



First Semester B.E. Degree Examination, July/August 2022 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. If $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (06 Marks)
- b. Find the angle of intersection of the curves $r = \sin \theta + \cos \theta$ and $r = 2 \sin \theta$. (05 Marks)
- c. Show that the radius of curvature at any point of:
 $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ is $4a \cos(\theta/2)$. (05 Marks)

OR

- 2 a. Find the n^{th} derivative of $\frac{x+1}{x-1} + e^{-2x} \cos^2 x$. (06 Marks)
- b. Find the pedal equation of $r^n = a^n \cos n\theta$. (05 Marks)
- c. Show that radius of curvature on the curve $y^2 = \frac{a^2(a-x)}{x}$ at $(a, 0)$ is $\frac{a}{2}$. (05 Marks)

Module-2

- 3 a. Expand $\log_e x$ in powers of $(x-1)$ upto fifth degree term. (06 Marks)
- b. If $u = \tan^{-1} \left[\frac{x^3 + y^3}{x-y} \right]$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ (05 Marks)
- c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$ and $w = x + y + z$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (05 Marks)

OR

- 4 a. Evaluate $\lim_{x \rightarrow \frac{\pi}{4}} (\tan x)^{\tan 2x}$. (06 Marks)
- b. If $Z = f(x, y)$ where $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$. (05 Marks)
- c. If $V = (x^2 + y^2 + z^2)^{-1/2}$, prove that $V_{xx} + V_{yy} + V_{zz} = 0$. (05 Marks)

Module-3

- 5 a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$ where t is the time. Find the Components of its velocity and acceleration at $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$. (06 Marks)
- b. If $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$, find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at $(1, 2, 3)$. (05 Marks)
- c. For any scalar point function ϕ and a vector point function \vec{A} , show that $\text{div}(\phi \vec{A}) = \phi(\text{div } \vec{A}) + \text{grad } \phi \cdot \vec{A}$. (05 Marks)

OR

- 6 a. If $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{R}| = r$, show that $\nabla r^n = nr^{n-2}\vec{R}$. (06 Marks)
- b. Show that for any scalar point function ϕ , $\nabla_n X(\nabla\phi) = 0$. (05 Marks)
- c. If $u = x^2 + y^2 + z^2$, $\vec{V} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\text{div}(u\vec{V}) = 5u$. (05 Marks)

Module-4

- 7 a. Obtain the reduction formula for $\int \sin^n x$ (06 Marks)
- b. Solve the differential equation : $(x^4 - 2xy^2 + y^4) dx - (2x^2y - 4xy^3 + \sin y) dy$. (05 Marks)
- c. Find the orthogonal trajectories of the family of coaxial circles $x^2 + y^2 + 2\lambda x + c = 0$, λ being the parameter. (05 Marks)

OR

- 8 a. Evaluate $\int_0^{2a} \frac{x^3}{\sqrt{2ax - x^2}} dx$ (06 Marks)
- b. Solve $x \frac{dy}{dx} + y = x^3 y^6$. (05 Marks)
- c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C . What will be the temperature of the body after 40 minutes from the original? (05 Marks)

Module-5

- 9 a. Find the rank of the matrix :

$$\begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$$
 (06 Marks)
- b. Using Gauss Seidel method solve :
 $20x + y - 2z = 17$
 $2x - 3y + 20z = 25$
 $3x + 20y - z + 18 = 0$
 in three iterations with $(x_0, y_0, z_0) = (0, 0, 0)$. (05 Marks)
- c. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. (05 Marks)

OR

- 10 a. Solve the system of equations :
 $x + y + z = 9$
 $2x - y + 2z = 15$
 $3x + 2y + z = 12$
 by Gauss elimination method. (06 Marks)
- b. Using Rayleigh's power method find the dominant eigen value of

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

 in five iterations, choosing $X_0 = [1 \ 0 \ 0]^T$. (05 Marks)
- c. Show that the transformation
 $y_1 = 2x_1 + x_2 + x_3$
 $y_2 = x_1 + x_2 + 2x_3$
 $y_3 = x_1 - 2x_3$
 is regular. Find the inverse transformation. (05 Marks)

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