SET 01/03

USN

Internal Assessment Test 1 – May. 2022

Sub:	HEAT TRANSFER					Sub Code:	18ME63	Branch:	ME
Date:	10.05.2022	Duration:	90 min	Max Marks:	50	Sem / Sec:	VI/A&B	OBE

	Answer All the Questions **Use of Heat Transfer Data handbook is permitted**	MARKS	CO	RBT
1	Derive general heat conduction equation for cartesian coordinate system.	[10]	CO1	L3
2	a) Describe the different modes of heat transfer in brief. (6M) b) Define the following terms in brief: (i) Thermal conductivity (ii) Thermal Diffusivity (4M)	[10]	CO1	L3
3	A furnace has a composite wall constructed of a refractory material for the inside layer and an insulating material on the outside. The total wall thickness is limited to 60cm. The mean temperature of the gases within the furnace is 850ºC, the external temperature is 30ºC and the material interface temperature is 500ºC. The thermal conductivities of refractory and insulating materials are 2 W/mK and 0.2 W/mK respectively. The combined coefficient of heat transfer by convection and radiation between gases and refractory surface is 200 W/m^{2}K and between outside surface and atmosphere is 40 W/m^{2}K. Find: (i) Thickness of each material. (ii) Rate of heat loss to atmosphere. (iii) Temperatures of external and internal surfaces.	[10]	CO1	
4	A furnace wall comprises three layers: 13.5 cm thick inside layer of fire brick (k=1.2W/mK), 7.5 cm thick middle layer of insulating brick (k=0.14W/mK) and 11.5 cm thick outside layer of red brick (k= 0.85W/mK). The furnace operates at 870°C and it is anticipated that the outside of this composite wall can be maintained at 40°C by circulation of air. Assuming close bounding of layers at their interfaces, find the rate of heat loss from the furnace and the temperatures at a distance of 10cm, 15cm and 25 cm from the inside of the furnace. The wall measures 5m and 2m.	[10]	CO1	
5	A wire of 8mm diameter at a temperature of 60ºC is to be insulated by a material having k=0.174 W/mK. Heat transfer coefficient between surface and atmosphere is 8W/m^{2}K and ambient temperature is 25ºC. For maximum heat loss find the minimum thickness of insulation. Find % increase in heat dissipation due to insulation.	[10]	CO1	

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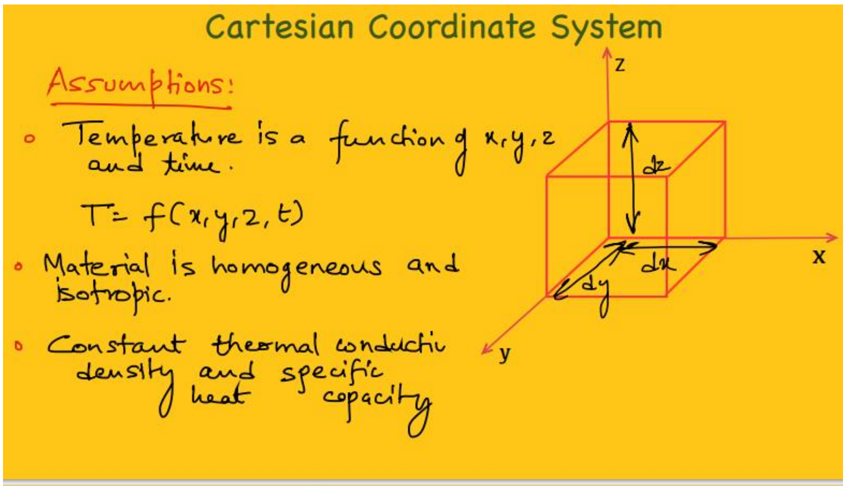
SOLUTION

1)



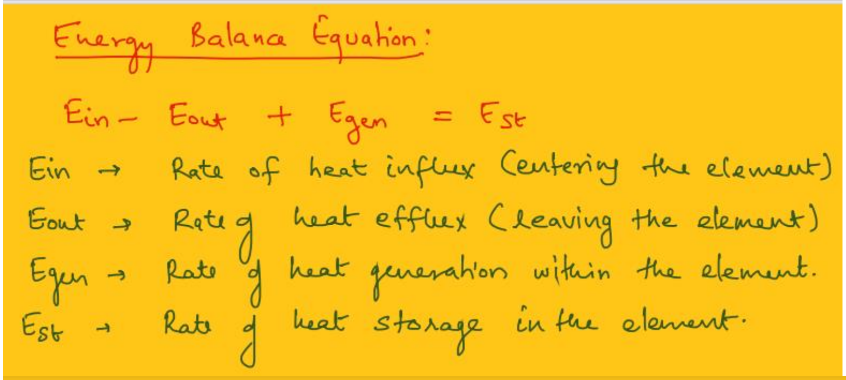
Cartesian Coordinate System

Assumptions:

- Temperature is a function of x, y, z and time.

  $T = f(x, y, z, t)$

- Material is homogeneous and isotropic.
- Constant thermal conductivity, density and specific heat capacity



Energy Balance Equation:

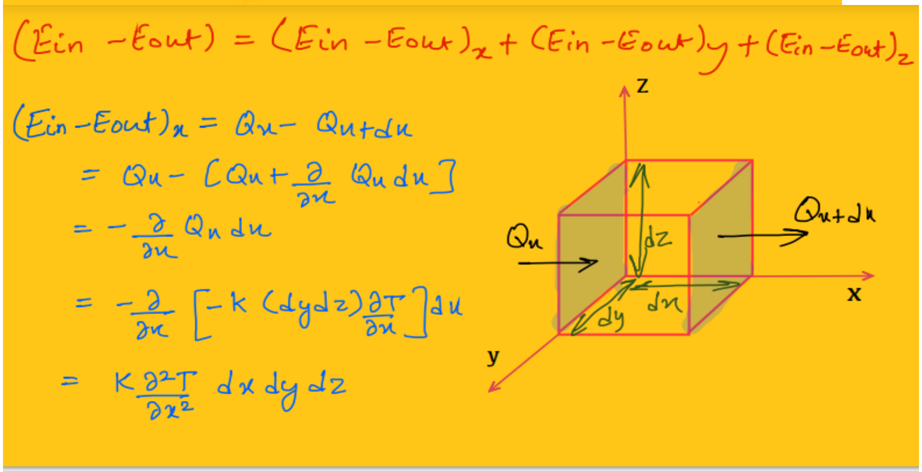
$$E_{in} - E_{out} + E_{gen} = E_{st}$$

E_{in} → Rate of heat influx (entering the element)

E_{out} → Rate of heat efflux (leaving the element)

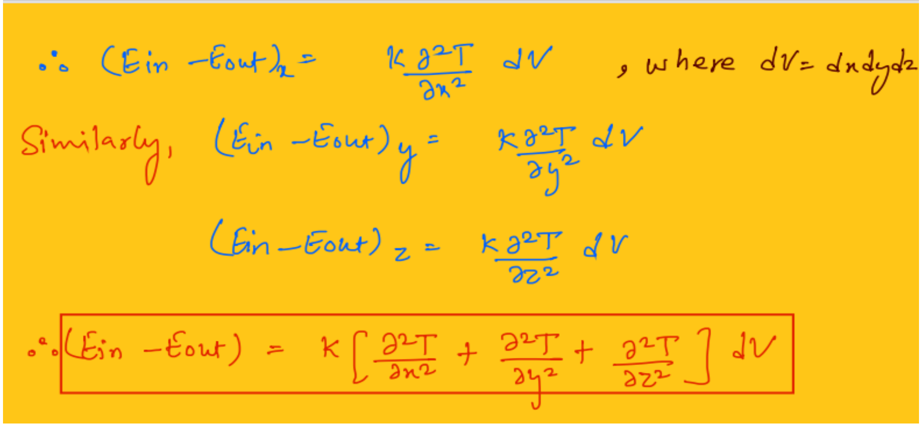
E_{gen} → Rate of heat generation within the element.

E_{st} → Rate of heat storage in the element.



$$(E_{in} - E_{out}) = (E_{in} - E_{out})_x + (E_{in} - E_{out})_y + (E_{in} - E_{out})_z$$

$$\begin{aligned}(E_{in} - E_{out})_x &= Q_x - Q_{x+dx}\\ &= Q_x - \left[Q_x + \frac{\partial}{\partial x} Q_x \, dx\right]\\ &= -\frac{\partial}{\partial x} Q_x \, dx\\ &= -\frac{\partial}{\partial x}\left[-k\,(dy\,dz)\frac{\partial T}{\partial x}\right] dx\\ &= k\frac{\partial^2 T}{\partial x^2}\, dx\, dy\, dz\end{aligned}$$



$$\therefore (E_{in} - E_{out})_x = k\frac{\partial^2 T}{\partial x^2}\, dV \quad , \text{where } dV = dx\,dy\,dz$$

Similarly, $(E_{in} - E_{out})_y = k\dfrac{\partial^2 T}{\partial y^2}\, dV$

$$(E_{in} - E_{out})_z = k\frac{\partial^2 T}{\partial z^2}\, dV$$

$$\therefore \boxed{(E_{in} - E_{out}) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] dV}$$

Let rate of heat generation per unit volume inside the element be q_g.

$$\therefore \quad \boxed{E_{gen} = q_g \cdot dV}$$

Let mass of element be m, density be ρ and specific heat capacity be c.

$$\therefore \quad E_{st} = mc\frac{\partial T}{\partial t}$$

$$\Rightarrow \quad \boxed{E_{st} = \rho c \frac{\partial T}{\partial t} \cdot dV}$$

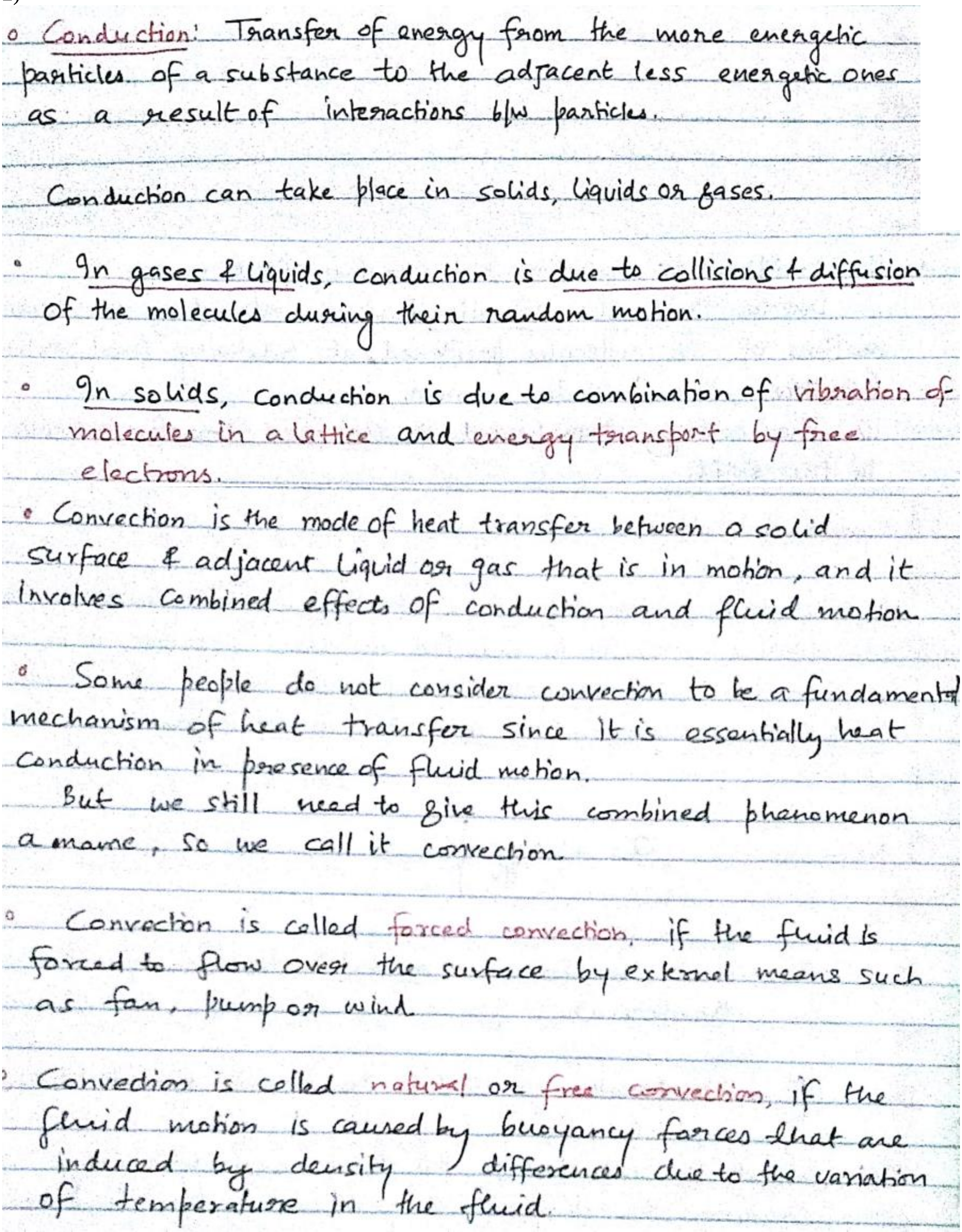
$$\therefore \quad k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] dV + q_g \cdot dV = \rho c \frac{\partial T}{\partial t} dV$$

$$\boxed{\frac{\rho c}{k} = \frac{1}{\alpha}}$$

$$\Rightarrow \quad \boxed{\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{q_g}{k} = \frac{\rho c}{k} \frac{\partial T}{\partial t}}$$

The above equation is called General Heat Conduction Equation in Cartesian Coordinate System.

2)



- Conduction: Transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions b/w particles.

  Conduction can take place in solids, liquids or gases.

- In gases & liquids, conduction is due to collisions & diffusion of the molecules during their random motion.

- In solids, conduction is due to combination of vibration of molecules in a lattice and energy transport by free electrons.

- Convection is the mode of heat transfer between a solid surface & adjacent liquid or gas that is in motion, and it involves combined effects of conduction and fluid motion

- Some people do not consider convection to be a fundamental mechanism of heat transfer since it is essentially heat conduction in presence of fluid motion.
  But we still need to give this combined phenomenon a name, so we call it convection.

- Convection is called forced convection, if the fluid is forced to flow over the surface by external means such as fan, pump or wind

- Convection is called natural or free convection, if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.

Radiation is energy emitted by matter in the form of electromagnetic waves (or photons) as a result of changes in the electronic configirations of the atoms or molecules.

- In heat transfer studies, we are interested in thermal radiations, which is the form of radiations emitted by bodies because of their temperature. It differs from other forms of electromagnetic radiations such as x-rays, gamma rays etc.

- Radiation is a volumetric phenomenon, & all solids, liquids & gases emit, absorb or transmit radiation to varying degrees.
  However, radiation is usually considered to be a Surface phenomenon for solids that are opaque to thermal radiations such as metals, wood & rocks etc.

K → thermal conductivity → Measure of the ability of material to conduct heat.

- Kinetic theory of gases predict & experiments confirm that the thermal conductivity of gases is proportional to the sq. root of thermodynamic temperature T, & inversely proportional to the sq. root of the molar mass M.
- Thermal conductivity of liquids is generally insensitive to pressure except near the thermodynamic critical point.
  In liquids thermal conductivity decreases with increasing temparature & with increasing molar mass, with water being an exception.

In solids, heat conduction is due to two effects -

i) Due to the lattice vibrational waves induced by the vibrational motions of the molecules positioned at relatively fixed positions (lattice), in a periodic manner

ii) The energy transported via the free flow of electrons in the solid.

Thermal Diffusivity is the property of a material which tells us about the rate of heat diffusion through the material. It is denoted by α.

The larger the value of α, the faster the heat will diffuse through the material and its temperature will change with time.

$$\alpha = \frac{k}{\rho c}$$

k → thermal conductivity.

ρ → Density

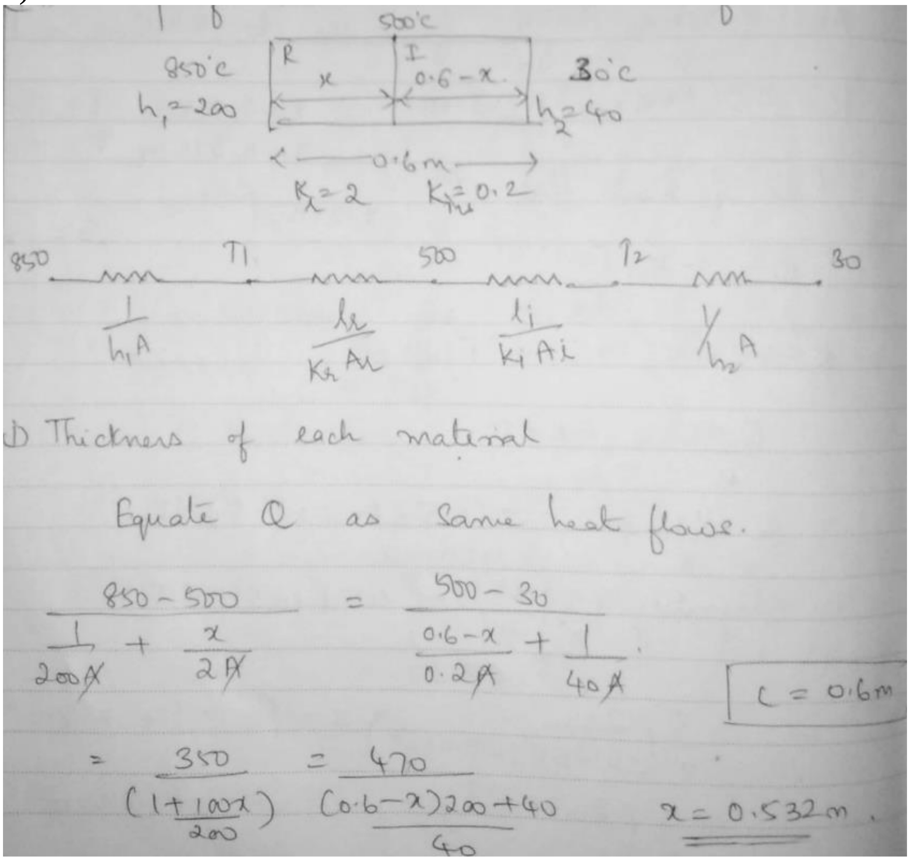
c → specific heat capacity

(ρc) → heat storage capacity.

k → A higher thermal conductivity means higher rate of heat conduction which results to larger value of α.

ρc → A low value of heat storage capacity means lesser amount of heat entering the material will be absorbed and will be used to raise its temperature and more heat will be available for onward transmission. This also results in higher value of α.

3)



i) Thickness of each material

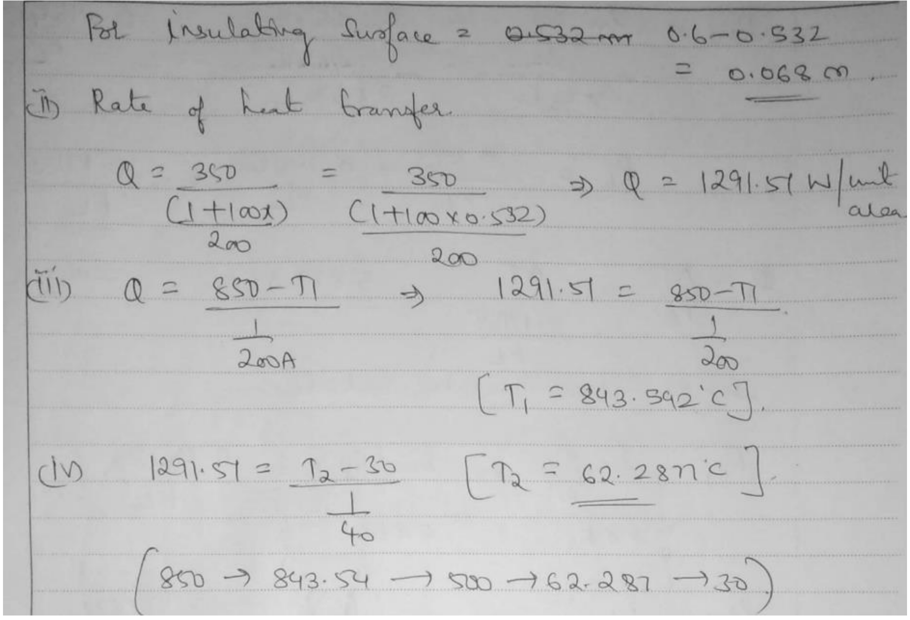
Equate Q as same heat flow.

$$\frac{850-500}{\frac{1}{200\cancel{A}}+\frac{x}{2\cancel{A}}} = \frac{500-30}{\frac{0.6-x}{0.2\cancel{A}}+\frac{1}{40\cancel{A}}}$$

$\boxed{L = 0.6m}$

$$= \frac{350}{\left(\frac{1+100x}{200}\right)} = \frac{470}{\frac{(0.6-x)200+40}{40}}$$

$\underline{\underline{x = 0.532 m}}$.



For insulating surface = ~~0.532 m~~ 0.6 − 0.532
= $\underline{\underline{0.068 m}}$.

ii) Rate of heat transfer.

$$Q = \frac{350}{\frac{(1+100x)}{200}} = \frac{350}{\frac{(1+100\times0.532)}{200}} \Rightarrow Q = 1291.51 \text{ W/unit area}$$

iii) $Q = \frac{850-T1}{\frac{1}{200A}} \Rightarrow 1291.51 = \frac{850-T1}{\frac{1}{200}}$.

$[T_1 = 843.542°c]$.

iv) $1291.51 = \frac{T_2-30}{\frac{1}{40}}$ $[T_2 = \underline{\underline{62.2877°c}}]$.

(850 → 843.54 → 500 → 62.287 → 30)

4)

$T_1 = 870^\circ C$; $T_2 = 40^\circ C$

$L_1 = 13.5\times10^{-2}\,m$; $L_2 = 7.5\times10^{-2}\,m$; $L_3 = 11.5\times10^{-2}\,m$;

$k_1 = 1.2\,W/mK$; $k_2 = 0.14\,W/mK$; $k_3 = 0.85\,W/mK$.

$A = 5\times2 = 10\,m^2$

$$Q = \frac{(870-40)}{\left(\frac{13.5\times10^{-2}}{1.2\times10} + \frac{7.5\times10^{-2}}{0.14\times10} + \frac{11.5\times10^{-2}}{0.85\times10}\right)}$$

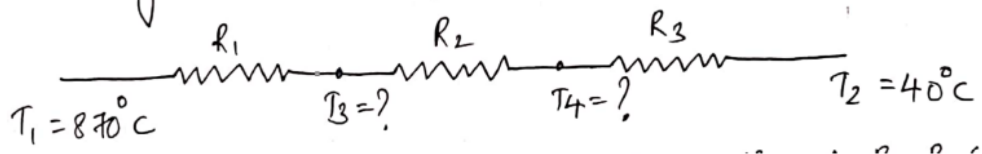
$$\boxed{Q = 10593.37713\,W}$$

To find temperatures at a distance of

(i) 10cm = $10\times10^{-2}\,m$ from the inside of the furnace.

$$\boxed{\frac{T-T_1}{T_2-T_1} = \frac{x}{L}}$$

* Finding intermediate temperatures.

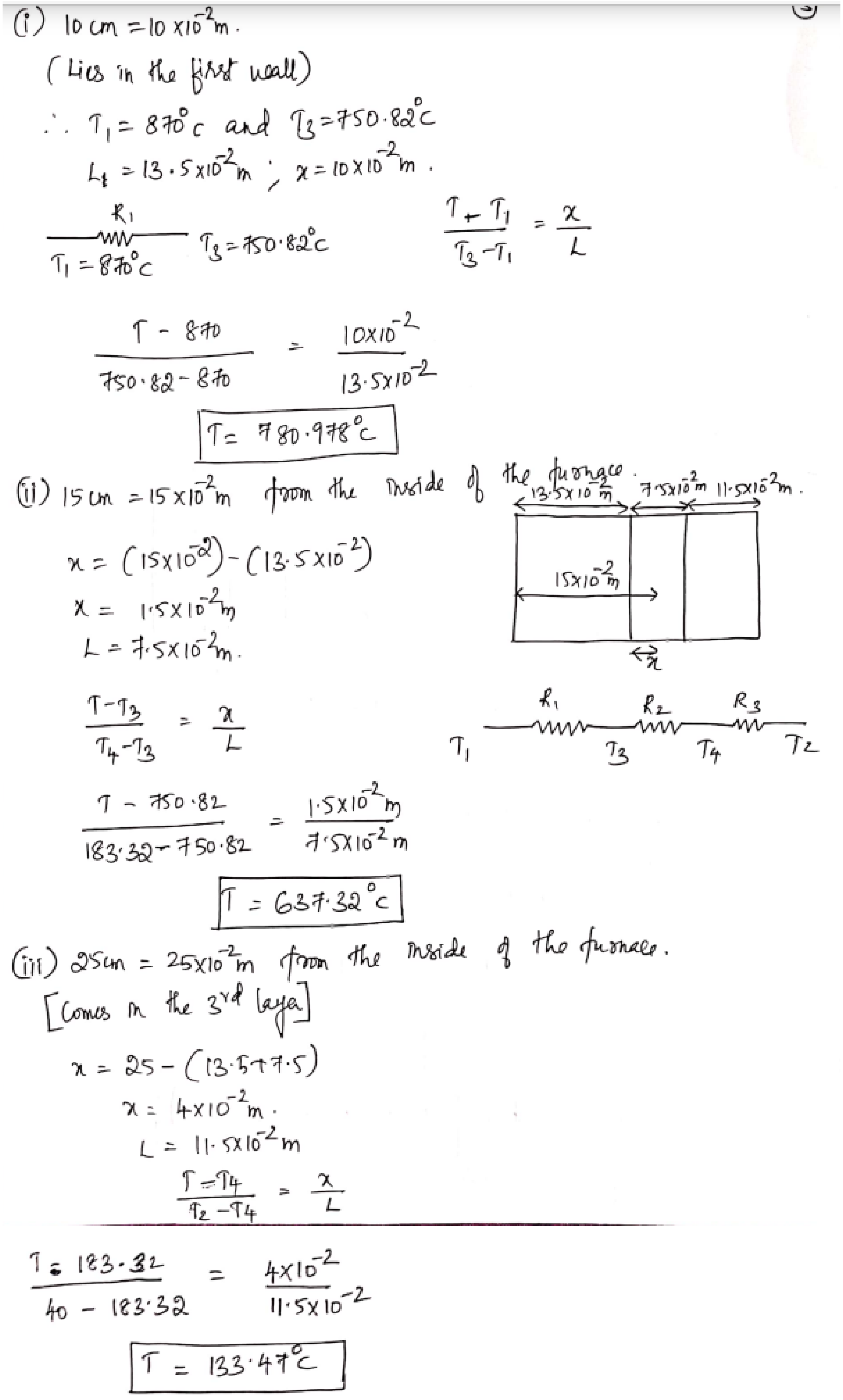


$$Q = \frac{T_1 - T_3}{R_1} \Rightarrow 10593.37713 = \frac{870 - T_3}{\left(\frac{13.5\times10^{-2}}{1.2\times10}\right)}$$

$$\boxed{T_3 = 750.8\ ^\circ C}$$

$$Q = \frac{T_4 - T_2}{R_3} \Rightarrow \frac{T_4 - 40}{\left(\frac{11.5\times10^{-2}}{0.85\times10}\right)} = 10593.37713$$

$$\boxed{T_4 = 183.32^\circ C}$$



(i) 10 cm $= 10\times10^{-2}$ m.

(Lies in the first wall)

$\therefore\ T_1 = 870^\circ c$ and $T_3 = 750.82^\circ c$

$L_1 = 13.5\times10^{-2}$ m ; $x = 10\times10^{-2}$ m.

R_1 ; $T_1 = 870^\circ c$; $T_3 = 750.82^\circ c$

$$\frac{T - T_1}{T_3 - T_1} = \frac{x}{L}$$

$$\frac{T - 870}{750.82 - 870} = \frac{10\times10^{-2}}{13.5\times10^{-2}}$$

$$\boxed{T = 780.978^\circ c}$$

(ii) 15 cm $= 15\times10^{-2}$ m from the inside of the furnace.

$x = (15\times10^{-2}) - (13.5\times10^{-2})$

$x = 1.5\times10^{-2}$ m

$L = 7.5\times10^{-2}$ m.

$$\frac{T - T_3}{T_4 - T_3} = \frac{x}{L}$$

$$\frac{T - 750.82}{183.32 - 750.82} = \frac{1.5\times10^{-2}\text{ m}}{7.5\times10^{-2}\text{ m}}$$

$$\boxed{T = 637.32^\circ c}$$

(iii) 25cm $= 25\times10^{-2}$ m from the inside of the furnace.

[Comes in the 3rd layer]

$x = 25 - (13.5 + 7.5)$

$x = 4\times10^{-2}$ m.

$L = 11.5\times10^{-2}$ m

$$\frac{T - T_4}{T_2 - T_4} = \frac{x}{L}$$

$$\frac{T - 183.32}{40 - 183.32} = \frac{4\times10^{-2}}{11.5\times10^{-2}}$$

$$\boxed{T = 133.47^\circ c}$$

5)

$T_1 = 60°c$ $T_2 = 25°c$ $K = 0.174$ w/mk $h = 8$ w/m^{2}K

$r = 4 \times 10^{-3}$ m

1. Convection => $\frac{(t_1 - t_2)}{1/(hA)} = \frac{60 - 25}{1/(8 \times 2\pi r l)}$ => $\frac{Q}{l} = 7.03716$ w.

2. Total Resistance => $\frac{\ln(r_2/r_1)}{2\pi K_1 l} + \frac{1}{(2\pi r l)h}$

=> r_c => $k/h = 0.02175$ => 21.75 mm Te => 13.75 mm

Total Resistance => 1.54 + 0.86 => 2.40 K/w.

Heat Transfer => 14.577 w.

% change => $\frac{14.577 - 7.03716}{7.03716}$ => 107.143%.