CBCS SCHEME

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15CS36

Third Semester B.E. Degree Examination, July/August 2022
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define the following with an example: i) Proposition ii) Logical connectives iii) Logical equivalence iv) Contradiction v) Open statement. **(05 Marks)**

b. Define tautology. Prove $[(r \wedge s) \rightarrow q] \leftrightarrow [N(r \wedge s) \vee q]$ is tautology. **(06 Marks)**

c. Verify the validity of the argument. Rita is baking a cake. If Rita is baking a cake, then she is not practicing her flute. If Rita is not practicing her flute, then her father will not buy her a car. Therefore Rita's father will not buy her a car. **(05 Marks)**

OR

2 a. Define dual of logical statement. Verify the principal of duality for $(N p \vee q) \wedge [p \wedge (p \wedge q)] \Leftrightarrow (p \wedge q)$. **(05 Marks)**

b. Verify the validity of the argument. All squares have four sides. Quadrilateral EFGH has four sides. Therefore quadrilateral EFGH is a square. **(06 Marks)**

c. Prove by contradiction, if m is an even integer then m + 7 is odd. **(05 Marks)**

Module-2

3 a. Prove by Mathematical Induction, $\forall n \in z^+, n > 3 \Rightarrow 2^n < \underline{|n}$. **(05 Marks)**

b. Define permutation with repetition. How many arrangements are there of all the letters in SOCIOLOGICAL? In how many of the arrangements are all the vowels adjacent? **(05 Marks)**

c. How many ways are there to place 12 marbles of the same size in five distinct jars if
i) The marbles are all black? ii) Each marble is a different color? **(06 Marks)**

OR

4 a. Prove by Mathematical Induction "if A has n-elements then power set of A has 2^n elements". Define well ordering principle. **(06 Marks)**

b. Determine the coefficient of
i) xyz^2 in $(x + y + z)^4$ ii) $w^3x^2yz^2$ in $(2w - x + 3y - 2z)^8$. **(05 Marks)**

c. Explain combination with repetition. Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 32$, where, $x_1, x_2 \geq 5$, $x_3, x_4 \geq 7$. **(05 Marks)**

Module-3

5 a. Verify whether f: $\mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x - 3$ is Bijective or not. **(05 Marks)**

b. Explain the properties of binary relation by stating the observations concerned with relation matrix and digraph of relation. **(06 Marks)**

c. Let $S \subseteq Z^+$, where $|S| = 37$. Then S contains two elements that have the same remainder upon division by 36. **(05 Marks)**

OR

6 a. Define composition of 2-functions. Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$, $g(x) = x + 5$, then $(gof)(x) = ?$, $(fog)(x) = ?$ **(05 Marks)**

b. Find the partition of z induced by R if $R = \{(x, y) / (x - y) \text{ is even}\}$ on z. **(06 Marks)**

c. Find the supremum and infimum of
$B = \{2, 6, 10\}$, $B \subseteq D_{30}$ where $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$ is poset with respect to divides relation. **(05 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-4

7 a. Out of 30 students in a hostel, 15 study History 8 study economics and 6 study geography. It is known that 3 students study all these subjects. Show that, 7 or more students study none of these subjects. **(05 Marks)**

b. Find the number of derangements of 1, 2, 3, 4 also write them. **(05 Marks)**

c. Find the root polynomial for the 3 × 3 board by using the expansion formula:

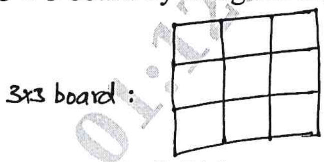

Fig.Q.7(c)

(06 Marks)

OR

8 a. Determine the number of positive integers n such that, $1 \le n \le 100$ and n is not divisible by 2, 3 or 5. **(06 Marks)**

b. There are 8 letters to 8 different people to be placed in 8 different addressed envelopes. Find the number of ways of doing this so that at least one letter gets to the right person. **(05 Marks)**

c. By using the expansion formula, find the rook polynomial for the Board C shown below: **(05 Marks)**

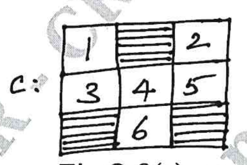

Fig.Q.8(c)

Module-5

9 a. Define isomorphism. Verify for isomorphism of G_1 and G_2:

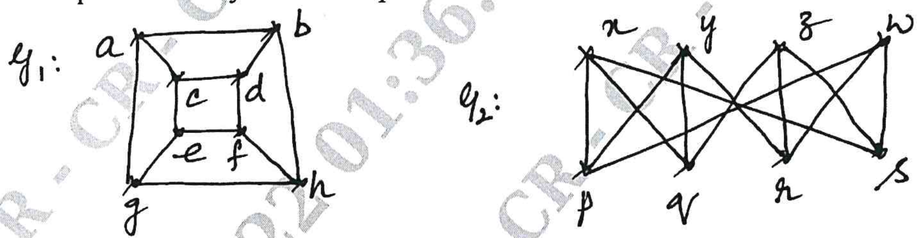

Fig.Q.9(a)

(05 Marks)

b. Define the following: i) Complete graph ii) $K_{m,n}$ iii) Hamiltonian graph iv) Eulerian graph iv) Hand shaking property. **(05 Marks)**

c. Define optimal tree. Find the weight of the optimal tree constructed for the weights, 20, 28, 4, 17, 12, 7. **(06 Marks)**

OR

10 a. i) A complete ternary tree T = (V, E) has 34 internal vertices. How many edges does T has? How many leaves?

ii) Discuss the properties of complete m-ary tree. **(05 Marks)**

b. i) Explain self-complementary graphs.

ii) Explain Konigsberg Bridge problem. **(05 Marks)**

c. Obtain an optimal prefix code for the message MISSION SUCCESSFUL. Indicate the code. **(06 Marks)**

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