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Internal Assessment Test 1 – July 2022

Sub:	Applied Hydraulics					Sub Code:	18CV43	Branch:	CV
Date:	09.07.2022	Duration:	90 mins	Max Marks:	50	Sem/Sec:	IV		OBE

Answer all questions. Provide neat sketches wherever necessary. Assume data wherever required.

		MARKS	CO	RBT
1	Explain three types of similarities in model analysis.	[10]	CO1	L2
2	Using Buckingham's Π - theorem, show that the velocity through a circular orifice is given by, $v = \sqrt{2gH}\phi\left[\frac{D}{H}, \frac{\mu}{\rho v H}\right]$ where H is head causing flow, μ is coefficient viscosity, ρ is mass density and g is gravitational acceleration.	[10]	CO1	L4
3	Derive the conditions for most economical trapezoidal section	[10]	CO2	L3
4	An open channel is to be constructed of trapezoidal section and with side slope 1V:1.5H. Find relationship between bottom width and depth of flow for min excavation. If flow is to be 2.7cumec, calculate the bottom width and depth of flow assuming C=44.5 and bed slope =1/4000.	[10]	CO2	L4
5	A circular open channel laid to a gradient of 1 in 9000 carries a discharge of 0.4m3/s. If the depth of flow is 1.25 times the radius of channel, find the diameter of the channel. Assume Manning's N or rugosity coefficient for channel as 0.015.	[10]	CO2	L4

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Solutions

1

SIMILITUDE

- It is defined as the similarity between the model and its prototype. It means it has similar properties. There are 3 types of similarities which must exist between model and prototype.

1. Geometric Similarity
2. Kinematic Similarity
3. Dynamic Similarity

GEOMETRIC SIMILARITY

- When the ratios of the linear dimensions in the model and prototype are equal, it is said to be geometrically similar.

Let

L_m = Length of model, L_p = Length of prototype

B_m = Breadth of model, B_p = Breadth of prototype

H_m = Height of model, H_p = Height of prototype

A_m = Area of model, A_p = Area of prototype

V_m = Volume of model, V_p = Volume of prototype

For geometric similarity,

$$\frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{H_p}{H_m} = L_r$$

L_r = scale ratio

For area

$$\frac{A_p}{A_m} = \frac{L_p}{L_m} * \frac{B_p}{B_m} = L_r^{\,2}$$

For volume

$$\frac{V_p}{V_m} = \frac{L_p}{L_m} * \frac{B_p}{B_m} * \frac{H_p}{H_m} = L_r^{\,3}$$

KINEMATIC SIMILARITY

- When the ratios of the velocity and acceleration at the corresponding points in the model and corresponding points in the prototype are same, it is said have kinematic similarity.
- The direction of the vector quantities (velocity and acceleration) should be same.

In the fluid, let

v_{m1} = velocity at pt 1 in model, v_{p1} = velocity at pt 1 in prototype

v_{m2} = velocity at pt 2 in model, v_{p_2} = velocity at pt 2 in prototype

a_{m1} = acc at pt 1 in model, a_{p_1} = acc at pt 1 in prototype

a_{m2} = acc at pt 2 in model, a_{p_2} = acc at pt 2 in prototype

For kinematic similarity

$$\frac{v_{p1}}{v_{m1}} = \frac{v_{p2}}{v_{m2}} = v_r$$

v_r is velocity ratio

$$\frac{a_{p1}}{a_{m1}} = \frac{a_{p2}}{a_{m2}} = a_r$$

a_r is acceleration ratio

DYNAMIC SIMILARITY

- When the ratios of the forces acting at the corresponding points in the model and in the prototype are same, it is said have dynamic similarity.
- The direction of the forces should be same.

At a point, let

$(F_i)_m$ = Inertial force in model, $(F_i)_p$ = Inertial force in prototype

$(F_v)_m$ = Viscous force in model, $(F_v)_p$ = Viscous force in prototype

$(F_g)_m$ = Gravity force in model, $(F_g)_p$ = Gravity force in prototype

For dynamic similarity

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r$$

F_r is force ratio

2.

$$V = \sqrt{2gH} * \phi\left[\frac{D}{H}, \frac{\mu}{\rho VH}\right]$$

1. $g = LT^{-2}$
2. $H = L$
3. $D = L$
4. $\rho = (kg/m^3) = ML^{-3}$
5. $\mu = ML^{-1}T^{-1}$
6. $V = LT^{-1}$

$V = f(g, H, D, \mu, \rho)$

$f_1(V, g, H, D, \mu, \rho) = 0$

There 6 variables and
3 fundamental dimensions

$\therefore 6 - 3 = 3\ \pi$ - terms

$f_1(\pi_1, \pi_2, \pi_3) = 0$

Repeating variables

H, g, ρ

$\pi_1 = H^{a_1} g^{b_1} \rho^{c_1}.V$

$\pi_2 = H^{a_2} g^{b_2} \rho^{c_2}.D$

$\pi_3 = H^{a_3} g^{b_3} \rho^{c_3}.\mu$

$\pi_1 = H^{a_1} g^{b_1} \rho^{c_1}.V$

$M^0L^0T^0 = L^{a_1}(LT^{-2})^{b_1}(ML^{-3})^{c_1}.LT^{-1}$

$a_1 = -1/2 \quad b_1 = -1/2 \quad c_1 = 0$

$\pi_1 = H^{-1/2} g^{-1/2} \rho^0.V \rightarrow \pi_1 = \frac{V}{\sqrt{gH}}$

Solving for a_i, b_i and c_i
in respective eqns

$\pi_2 = H^{a_2} g^{b_2} \rho^{c_2}.D$

$a_2 = -1 \quad b_2 = 0 \quad c_2 = 0$

$\pi_2 = H^{-1} g^0 \rho 0.D \rightarrow \pi_2 = \frac{D}{H}$

$\pi_3 = H^{a_3} g^{b_3} \rho^{c_3}.\mu$

$a_3 = -3/2 \quad b_3 = -1/2 \quad c_3 = -1$

$\pi_3 = H^{-3/2} g^{-1/2} \rho^{-1}.\mu \rightarrow \pi_3 = \frac{\mu}{\rho H^{3/2}\sqrt{g}}$

$f_1(\pi_1, \pi_2, \pi_3) = 0$

$\pi_1 = \frac{V}{\sqrt{gH}} = \frac{V}{\sqrt{2gH}}$ (*applying* principles of $\pi - term$)

$\pi_2 = \frac{D}{H}$

$$\pi_3 = \frac{\mu}{\rho H^{3/2}\sqrt{g}} = \frac{\mu}{\rho H\sqrt{gH}} * \frac{V}{V} = \frac{\mu}{\rho HV} * \frac{V}{\sqrt{gH}} = \frac{\mu}{\rho HV} * \pi_1$$

$\pi_3 = \frac{\mu}{\rho HV} * \pi_1 \rightarrow \pi_3 / \pi_1 = \frac{\mu}{\rho HV}$ (*applying* principles of $\pi - term$)

$$f_1\left(\frac{V}{\sqrt{2gH}}, \frac{D}{H}, \frac{\mu}{\rho HV}\right) = 0 \quad \rightarrow \quad \frac{V}{\sqrt{2gH}} = \phi\left(\frac{D}{H}, \frac{\mu}{\rho VH}\right)$$

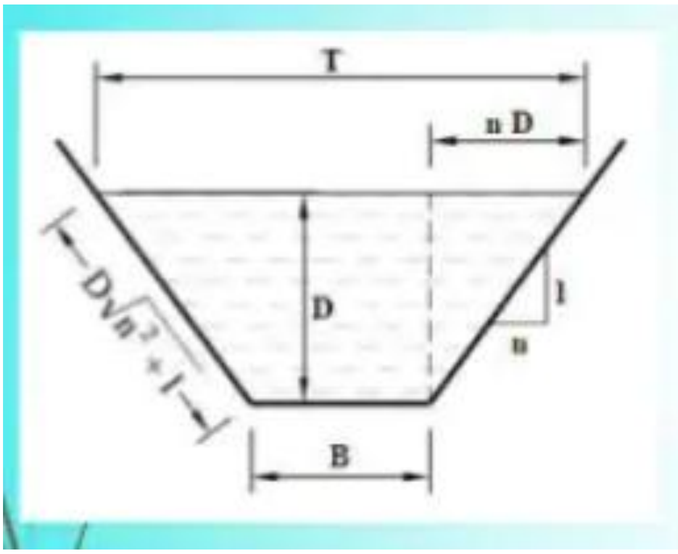
$V = \sqrt{2gH} * \phi\left(\frac{D}{H}, \frac{\mu}{\rho VH}\right)$ *Hence* proved.

3. Let

Depth of flow - D

Bed width - B

Side slope - 1/n



Wetted area, $A = D*(B + nD)$

Re writing the above eqn (B in terms of A, D, n)

$$\frac{A}{D} = B + nD \rightarrow B = \frac{A}{D} - nD.....(1)$$

Wetted Perimeter - $P = B + 2D\sqrt{1+n^2}$.....(2)

Sub (1) in (2)

$$P = \frac{A}{D} - nD + 2D\sqrt{1+n^2}$$

For most economical section, P is min

$$\frac{dP}{dD} = 0$$

$$\frac{dP}{dD} = -\frac{A}{D^2} - n + 2\sqrt{1+n^2} = 0$$

$$-\frac{A}{D^2} - n + 2\sqrt{1+n^2} = 0$$

$$\frac{A}{D^2} + n = 2\sqrt{1+n^2}$$

Sub the value of A

$$\frac{D*(B+nD)}{D^2} + n = 2\sqrt{1+n^2}$$

$$\frac{(B+nD)}{D} + n = 2\sqrt{1+n^2}$$

$$(B+nD) + nD = 2D\sqrt{1+n^2}$$

$$(B+2nD) = 2D\sqrt{1+n^2}$$

$$\frac{(B+2nD)}{2} = D\sqrt{1+n^2}$$

1. Half of the top width is equal to one of the sloping sides

Calculation of R

$$R = \frac{A}{P} = \frac{D*(B+nD)}{B+2D\sqrt{1+n^2}}....(3)$$

$$D\sqrt{1+n^2} = \frac{B+2nD}{2}....(4)$$

Sub (4) in (3)

$$R = \frac{A}{P} = \frac{D*(B+nD)}{B+2(\frac{B+2nD}{2})} = \frac{D*(B+nD)}{B+B+2nD} = \frac{D*(B+nD)}{2(B+nD)}$$

$$R = \frac{D}{2}$$

2. For most economical trapezoidal section, the hydraulic radius is equal to half of the depth of flow.

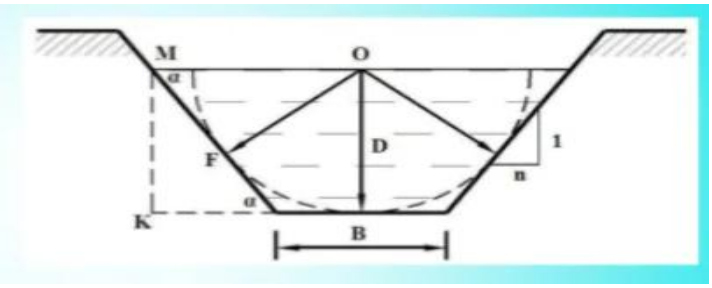
Let

α - angle made by the sloping side wrt to x - axis(horizontal axis)

O - centre of the top width

OF - perpendicular line to the sloping side MN

Taking ΔOFM(right angled triangle)



$$\sin\alpha = \frac{OF}{OM} \rightarrow OF = OM * \sin\alpha .....(i)$$

Taking ΔMKN (right angled triangle)

$$\sin\alpha = \frac{MK}{MN} = \frac{D}{D*\sqrt{1+n^2}} .....(ii)$$

Sub (ii) in (i)

$$OF = OM * \sin\alpha$$

$$OF = OM * \frac{D}{D*\sqrt{1+n^2}}$$

$Sub\ OM = half$ of Top width, $T = D*\sqrt{1+n^2}$ ($first$ condition)

$$OF = D*\sqrt{1+n^2} * \frac{D}{D*\sqrt{1+n^2}} = D$$

3. Thus for a most economical trapezoidal section, a semi-circle with centre O(centre of top width) and radius equal to the depth of flow, D will be tangential to the three sides of the most economical trapezoidal section.

4. Let B be the bottom width and y be the depth of flow

Most eonomical trapezoidal section

$$1. \frac{B+2ny}{2} = y*\sqrt{1+n^2} ....(1)$$

2. R = y/2

n = 1.5

$$\frac{B+2*1.5y}{2} = y*\sqrt{1+1.5^2}$$

$$B+3y = y*2*\sqrt{1+1.5^2}$$

$$B = 0.605y$$

$$Q = 2.7m^3/s$$

$$C = 44.5$$

$$S = 1/4000$$

$$Q = A*C*\sqrt{RS}$$

$$A = y(B+ny)$$

$$2.7 = y(0.605y+1.5y)*44.5*\sqrt{\frac{y}{2}*\frac{1}{4000}}$$

$$\frac{2.7*20*\sqrt{2}}{44.5} = 2.105y^2*\sqrt{y}$$

$$\frac{2.7*20*\sqrt{20}}{44.5*2.105} = y^2*\sqrt{y} = y^{(2+1/2)} = y^{5/2}$$

$$y = \left(\frac{2.7*20*\sqrt{20}}{44.5*2.105}\right)^{2/5} = 1.46m$$

$$B = 0.605y = 0.605*1.46 = 0.885m$$

5.

$$S = 1/9000$$

$$Q = 0.4 m^3/s$$

$$y = 1.25r = 0.625D$$

$$N = 0.015$$

Calculation of θ

$$\text{OC} = \text{CD} - \text{OD}$$

$$= (0.625 - 0.5)\text{D} = 0.125\text{D}$$

$$\cos\alpha = \frac{0.125\text{D}}{0.5\text{D}} \rightarrow \alpha = 75.52°$$

$$\theta = 180° - 75.52° = 104.48°$$

$$\theta \text{ in radians} = 104.48°/180° * \pi(3.14) = 1.824$$

$$A = \frac{D^2 * (2\theta - \sin 2\theta)}{8}$$

$$A = \frac{D^2 * (2*1.824 - \sin 2*104.48)}{8}$$

$$A = 0.5165D^2$$

$$P = 2\theta * D/2 = 1.824 * D = 1.824D$$

$$R = \frac{A}{P} = \frac{0.5165D^2}{1.824D} = 0.283D$$

$$Q = A * \frac{1}{N} * R^{2/3} S^{1/2}$$

$$0.4 = 0.5165D^2 * \frac{1}{0.015} * 0.283^{2/3} (1/9000)^{1/2}$$

$$D = 1.4219m$$