<u>Internal Assessment Test 1 – May 2022</u>

Sub:	Finite Element Methods					**Code:**	18ME61
Date:	06/ 05 / 2022	Duration: 90 mins	Max Marks:	50	**Sem:** VI	**Branch:**	MECH

Note: Answer all questions.

		Marks	OBE CO	OBE RBT
1	Explain the basic steps involved in FEM.	8	CO1	L2
2	Write the equilibrium equation for 2D state of stress.	2	CO1	L1
3	Derive stiffness matrix for one dimensional bar element.	10	CO3	L2
4	A cantilever beam of span 'L' is subjected to a point load at free end. Derive an equation for the deflection at free end by using RR method. Assume polynomial displacement function.	10	CO2	L3

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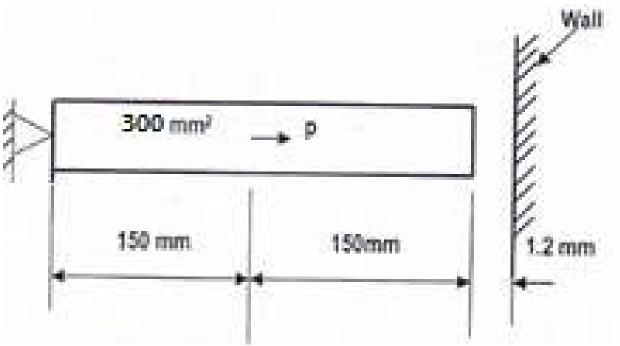

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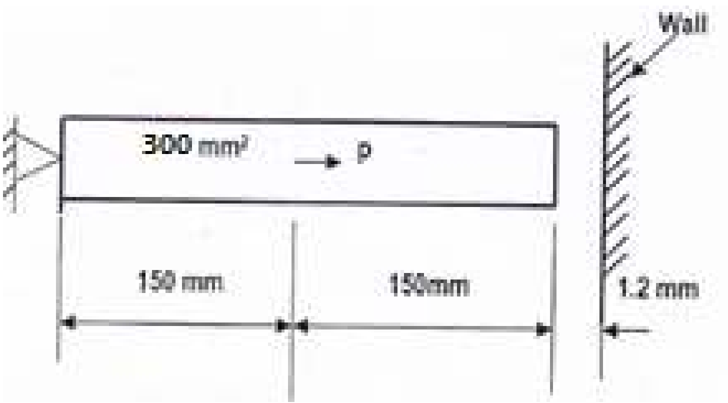


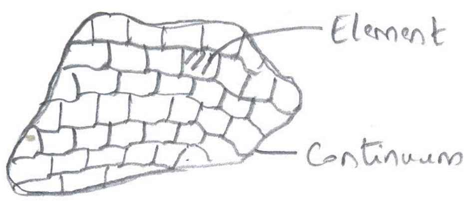
Sub :- FEM

1st IAT Solution

1 STEPS IN FEA

1. Discritization of given Continuum

In this step, the continuum is subdivided into number of parts called finite elements. The type of element selected depends on the kind of analysis namely 1D, 2D or 3D.



2. Selection of displacement model for each element

The displacement variation for each element is unknown. Hence a mathematical model is represented for each finite element. They can be either polynomial or trignometric function.

3. Generation of stiffness matrix for each finite element

4. To derive global stiffness matrix

$$K = K_1 + K_2 ----$$

5) Imposing the equilibrium equation

$$[K][q] = [F]$$

6) Enforcing the boundary conditions of the given problem

There are two methods to enforce the boundary condition

i) Elimination method - We eliminate the corresponding rows & columns were we have constraints

ii) Penalty Method - Add a constant $C = |\max k_{ij}| \times 10^4$ to the first & last element of diagonal.

7) Determination of unknowns.

In this step, the unknowns such as stresses, displacements, strain etc are found.

2 . Equilibrium equation for 2D state of stress

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + Y = 0$$

3. **Stiffness matrix for one dimensional bar element**

Strain energy for 3 D element $U_e = \frac{1}{2}\int_V \sigma^T \epsilon \, dv$

Strain energy for 1D element $U_e = \frac{1}{2}\int_{l_e} \sigma^T \epsilon \, A \, dx$

Strain $\in = Bq$

$$where\ B = \frac{1}{l_e}[-1 \quad 1]$$

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Strain energy for 1D element $U_e = \frac{1}{2}\int_{l_e} [EBq]^T \, Bq \, A \, dx$

Relation between natural and Cartesian coordinate is

$$\xi = \frac{x - x_1}{x_2 - x_1} - 1$$

$$\frac{d\xi}{dx} = \frac{2}{l_e}$$

$$U_e = \frac{1}{2}\int_{l_e} q^T \, B^T E \, Bq \, A \, \frac{l_e}{2} \, d\xi$$

$$U_e = \frac{1}{2} q^T [EA \, \frac{l_e}{2} \int_{l_e} B^T \, Bd\xi \,] \, q$$

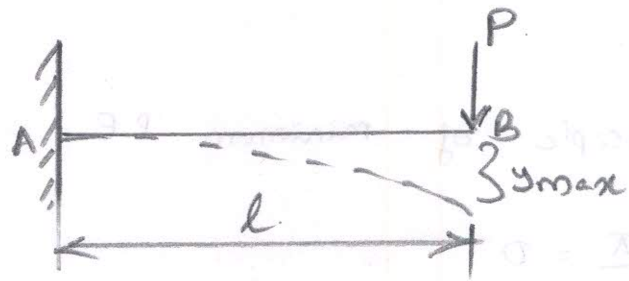
$$U_e = \frac{1}{2} q^T k_e \, q$$

$$Where\ k_e = stiffness\ matrix$$

$$k_e = EA \, \frac{l_e}{2} \int_{l_e} B^T \, Bd\xi$$

$$K = \frac{EA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

4) A Cantilever beam Subjected to point load at free end. Derive an eqn for max' deflection using R.R method.



Sol 1) Potential Energy functional

$$\pi = \underset{(+ve)}{S.E} + \underset{(-ve)}{W.P}$$

$$S.E = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx$$

$$W.P = P . y_{max}$$

2) Assume displacement function

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

B.C At A, $x=0$, $y=0$ $\quad$ At A, $x=0$, $\frac{dy}{dx}=0$

$$\boxed{a_0 = 0}$$

$$\frac{dy}{dx} = a_1 + 2a_2 x + 3a_3 x^2$$

$$\boxed{a_1 = 0}$$

$$y = a_2 x^2 + a_3 x^3$$

$$\frac{dy}{dx} = 2a_2 x + 3a_3 x^2$$

$$\frac{d^2y}{dx^2} = 2a_2 + 6a_3 x$$

$y = y_{max}$ at $x = l$

$$y = a_2 x^2 + a_3 x^3$$

$$y_{max} = a_2 l^2 + a_3 l^3$$

3) P.E functional

$$\pi = \frac{EI}{2}\int_0^l (2a_2 + 6a_3 x)^2 \, dx - P(a_2 l^2 + a_3 l^3)$$

$$= \frac{EI}{2}\int_0^l [4a_2^2 + 36a_3^2 x^2 + 24 a_2 a_3 x] \, dx - P(a_2 l^2 + a_3 l^3)$$

$$= \frac{EI}{2}\left[4a_2^2 x + 36a_3^2 \frac{x^3}{3} + 24 a_2 a_3 \frac{x^2}{2}\right]_0^l - P(a_2 l^2 + a_3 l^3)$$

$$\pi = \frac{EI}{2}\left[4a_2^2 l + 36a_3^2 \frac{l^3}{3} + 24 a_2 a_3 \frac{l^2}{2}\right] - P(a_2 l^2 + a_3 l^3)$$

4) Using Principle of minimum P.E

$$\frac{\partial \pi}{\partial a_2} = 0 \quad ; \quad \frac{EI}{2}\left[8a_2 l + 12a_3 l^2\right] - Pl^2 = 0 \quad \rightarrow (1)$$

$$\frac{\partial \pi}{\partial a_3} = 0 \quad ; \quad \frac{EI}{2}\left[24 a_3 l^3 + 12 a_2 l^2\right] - Pl^3 = 0 \quad \rightarrow (2)$$

Multiply eqn (1) by $2l$

$$\frac{EI}{2}\left[16 a_2 l^2 + 24 a_3 l^3\right] - 2Pl^3 = 0$$

$$\frac{EI}{2}\left[12 a_2 l^2 + 24 a_3 l^3\right] - Pl^3 = 0$$

$$-\qquad\qquad -\qquad\qquad +$$

$$\frac{EI}{2}\left[4 a_2 l^2\right] - Pl^3 = 0$$

$$a_2 = \frac{2Pl^3}{4l^2 \cdot EI}$$

$$\boxed{a_2 = \frac{Pl}{2EI}}$$

Sub. a_2 in eqn (2)

$$\frac{EI}{2}\left[12 \cdot \frac{Pl}{2EI} \cdot l^2 + 24 a_3 l^3\right] - Pl^3 = 0$$

$$\frac{6Pl^3}{EI} + 24 a_3 l^3 = \frac{2Pl^3}{EI}$$

$$24 a_3 l^3 = \frac{2Pl^3}{EI} - \frac{6Pl^3}{EI} = \frac{-4Pl^3}{EI}$$

$$a_3 = \frac{-4Pl^3}{24 l^3 \cdot EI} = \frac{-P}{6EI}$$

$$\boxed{a_3 = \frac{-P}{6EI}}$$

Max' deflection

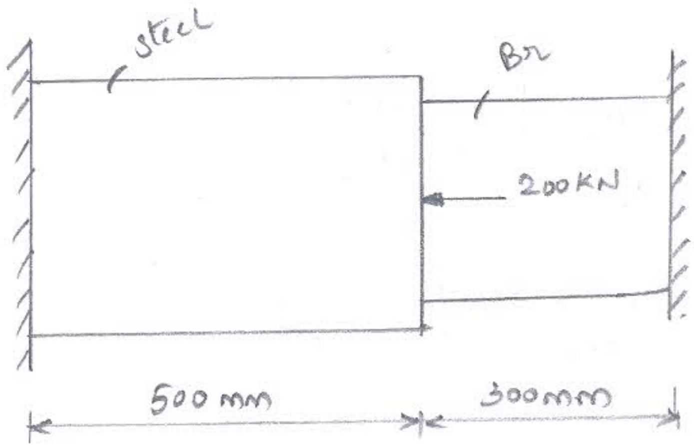
$$y_{max} = a_2 l^2 + a_3 l^3$$

$$= \frac{P.\, l^3}{2EI} - \frac{P l^3}{6EI}$$

$$\boxed{y_{max} = \frac{P l^3}{3EI}}$$

5 A Compound bar 800 mm long is made of Steel of 500mm length with an area of 400 mm^2 with the remaining length made of brass having an area of 300 mm^2. At the junction it is Subjected to an axial load of 200 KN which is in Compression to Steel. Both the ends are fixed. $E_{St} = 200\ GPa$; $E_{Br} = 70\ GPa$. Using Penalty method find the foll. nodal displacement, Stress in each element, reaction at supports.

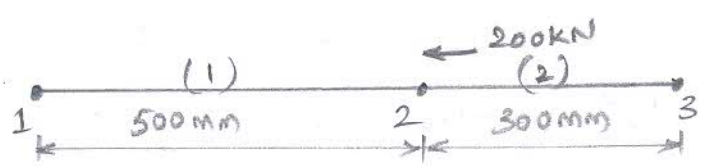
Sol



$E_{St} = 200 GPa$ $\quad$ $E_{Br} = 70\ GPa$

$A_1 = 400 mm^2$ $\quad$ $A_2 = 300 mm^2$

F.E. Model



Stiffness matrix for each element

For element 1

$$K_1 = \frac{E_{st} A_1}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{200 \times 10^3 \times 400}{500} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_1 = 10^5 \begin{array}{cc} 1 & 2 \\ \begin{bmatrix} 1.6 & -1.6 \\ -1.6 & 1.6 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

For element 2

$$K_2 = \frac{E_{Br} \cdot A_2}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{70 \times 10^3 \times 300}{300} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^5 \begin{array}{cc} 2 & 3 \\ \begin{bmatrix} 0.7 & -0.7 \\ -0.7 & 0.7 \end{bmatrix} & \begin{array}{c} 2 \\ 3 \end{array} \end{array}$$

Global stiffness matrix

$$K = K_1 + K_2$$

$$= 10^5 \begin{bmatrix} 1.6 & -1.6 & 0 \\ -1.6 & 2.3 & -0.7 \\ 0 & -0.7 & 0.7 \end{bmatrix}$$

Using Penalty method

$$C = |\max K_{ij}| \times 10^4$$

$$= 2.3 \times 10^5 \times 10^4$$

$$C = 2.3 \times 10^9$$

'C' should be added to first & last element of the diagonal.

$$K = 10^5 \begin{bmatrix} 1.6 + C & -1.6 & 0 \\ -1.6 & 2.3 & -0.7 \\ 0 & -0.7 & 0.7 + C \end{bmatrix}$$

$$K = \begin{bmatrix} 2.30016 \times 10^9 & -1.6 \times 10^5 & 0 \\ -1.6 \times 10^5 & 2.3 \times 10^5 & -0.7 \times 10^5 \\ 0 & -0.7 \times 10^5 & 2.30007 \times 10^5 \end{bmatrix}$$

Equilibrium eqn.

$$[K][q] = [F]$$

$$\begin{bmatrix} 2.30016 \times 10^9 & -1.6 \times 10^5 & 0 \\ -1.6 \times 10^5 & 2.3 \times 10^5 & -0.7 \times 10^5 \\ 0 & -0.7 \times 10^5 & 2.30007 \times 10^5 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -200 \times 10^3 \\ 0 \end{bmatrix}$$

Solving
Equilibrium eqn.

$$\boxed{q_1 = -6.0491 \times 10^{-5}\ mm} \quad \boxed{q_2 = -0.8696\ mm}$$

$$\boxed{q_3 = -2.6466 \times 10^{-5}\ mm}$$

Stresses in each element

For element 1

$$\sigma_1 = EBQ$$

$$= E_{st} \frac{1}{l_1} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$= \frac{200 \times 10^3}{300} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} -6.0491 \times 10^{-5} \\ -0.8696 \end{bmatrix}$$

$$\boxed{\sigma_1 = -347.82\ N/mm^2}$$

For element 2

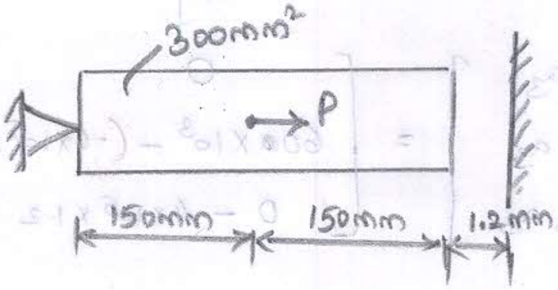
$$\sigma_2 = E\ B.\ Q$$

$$= E_{Br}.\ \frac{1}{l_2} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} q_3 \\ q_4 \end{bmatrix}$$

$$= 70 \times 10^3 \times \frac{1}{300} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} -0.8696 \\ -2.6466 \times 10^{-5} \end{bmatrix}$$

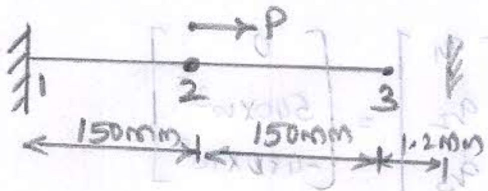
$$\boxed{\sigma_2 = 202.88\ N/mm^2}$$

6



$E = 200\ GPa$

F.E. Model



Elemental Stiffness matrix

$$K = \frac{EA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

For element 1

$$K_1 = \frac{200\times10^3 \times 300}{150}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_1 = 10^5 \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

For element 2

$$K_2 = \frac{200\times10^3 \times 300}{150}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_2 = 10^5 \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 3 \end{array}$$

Global Stiffness matrix

$$K = 10^5 \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 4 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 4 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

Load Vector

$$F = \begin{bmatrix} F_1 - K_{13}\, a_3 \\ F_2 - K_{23}\, a_3 \\ F_3 - K_{33}\, a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 600 \times 10^3 - (-4\times10^5 \times 1.2) \\ 0 - 4\times10^5 \times 1.2 \end{bmatrix} = \begin{bmatrix} 0 \\ 540\times10^3 \\ -480\times10^3 \end{bmatrix}$$

Equilibrium eqn. $[K][q] = [F]$

$$10^5 \begin{bmatrix} 4 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 04 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 540\times10^3 \\ -480\times10^3 \end{bmatrix}$$

Applying B.C : Using Elimination method

$$10^5 \begin{bmatrix} 4 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 4 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 540\times10^3 \\ -480\times10^3 \end{bmatrix}$$

$q_1 = 0$; $q_2 = 0.675$ mm ; $q_3 = 1.2$ mm.

Displacement at node 1 : $q_1 = 0$

node 2 : $q_2 = 0.675$ mm

node 3 : $q_3 = 1.2$ mm //

Stress in each element

$$\sigma = E \cdot B \cdot q = E \cdot \frac{1}{l_e} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Element 1

$$\sigma_1 = 200\times10^3 \times \frac{1}{150} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0.675 \end{bmatrix}$$

$$\sigma_1 = 900 \text{ N/mm}^2 //$$

Element 2

$$\sigma_2 = 200\times10^3 \times \frac{1}{150} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 0.675 \\ 1.2 \end{bmatrix}$$

$$\sigma_2 = 700 \text{ N/mm}^2 //$$