CBCS SCHEME

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18ELD11

First Semester M.Tech. Degree Examination, July/August 2022
Advanced Engineering Mathematics

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. If a, b, c are fixed elements of a field F, then shown that the Set W of all ordered triads (x, y, z) of elements of F, such that ax+by+cz = 0 is a subspace of $V_3(F)$. **(08 Marks)**

b. If w_1 and w_2 are subspaces of a vector space V(F), then prove that $w_1 + w_2$ is also a sub space of V(F). **(12 Marks)**

OR

2 a. Prove that a mapping $T : U \to V$ from a vector space U(F) into a vector space V(F) is a linear transformation if and only if $T(C_1\alpha + C_2\beta) = C_1T(\alpha) + C_2T(\beta)\ \forall C_1, C_2 \in F$ and $\alpha, \beta \in U$. **(08 Marks)**

b. Let T be a linear transformation on R^3 defined by $T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$. Find the matrix T of the linear transformation in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$, where $\alpha_1 = \{1, 0, 1\}$, $\alpha_2 = \{-1, 2, 1\}$, $\alpha_3 = \{2, 1, 1\}$. **(12 Marks)**

Module-2

3 a. If $S = \{u_1, u_2, u_3 .... u_p\}$ is an orthogonal set of non zero vectors in R^n. Prove that S is linearly independent and hence is a basis for the subspace spanned by S. **(08 Marks)**

b. Use Given's method to find eigen values of the matrix, $\begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{pmatrix}$. **(12 Marks)**

OR

4 a. Use Gram-Schmidt orthogonalization process to construct an orthogonal basis for R^4. Given $V_1 = (1, 1, 1, 1)$, $V_2 = (1, 1, 1, 0)$, $V_3 = (1, 1, 0, 0)$, $V_4 = (1, 0, 0, 0)$ **(12 Marks)**

b. Let $S = \{u_1, u_2, u_3\}$ where $u_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$, $u_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ and $u_3 = \begin{pmatrix} -1/2 \\ -2 \\ 7/2 \end{pmatrix}$. Show that S is an orthogonal set and Express $Y = \begin{pmatrix} 6 \\ 1 \\ -8 \end{pmatrix}$ as a linear combination of the vectors in S. **(08 Marks)**

Module-3

5 a. Show that the equation of the curve joining the points (1, 0) and (2, 1) for which $I = \int_1^2 \frac{\sqrt{1+(y')^2}}{x} dx$ is an extremum is a circle. **(12 Marks)**

b. Show that the curve which extremizes the functional $\int_0^{\pi/2} \left[(y'')^2 - y^2 + x^2\right] dx$ under the conditions $y(0) = 1$, $y'(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$, $y'\left(\frac{\pi}{2}\right) = -1$ is $y = \cos x$. **(08 Marks)**

OR

6 a. Find the function y(x) for which $\int_0^{\pi} \left\{(y')^2 - y^2\right\} dx$ is stationary. Given that $\int_0^{\pi} y dx = 1$ and $y(0) = 0$, $y(\pi) = 1$. **(08 Marks)**

b. Find the extremals of $I = \int_0^1 \left[2yz - 2y^2 + (y')^2 - (z')^2\right] dx$, given that $y(0) = 0$, $y(1) = 1$, $z(0) = 0$, $z(1) = 1$. **(12 Marks)**

Module-4

7 a. Define moment generating function for discrete and a continuous random variable. A random variable X has the following p.d.f $f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$. Determine the moment generating function and also the first four moments about the origin. **(08 Marks)**

b. A random variable X has the following probability distribution:

x	−2	−1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	3K

Find K, $P(x < 2)$, $P(-2 < x < 2)$, cdf of x and mean of x. **(12 Marks)**

OR

8 a. The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of drivers with
(i) No accidents in a year.
(ii) More than 3 accidents in a year. **(08 Marks)**

b. In an engineering examination a student is considered to have failed, secured second class, first class and distinction according as he scores less than 45%, between 45% and 60%, between 60% and 75% and above 75% respectively. In a particular year 10% of the students failed in the examination and 5% of the students got distinction. Find the percentage of the students who have got First and Second class. Given $\phi(1.28) = 0.4$, $\phi(1.64) = 0.45$, $\phi(0.18) = 0.0714$. **(12 Marks)**

Module-5

9 a. If $\{X(t)\}$ is a wide-sense stationary process with auto correlation $R(\tau) = Ae^{-\alpha|\tau|}$. Determine the second order moment of the random variable $X(8) - X(5)$. **(08 Marks)**

b. Define stationary process, wide sense stationary process and Ergodic process. Given that the auto correlation function for a stationary Ergodic process with no periodic components is

$$R_{XX} = 25 + \frac{4}{1+6\tau^2}.$$

Find mean and variance of the process $\{X(t)\}$. **(12 Marks)**

OR

10 a. Define Gaussian process. Consider a random process $X(t) = (A\cos\omega t + B\sin\omega t)$ where A and B are Gaussian random variables with zero mean and variance σ^2. Find the mean and variance of this process. **(08 Marks)**

b. If $X(t)$ is a Gaussian process with $\mu(t) = 10$, $C(t_1, t_2) = 16e^{-|t_1 - t_2|}$. Find the probability that (i) $X(10) \le 8$ (ii) $|X(10) - X(6)| \le 4$. Given that $\phi(0.71) = 0.2611$, $\phi(0.5) = 0.1915$. **(12 Marks)**

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