CMR INSTITUTE OF TECHNOLOGY

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Internal Assesment Test - I

Sub:	Electric Motors Code:					18EE44		44			
Date:	09.07.2022 (02.00 -3.30PM)	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branc	eh: EEE		
	Answer Any FIVE FULL Questions										
								Marks	OBE		
									Marks	CO	RBT
1a	Derive the torque e	equation of a l	O.C. Moto	r.					6	CO1	L2
1b	State the application	ns of various	types of D	OC motor					4	CO2	L1
2	Explain the different methods of speed control of DC shunt and series motor.					r.	10	CO4	L2		
3	What is the necessity of starter? With neat sketch describe the working of three point starter. List the limitations.						three	10	CO1	L2	
4	While conducting Swinburne's test on a 500 V shunt motor, it is noted that it takes a current of 5A on no-load. The resistances of the armature and field circuits are 0.22 ohms and 250 ohms respectively. Estimate the efficiency when the motor current is 100 A. What is the percentage change of speed between no-load and full-load?					field when	10	CO3	L4		
5	Explain back to back test of two identical dc machines and find the efficiency of motor and generator. State its advantages and disadvantages.					10	CO3	L2			
6						CO3	L4				
7	A 220 V shunt motor with an armature resistance of 0.5 ohm is excited to give constant main field. At full load the motor runs at 500rpm and takes an armature current of 30A. If a resistance of 1.0 ohm is placed in the armature circuit, find the speed at (a) full-load torque (b) double full-load torque.						10	CO2	L3		





Internal Assesment Test – I – Solutions

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Date:	09.07.2022 (02.00 -3.30PM)	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	EEE

Answer Any FIVE FULL Questions

M	OB						
a	Е						
r	С	R					
k	O	В					
S		T					
6	CO	L2					

1

1a Derive the torque equation of a D.C. Motor.

Torque Equation

Torque $T = F \times r (Nm)$

Let in a d.c. motor

r = average radius of armature in m

I = effective length of each conductor in m

Z = total number of armature conductors

A = number of parallel paths

i = current in each conductor = Ia/A

B = average flux density in Wb/m2

Φ = flux per pole in Wb

P = number of poles

Force on each conductor, F = B i I newtons

Torque due to one conductor = F x r newton- metre

Total armature torque, Ta = Z F r newton-metre

Now $i=I_a/A,\ B=\phi/a$ where a is the x-sectional area of flux path per pole at radius r. Clearly, $a=2\pi r\,\ell\,/P.$

$$\begin{split} \therefore \qquad & T_a = Z \times \left(\frac{\varphi}{2}\right) \times \left(\frac{I_a}{\Lambda}\right) \times \ell \times r \\ & = Z \times \frac{\varphi}{2\pi \; r \, \ell / P} \times \frac{I_a}{\Lambda} \times \ell \times r = \frac{Z \varphi \, I_a \, P}{2\pi \; \Lambda} \; \; N \cdot m \end{split}$$

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$$T_a = 0.159 Z \phi I_a \left(\frac{P}{A}\right) N - m \tag{i}$$

Since Z, P and A are fixed for a given machine,

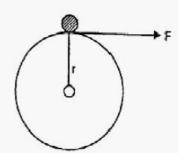
Hence torque in a d.c. motor is directly proportional to flux per pole and armature current.

(i) For a shunt motor, flux φ is practically constant.

$$T_a \propto I_a$$

(ii) For a series motor, flux φ is directly proportional to armature current I_a provided magnetic saturation does not take place.

$$T_a \propto I_a^2$$



- 4 CO L1 1b State the applications of various types of DC motor Applications 2 ✓ Shunt motors - Lathes, drills, boring mills, shapers, spinning and weaving machines ✓ Series motors - Electric traction, cranes, elevators, air compressors, vacuum cleaners, hair drier, sewing machines. ✓ Compound motors -Presses, shears, reciprocating machines 2 Explain the different methods of speed control of DC shunt and series motor. 10 CO L2 The speed of a dc motor is given by the relation, $N = K \frac{(V - I_a R_a)}{\phi} = \frac{K E_b}{\phi}$ 1. Flux / pole Φ (Flux Control) Resistance R_a of armature circuit (Rheostatic control) 2. Applied voltage V (Voltage Control) Speed Control of Shunt Motors Flux Control Method Speed with field rheostat in field circuit Speed without field rheostat in field circuit Rheostat ✓ It is based on the fact that by varying the flux Φ , the motor speed (N α 1/ Φ) can be changed and hence the name flux control method. ✓ In this method, a variable resistance (known as shunt field rheostat) is placed in series with shunt field winding as shown in Fig.
 - √The shunt field rheostat reduces the shunt field current I_{sh} and hence the flux Φ.
 - ✓ Therefore, we can only raise the speed of the motor above the normal speed.
 - ✓ Wider speed ranges tend to produce instability and poor commutation.
 - ✓ In non interpolar machine the speed ratio is 2 : 1.
 - ✓ In machines with interpoles the speed ratio is 6 : 1

Advantages & Disadvantages of Flux control method

Advantages

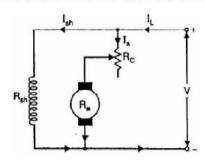
- (i) This is an easy and convenient method.
- (ii) It is an inexpensive method since very little power is wasted in the shunt field rheostat due to relatively small value of lsh.
- (iii) The speed control exercised by this method is independent of load on the machine.

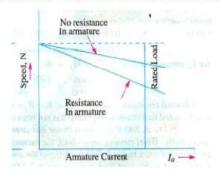
Disadvantages

- (i) Only speeds higher than the normal speed can be obtained since the total field circuit resistance cannot be reduced below R_{sh}—the shunt field winding resistance.
- (ii) There is a limit to the maximum speed obtainable by this method. It is because if the flux is too much weakened, commutation becomes poorer.

Note. The field of a shunt motor in operation should never be opened because its speed will increase to an extremely high value.

ii. Armature or Rheostatic control method





- √This method is based on the fact that by varying the voltage available across the
 armature, the back e.m.f and hence the speed of the motor can be changed.
- √This is done by inserting a variable resistance RC (known as controller resistance)
 in series with the armature as shown in Fig.

$$N \propto V - I_a \left(R_a + R_C \right)$$

- ✓ Due to voltage drop in the controller resistance, the back e.m.f. (E_b) is decreased.
- ✓ Since N a Eb, the speed of the motor is reduced.
- ✓ The highest speed obtainable is that corresponding to R_C = 0 i.e., normal speed.
- ✓ Hence, this method can only provide speeds below the normal speed

Disadvantages of Armature control method

Disadvantages

- (i) A large amount of power is wasted in the controller resistance since it carries full armature current la.
- (ii) The speed varies widely with load since the speed depends upon the voltage drop in the controller resistance and hence on the armature current demanded by the load.
- (iii) The output and efficiency of the motor are reduced.
- (iv) This method results in poor speed regulation.

Due to above disadvantages, this method is rarely used to control tie speed of shunt motors.

Note.

- ✓ The armature control method is a very common method for the speed control of d.c. series motors.
- ✓ The disadvantage of poor speed regulation is not important in a series motor which is used only where varying speed service is required.

iii. Voltage Control Method

- (i) Multiple voltage control.
- ✓ In this method, the shunt field of the motor is connected permanently across a-fixed voltage source.
- √ The armature can be connected across several different voltages through a suitable switchgear.
- ✓ In this way, voltage applied across the armature can be changed.
- The speed will be approximately proportional to the voltage applied across the armature.
- Intermediate speeds can be obtained by means of a shunt field regulator.

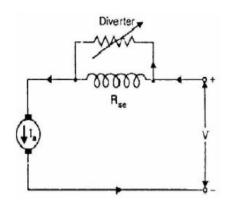
Speed Control of DC Series Motor 1. Flux Control Method

(i) Field divertors.

In this method, a variable resistance (called field diverter) is connected in parallel with series field winding as shown in Fig.

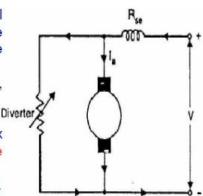
Its effect is to shunt some portion of the line current from the series field winding, thus weakening the field and increasing the speed (N α 1 / Φ).

- √ The lowest speed obtainable is that corresponding to zero current in the diverter (i.e., diverter is open).
- Obviously, the lowest speed obtainable is the normal speed of the motor.
- Consequently, this method can only provide speeds above the normal speed.
- The series field diverter method is often employed in traction work.



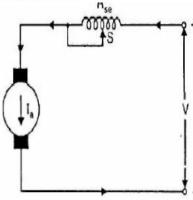
Flux Control Method ii) Armature Diverter

- ✓ In order to obtain speeds below the normal speed, a variable resistance (called armature diverter) is connected in parallel with the armature as shown in Fig.
- ✓ The diverter shunts some of the line current, thus reducing the armature current.
- Now for a given load, if I_a is decreased, the flux
 Φ must increase (T_a α ΦI_a) because the torque is maintained as a constant.
- ✓ Since N α 1/ Φ, the motor speed is decreased.
- By adjusting the armature diverter, any speed lower than the normal speed can be obtained.



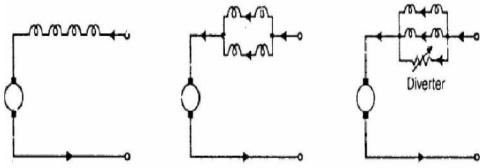
Flux Control Method iii) Tapped field control

- In this method, the flux is reduced (and hence speed is increased) by decreasing the number of turns of the series field winding as shown in Fig.
- ✓ The switch S can short circuit any part of the field winding, thus decreasing the flux and raising the speed.
- With full turns of the field winding, the motor runs at normal speed and as the field turns are cut out, speeds higher than normal speed are achieved.

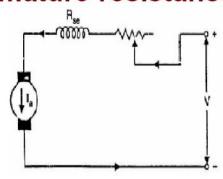


iv) Paralleling field coils

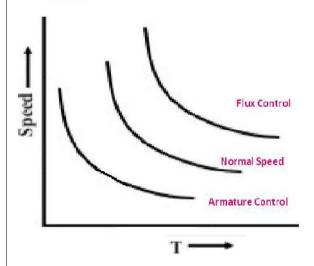
- ✓ This method is usually employed in the case of fan motors.
- ✓ By regrouping the field coils as shown in Fig.
- ✓ several fixed speeds can be obtained.



2. Armature-resistance control



- ✓ In this method, a variable resistance is directly connected in series with the supply as shown in Fig.
- ✓ This reduces the voltage available across the armature and hence the speed falls.
- By changing the value of variable resistance, any speed below the normal speed can be obtained.
- ✓ This is the most common method employed to control the speed of d.c. series motors.



Necessity of D.C. Motor Starter

- ✓ At starting, when the motor is stationary, there is no back e.m.f. in the
 armature.
- ✓ Consequently, if the motor is directly switched on to the mains, the armature will draw a heavy current $(I_a = V/R_a)$ because of small armature resistance.
- \checkmark As an example, 5 H.P., 220 V shunt motor has a full-load current of 20 A and an armature resistance of about 0.5 Ω.
- ✓ If this motor is directly switched on to supply, it would take an armature current of 220/0.5 = 440 A which is 22 times the full-load current.
- ✓ This high starting current may result in:
- (i) burning of armature due to excessive heating effect,
- (ii) damaging the commutator and brushes due to heavy sparking,
- (iii) excessive voltage drop in the line to which the motor is connected.
- ✓ In order to avoid excessive current at starting, a variable resistance (known as starting resistance) is inserted in series with the armature circuit.
- √ This resistance is gradually reduced as the motor gains speed (and hence E_b increases) and eventually it is cut out completely when the motor has attained full speed.
- ✓ The value of starting resistance is generally such that starting current is limited to 1.25 to 2 times the full-load current.

Three Point Starter

✓ Important Parts

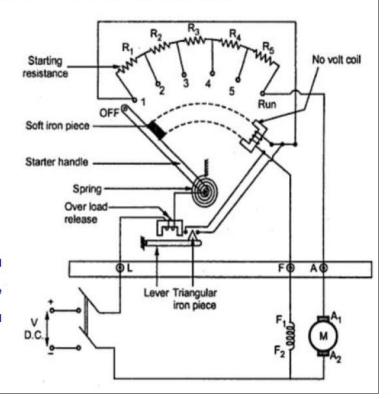
- √ Resistance stud
- √ Handle

3

- √ Spring
- ✓ Soft Iron Piece

√ Protective Devices

- √ No volt release coil
- ✓ Over load release coil
- 1. 'L' Line terminal to be connected to positive of supply.
- 2. 'A' To be connected to the armature winding.
- 3. 'F' To be connected to the field winding.



Function of No Volt Coil

- 1. The supply to the field winding is derived through NVC. So when field current flows, it magnetizes the NVC. When the handle is in the 'RUN' position, soft iron piece connected to the handle gets attracted by the magnetic force produced by NVC. Design of NVC is such that it holds the handle in 'RUN' position against the force of the spring as long as supply to the motor is proper. Thus NVC holds the handle in the 'RUN' position and hence also called hold on coil.
- 2. Whenever there is supply failure or if field circuit is broken, the current through NVC gets affected. It looses its magnetism and hence not in a position to keep the soft iron piece on the handle, attracted. Under the spring force, handle comes back to OFF position, switching off the motor. So due to the combination of NVC and the spring, the starter handle always comes back to OFF position whenever there is any supply problems. The entire starting resistance comes back in series with the armature when attempt is made to start the motor every time. This prevents the damage of the motor caused due to accidental starting.
- 3. NVC performs the similar action under low voltage conditions and protects the motor from such dangerous supply conditions as well.

Action of Overload Release

- ✓ The current through the motor is taken through the OLR, an electromagnet.
- ✓ Under overload condition, high current is drawn by the motor from the supply which passes through OLR.
- ✓ Below this magnet, there is an arm which is fixed at its fulcrum and normally resting in horizontal position.
- ✓ Under overloading, high current through OLR produces enough force of attraction to attract the arm upwards.
- ✓ Normally magnet is so designed that up to a full load value of current, the force of attraction produced is just enough to balance the gravitational force of the arm and hence not lifting it up.
- ✓ At the end of this arm, there is a triangular iron piece fitted. When the arm is pulled upwards the triangular piece touches the two points which are connected to the two ends of NVC.
- ✓ This shorts the NVC and voltage across NVC becomes zero due to which NVC looses its magnetism.
- ✓ So under the spring force, handle comes back to the OFF position, disconnecting the motor from the supply.
- Thus motor gets saved from the overload conditions.

Functions of Protective Devices

- (i) when the supply fails, thus preventing the armature being directly across the mains when this voltage is restored. For this purpose, we use no-volt release coil.
- (ii) when the motor becomes overloaded or develops a fault causing the motor to take an excessive current. For this purpose, we use overload release coil.

Operations of Three Point Starter

- (i) The d.c. supply is switched on with handle in the OFF position.
- (ii) The handle is now moved clockwise to the first stud. As soon as it comes in contact with the first stud, the shunt field winding is directly connected across the supply, while the whole starting resistance is inserted in series with the armature circuit.
- (iii) As the handle is gradually moved over to the final stud, the starting resistance is cut out of the armature circuit in steps. The handle is now held magnetically by the no-volt release coil which is energized by shunt field current.
- (iv) If the supply voltage is suddenly interrupted or if the field excitation is accidentally cut, the no-volt release coil is demagnetized and the handle goes back to the OFF position under the pull of the spring. If no-volt release coil were not used, then in case of failure of supply, the handle would remain on the final stud. If then supply is restored, the motor will be directly connected across the supply, resulting in an excessive armature current.
- (v) If the motor is over-loaded (or a fault occurs), it will draw excessive current from the supply. This current will increase the ampere-turns of the over-load release coil and pull the armature C, thus short-circuiting the no volt release coil. The no-volt coil is demagnetized and the handle is pulled to the OFF position by the spring. Thus, the motor is automatically disconnected from the supply.

Drawback of 3 Point Starter

- ✓ In a three-point starter, the no-volt release coil is connected in series with the shunt field circuit so that it carries the shunt field current.
- ✓ While exercising speed control through field regulator, the field current may be weakened to such an extent that the no-volt release coil may not be able to keep the starter arm in the ON position.
- ✓ This may disconnect the motor from the supply when it is not desired.
- This drawback is overcome in the four point starter.

Surin bunne's Test:
$$V = 300V$$
; $T_{10} = 5A$; $R_{0} = 0.22$ N.

$$T sh = \frac{V}{R sh} = \frac{500}{250} = 2A$$
. $Tao = Tlo - Tsh$
= 5 - 2 = 3A

$$OP = 1 - \frac{OP}{IP} = \frac{IP}{IP} - \frac{10000}{50000} = \frac{50000 - 4610.9}{50000}$$

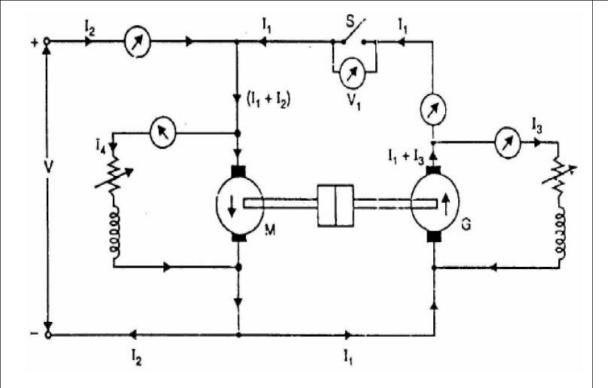
Y. Change in Speed. =
$$N_1 - N_2$$
 x100 (2)
 $Eb1 = V - Tao Ra = 500 - 3x0.22 = 499.34V$
 $Eb2 = V - Ta Ra = 500 - 98 x0.22 = 478.44V$
 $\frac{N_2}{N_1} = \frac{Eb2}{Eb_1} = \frac{478.44}{499.34} = 6.96$.
 $N_2 = 0.96 N_1$
Y. Change in Speed = $N_1 - N_2$
 $N_2 = 0.96 N_1$
 $= \frac{N_1 - 0.96 N_1}{0.96 N_1} = 4.177$.

5 Explain back to back test of two identical dc machines and find the efficiency of motor and generator. State its advantages and disadvantages.

10 CO L2

Hopkinson's Test or Regenerative Test or Back to Back Test

- ✓ This method of determining the efficiency of a d.c. machine saves power and gives more accurate results.
- ✓ In order to carry out this test, we require two identical d.c. machines and a source of electrical power.
- ✓ This is Full load test.



- ✓ Two identical d.c. shunt machines are mechanically coupled and connected in parallel across the d.c. supply.
- ✓ By adjusting the field excitations of the machines, one is run as a motor and the other as a generator.
- ✓ The electric power from the generator and electrical power from the d.c. supply are fed to the motor.
- ✓ The electric power given to the motor is mostly converted into mechanical power, the rest going to the various motor losses.
- ✓ This mechanical power is given to the generator.
- ✓ Two identical d.c. shunt machines are mechanically coupled and are connected in parallel across the d.c. supply.
- ✓ By adjusting the field strengths of the two machines, the machine M is made to run as a motor and machine G as a generator.
- ✓ The motor M draws current I_1 from the generator G and current I_2 from the d.c. supply so that input current to motor M is $(I_1 + I_2)$.
- ✓ Power taken from the d.c. supply is VI₂ and is equal to the total motor and generator losses.
- ✓ The field current of motor M is I₄ and that of generator G is I₃.

- ✓ If V be the supply voltage, then,
- ✓ Motor input = $V(I_1 + I_2)$
- √ Generator output = VI₁
- ✓ We shall find the efficiencies of the machines considering two cases viz.
- (i) assuming that both machines have the same efficiency n
- (ii) assuming iron, friction and windage losses are the same in both machines.

Stray loss = iron loss + friction loss + windage loss

(i) Assuming that both machines have the same efficiency η

Motor output = $\eta \times \text{motor input} = \eta \ V(I_1 + I_2) = \text{Generator input}$

Generator output = $\eta \times \text{generator input} = \eta \times \eta \ V(I_1 + I_2) = \eta^2 \ V(I_1 + I_2)$

But generator output is VI₁

$$\therefore \quad \eta^2 V(I_1 + I_2) = VI_1$$

or $\eta = \sqrt{\frac{I_1}{I_1 + I_2}}$

This expression gives the value of efficiency sufficiently accurate for a rough test. However, if accuracy is required, the efficiencies of the two machines should be calculated separately

	(ii) Assuming that iron, friction and windage losses are same in both machines.
	Let R_a = armature resistance of each machine I_3 = field current of generator G I_4 = field current of motor M
	Armature Cu loss in generator = $(I_1 + I_3)^2 R_a$
	Armature Cu loss in motor = $(I_1 + I_2 - I_4)^2 R_a$
	Shunt Cu loss in generator = $V I_3$
	Shunt Cu loss in motor = $V I_4$
	Power drawn from the d.c. supply is VI ₂ and is equal to the total losses of the motor and generator
	VI_2 = Total losses of motor and generator
	If we subtract armature and shunt Cu losses of the two machines from VI ₂ , we get iron, friction windage losses of the two machines.
	Iron, friction and windage losses of two machines (M and G)
	$= VI_2 - \left[(I_1 + I_3)^2 R_a + (I_1 + I_2 - I_4)^2 R_a + VI_3 + VI_4 \right] = W \text{ (say)}$
	∴ Iron, friction and windage losses of each machine = W/2
6	Two shunt motors loaded for the Hopkinson's test take 15 A at 200 V supply. The motor current is 100 A and the shunt currents are 3 A & 2.5 A. If the resistance of each armature is 0.05 ohm, calculate the efficiency of each machine for the given operating conditions.

```
Hopkinson's Test.
                    Currents
  I = 15A.
                    ILM = Iam + Irhm.
  V = 200V.
                    Iam = ILm - Ishm.
  ILM = 100A.
                         = 100 - 2.5
  Ishg = 3A.
                    Jam = 97.5 A.
  Ishm = 2.5A.
 Ra = 0.05/L.
                   IL9 = ILm - I
To find Constant = 100-15 ILg=ILm-I

Loss(Ws) TLg = 85A)

Ing: ILm-I
 Copper Loss Wc Iag = Izg + Ishg
                          = 85+3 = 88A.
1. Wcum = Iam Ra + | Iag = 88 A)
             V Ishm
     =(97.5^2 \times 0.05) + (200 \times 2.5)
                     Weum= 975.31 walls)
2. Wang = Tag2 Ra + V Ishg.
  =(882x0.05)+(200 x3)
Weng = 987.2 W
 Wea= Wang + Wan = 975.31 + 987.2
    Wen= 1962.51 walts
 Constant Loss Ws
     We= VI - Wey
        = (200 x 15) - 1962.51
    Ws = 1037. 49 waits).
Constant los for each machine
    Ws/2 = 1037.49 = 518.745 walts
```

Ethiciency of Motor
$$y_m$$
.

 $y_m = \frac{O|p}{I|p} = \frac{I|p - lones}{I|p} = \frac{I|p}{I|p}$
 $I|p = V I Lm = 200 \times 100 = 20000 \text{ water}$.

 $y_m = 20000 - (Wcum + Ws_2)$
 $= 20000 - (975.31 + 518.74)$
 $= 20000$
 $y_m = 92.537$.

Ethiciency of Generator. y_g .

 $y_g = \frac{U|p}{I|p} = \frac{O|p}{O|p+lones}$
 $y_g = \frac{U|p}{I|p} = \frac{O|p}{O|p+lones}$
 $y_g = \frac{V|p}{I|p} = \frac{O|p}{O|p+lones}$
 $y_g = \frac{V|p}{I|p} = \frac{O|p}{O|p+lones}$
 $y_g = \frac{17000}{17000 + 1505.945}$
 $y_g = \frac{17000}{17000 + 1505.945}$
 $y_g = \frac{17000}{17000 + 1505.945}$
 $y_g = \frac{17000}{17000 + 1505.945}$

A 220 V shunt motor with an armature resistance of 0.5 ohm is excited to give constant main field. At full load the motor runs at 500rpm and takes an armature current of 30A. If a resistance of 1.0 ohm is placed in the armature circuit, find the speed at (a) full-load torque (b) double full-load torque.

10 CO L3

2

(a) Speed at Full load Tongue.

Eb1 =
$$V - Ia_1 Ra$$

Eb1 = $220 - (30 \times 0.5) = 205 V$.

 $N1 = 500 rpm$.

 $Eb2 = V - Ia_2 (Ra + Rc)$
 $= 220 - 30 (0.5 + 1) = 175 V$.

 $\frac{N_2}{N_1} = \frac{Eb_2}{Eb_1} = \frac{177}{205}$
 $N_2 = \frac{175}{205} \times 500 = 426.83 rpm$

(b) Speed at dowble the full load Tongue.

 $Rc = 10L$; $Ra = 0.50L$
 $Ta2 = 2 Ta_1$ $Ta_1 \rightarrow Full load$
 $Ta2 = 1a_1$
 $Ta2 = Ta2 \times Ia_1 = 2 Ta1 \times Ia_1$
 $Ia2 = Ta2 \times Ia_1 = 2 Ta1 \times Ia_1$
 $Ia2 = Ta2 \times Ia_1 = 2 Ta1 \times Ia_1$
 $Ia2 = 1a_1 \times Ia_1 = 2 Ta_1 \times Ia_1$
 $Ia2 = 1a_1 \times Ia_1 = 2 Ta_1 \times Ia_1$
 $Ia3 = 1a_1 \times Ia_1 = 1a_1 \times Ia_1$
 $Ia4 = 1a_1 \times Ia_1 = 1a_1 \times Ia_1$
 $Ia5 = 1a_1 \times Ia_1 = 1a_1 \times Ia_1$
 $Ia5 = 1a_1 \times Ia_1 = 1a_1 \times Ia_1$
 $Ia7 = 1a_1 \times Ia_1 = 1a_1 \times Ia_1$
 $Ia8 = 1a_1 \times Ia_1 = 1a_1 \times Ia_1$
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