CBCS SCHEME

USN

17MAT11

First Semester B.E. Degree Examination, July/August 2022
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the n^{th} derivative of i) x^2e^{5x} ii) cos2x cosx. **(06 Marks)**

b. Find the angle between the curves $r = a\theta$ and $r = \dfrac{a}{\theta}$. **(07 Marks)**

c. Find the pedal equation of the curve $r = a(1 + \cos\theta)$. **(07 Marks)**

OR

2 a. If $x = \tan(\log y)$ prove that $(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-1} = 0$. **(06 Marks)**

b. Prove with usual notation $\tan\phi = r\left(\dfrac{d\theta}{dr}\right)$. **(07 Marks)**

c. Find the radius of curvature of the curve $y = 4\sin x - \sin 2x$ at $x = \pi/2$. **(07 Marks)**

Module-2

3 a. Expand $2x^3 + 7x^2 + x - 6$ in power of $(x-2)$ using Taylor's theorem. **(06 Marks)**

b. Evaluate: i) $\underset{x\to 0}{\text{Lt}} \dfrac{a^x - b^x}{x}$ ii) $\underset{x\to 0}{\text{Lt}} \left(x^x\right)$ **(07 Marks)**

c. If $u = \dfrac{2yz}{x}$: $V = \dfrac{3zx}{y}$ $W = \dfrac{4xy}{z}$ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **(07 Marks)**

OR

4 a. If $u = \sin^{-1}\left(\dfrac{x^2y^2}{x+y}\right)$ prove that $xu_x + yu_y = 3\tan u$. **(06 Marks)**

b. Using Maclaurin's series prove that $\dfrac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n$. **(07 Marks)**

c. Find $\dfrac{dU}{dt}$, if $u = x^3y^2 + x^2y^3$ where $x = at^2$, $y = 2at$ using partial derivatives. **(07 Marks)**

Module-3

5 a. A particle moves along the curve C: $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$ where 't' denotes time. Find the components of its acceleration at t = 2 along the tangent. **(06 Marks)**

b. Find the value of "a" such that $\vec{F} = (axy - z^3)\hat{i} + (a-2)x^2\hat{j} + (1-a)xz^2\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\text{div}(\text{curl}\vec{A}) = 0$. **(07 Marks)**

OR

6 a. Find the divergence and curl of vector $\vec{F} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at $(2,-1,1)$. **(06 Marks)**

b. Find the directional derivative of $\vec{F} = xy^2 + yz^3$ at (2, -1, 1) in the direction of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$. **(07 Marks)**

c. Prove that curl (grad ϕ) = $\vec{0}$. **(07 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^n x dx$. **(06 Marks)**

b. Solve: $x\frac{dy}{dx} + y(\log y) = xye^x$. **(07 Marks)**

c. Water at temperature 30°C takes 5 minutes to warm upto 50°C in a room temperature of 60°C. Find the temperature after 20 minutes. **(07 Marks)**

OR

8 a. Evaluate $\int_0^\infty \frac{x^2}{(1+x^2)^{7/2}} dx$. **(06 Marks)**

b. Solve: $(2xy + e^y)dx + (x^2 + xe^y)dy = 0$. **(07 Marks)**

c. Find the orthogonal trajectory of the family of curve $r^2 = a\sin 2\theta$. **(07 Marks)**

Module-5

9 a. Solve the system of equations by Gauss Seidal method.
$30x - 2y + 3z = 75$; $2x + 2y + 18z = 30$; $x + 17y - 2z = 48$, carry out three iteration. **(06 Marks)**

b. Diagonalise the matrix $A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$. **(07 Marks)**

c. Using Rayleigh's power method, find largest Eigen value and corresponding Eigen vector of the matrix $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ choosing $X_0 = [1\ 0\ 0]^T$. **(07 Marks)**

OR

10 a. Find the Rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$. **(06 Marks)**

b. Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into canonical form, using orthogonal transformation. **(07 Marks)**

c. Apply Gauss-Jordan method to find the solution of system of equations
$x + 2y + z = 3$, $3x - y + 2z = 13$, $2x + 3y + 3z = 10$. **(07 Marks)**

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