $\frac{d^2y}{dx^2} = 1 - 2y \frac{dy}{dx}$ and the following table of initial values. (06 Marks)

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

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- b. Find the extremal of the functional $\int_a^b (x^2 y'^2 + 2y^2 + 2xy) dx$. (07 Marks)
- c. Prove that Geodesics on a plane are straight line. (07 Marks)
